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Dimension of strange attractors : an experimental determination for the chaotic regime of two convective systems (*)

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Résumé. — Les caractéristiques dynamiques de régimes chaotiques rencontrés dans deux instabilités convectives (Rayleigh-Bénard et injection unipolaire d'ions) sont déterminées en étudiant la trajectoire reconstituée dans l'espace des phases à partir des enregistrements expérimentaux. On montre que ces trajectoires correspondent à des attracteurs dont la dimension fractale, de faible valeur, est clairement mise en évidence.

Abstract. — We have determined the dynamical characteristics of the chaotic regimes encountered in two convective instability problems (Rayleigh-Bénard and E.H.D. unipolar ion injection) by studying the reconstructed trajectory in the phase space obtained from experimental data. We show that these trajectories do indeed correspond to attractors, whose fractal dimension low value is clearly evidenced.

A number of experiments have shown that in different dynamical systems a chaotic state may be obtained after a small number of bifurcations corresponding to the appearance of a periodic or a quasi periodic state. The crucial point is now to determine if this chaos has its origin in a large number of degrees of freedom (and thus is entirely stochastic) or if it is deterministic i.e. related to a small number of degrees of freedom. In this second case, we expect that the phase space trajectories which are characteristic of the dynamics of the system, should lie on attractors with a fractal dimension and particular properties which qualify them as being « strange » [1, 2].

In a dissipative system which is the only case considered here, the chaotic state has been essentially characterized, until very recently, by the properties of the Fourier spectra of one of the dynamical variables of the system. These spectra present a broad band noise, superposed or not on discrete peaks. However they give practically no information on the actual nature of the chaotic state that they relate to. So, some methods have been proposed to identify the presence or not of deterministic chaos by looking directly either at the reconstructed physical trajectories in phase space [3] or at the experimental Poincaré sections of these trajectories [4-7]. In this way, strange attractors have been evidenced in some cases. Unfortunately, these methods are only

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useable when the attractor lies in at the most a 3-d phase space. Also they do not allow easy measurements of the fractal dimension of the attractor.

To determine this dimension, the algorithm proposed by Takens [8] is not efficient [9]. On the contrary, the method proposed recently by Grassberger and Procaccia [10] furnishes a practical method of approximating the dimensionality of the attractor measure : the dimension ν is given by the asymptotic behaviour r^ν of the integral correlation function

$$C(r) = \lim_{N \rightarrow \infty} \frac{1}{N^2} \sum_{i,j=1}^N H(r - \|X_i - X_j\|) \quad (1)$$

(H : Heaviside function) ; r is the distance variable in phase space. We report here that, by applying this method, we have effectively characterized attractors corresponding to given chaotic regimes, observed in Rayleigh-Bénard convection [12] and with strong ionic unipolar injection in electrohydrodynamical instability.

In both experiments, we have calculated $C(r)$ from a time series of one physical signal $u(t)$. The trajectories in a n -d phase space have then been reconstructed by looking at the evolution of the representative point M , with coordinates $u(t)$, $u(t - \tau)$, $u(t - 2\tau)$, ..., $u(t - (n - 1)\tau)$; $C(r)$ is calculated for a finite number of r values and may be interpreted as being proportional to the mean statistical number of the points of the trajectory lying inside an hypersphere of radius r centred at any point of the attractor. As has been remarked by Rammal [11], if the attractor has a fractal dimension, each $C_i(r)$, obtained by summing only on j in the relation 1, shows the same asymptotic behaviour r^ν . This remark is of importance : we can obtain a good approximation for $C(r)$, just by carrying out the average over a limited number of functions $C_i(r)$. In practice, the calculation of $C(r)$ has been made with 100 i values, the total number of points of the experimental data being $N = 15\,000$.

This simplified method has been successfully checked for known attractors (Henon, Lorenz) where discrete values of the variable (12 bits) have been taken just as for the sampling operation in an experimental set up. Different norms have been used ; in the following, we used the norm

$\|X_i - X_j\| = \sum_{m=0}^{n-1} |u_i(t + m\tau) - u_j(t + m\tau)|$ which is the sum of the absolute value of the components.

As we want to discriminate deterministic chaos from noise, the first step was to study a stochastic signal (generated numerically) or a random noise. In this case, the points of the corresponding trajectories have to be distributed uniformly in a part of the phase space and we expect that the asymptotic behaviour of $C(r)$ would be in r^n . On figure 1, we can see the curves $\log C(r) = f(\log r)$ for sufficiently low r values. The exponent ν does not depend on $\tau = p \Delta t$ (p is an integer, Δt is the period sampling).

For $n \leq 4$, we find $\nu \simeq n$. For $n > 4$, ν departs progressively from n and continues to grow without reaching a limit (Fig. 2). This difference between ν and n is probably due to the finite size of the phase space which is visited at random (one would have probably found $\nu = n$ by working with lower values of r (and then of $C(r)$) but this needs $N \gg 15\,000$).

Our attempt to characterize strange attractors is performed with two different convective systems in confined geometry where the strong stress imposed by the boundaries on the spatial modes, presumably separate the dynamical and the spatial effects.

In the first system (Rayleigh-Bénard convection with test fluid of $Pr = 40$), the convective structure is very likely characterized by the existence of two rolls perpendicular to the larger side wall Ox (aspect ratio of the cell $\Gamma_x = 2$) and one or two rolls perpendicular to Oy ($\Gamma_y = 1.2$) [12]. The time-dependent regimes are studied by looking at the deflection of a narrow light beam crossing the cell parallel to the Oy direction (semi-local measurement). By increasing the temperature difference ΔT , a periodic regime sets in and chaos follows after a period doubling cascade.

Fig. 1. — Curves $C(r)$ corresponding to random white noise, a) for different n values, n is the dimension of the phase space; b) for different p values with $n = 7$ (the successive curves are regularly shifted by an arbitrary value along the r axis).

Fig. 2. — Dependence of the measured exponent ν versus n for the different analysed signals.

The data which are analysed here ($Ra/Ra_c = 235$) correspond to this chaotic regime in the inverse cascade : one can still identify the frequency f_1 and first sub harmonic $f_1/2$, superimposed on the continuous spectrum (Fig. 3a). The shape of the curves $C(r)$ is, to a first approximation, independent of the dimension of the phase space when $n > 4$ (Fig. 3b) and independent of the time delay (Fig. 3c) for $2 < p \leq 10$. These two facts obviously characterize a well defined attractor ; its dynamical dimension is deduced from the linear variation over a wide range in log-log coordinates. The value $\nu = 2.8 \pm 0.1$ evidences a fractal structure (Fig. 2).

Similar results are obtained in the problem of strong ionic unipolar injection in an insulating liquid (critical voltage $U_c \simeq 50$ V) confined in a cylindrical container of radius nearly equal to the depth ($\Gamma \simeq 1$ of [13]). One assumes that only one convective cell is present and the transition to chaos is very likely produced through a periodic, followed by a biperiodic regime [13]. The state analysed here, is obtained by applying a voltage ($U = 270$ V) which is 3 to 4 times the voltage corresponding to the onset of the chaotic behaviour and the Fourier spectrum (Fig. 3d) shows a broadened peak (fundamental frequency of the main oscillations) with an exponential decay [14]. In this case, a macroscopic quantity is measured, namely the total current crossing the fluid. The curves $C(r)$, determined from the current fluctuations, are very similar to the previous ones : the shape is independent of the dimension n (Fig. 3e) for $n \geq 6$, and of the delay p (Fig. 3f) for $3 < p \leq 7$. Furthermore the presence of a good linearity provides a dynamical dimension $\nu = 5.1 \pm 0.3$ (see Fig. 2).

For both convective systems, the results obtained for the two chaotic regimes evidence the existence of an attractor of low dimension, the fractal character of which allows us to suppose that this attractor is of the strange type. However this conclusion does not appear to be general : we have observed chaotic regimes for which the curves $\log C(r)$ did not show a defined slope, like that, for the example shown on figure 4. In this case, it was not possible to measure the dimension of an attractor, which, if it exists, seems nevertheless to have a bounded dimension.

Fig. 4. — Time behaviour and curve $C(r)$ in a given chaotic state observed in Rayleigh-Bénard convection ($n = 6$; $p = 5$).

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- Fig. 3. — Fourier spectra and corresponding curves $C(r)$ obtained from experimental chaotic signals. In Rayleigh-Bénard convection : (a) Fourier spectrum of the analysed signal ; (b) $C(r)$ with various n ; (c) $C(r)$ with various p and with $n = 6$. In electro convection : (d) Fourier spectrum ; (e) $C(r)$ with various n ; (f) $C(r)$ with various p and with $n = 7$.

Note. — While this paper was in preparation, we learned that two other groups were performing similar studies of dimension, namely the group of the reference 15 and Guckenheimer, G. Buzyna and R. Pfeffer.

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