Classification

Physics Abstracts

67.40 — 68.10 — 51.10 — 34.50

Kapitza resistance at the $H\downarrow$ /liquid He interface

B. Castaing and M. Papoular

Centre de Recherches sur Les Très Basses Températures, C.N.R.S.,
B.P. 166 X, centre de Tri, 38042 Grenoble cedex, France

(Reçu le 17 décembre 1982, révisé le 11 avril 1983, accepté le 16 mai 1983)

Résumé. — Dans l'hydrogène atomique polarisé, l'accès aux basses températures et fortes densités est bloqué par la résistance thermique de Kapitza. Cette résistance est calculée, ainsi que le temps caractéristique associé, pour un canal significatif : du gaz hydrogène aux modes de surface de l'hélium liquide.

Abstract. — In spin-polarized atomic hydrogen, the way to low T and high density is restricted by the Kapitza resistance. This is calculated, together with the corresponding characteristic time, for one channel : from H gas to liquid helium ripplons.

1. In the past three years a set of important results, both experimental and theoretical, have been gathered on the physics of spin-polarized atomic hydrogen [1] : recombination and relaxation rates, including the B^{-2} magnetic field dependence, adsorption energies on liquid ^4He and ^3He , sticking probabilities [2], etc. The relaxation bottleneck [3] in the population time-evolution has been observed [4], as well as the anisotropy in the surface dipolar relaxation rate [5]. As a result, higher and higher $H\downarrow$ densities have been stabilized on long time scales, at lower temperatures — but these achievements seem to be levelling off at about 3×10^{17} at/cm³ and 200 mK, which is still way out of the Bose-condensation border (16 mK for 10^{18} cm⁻³, 75 mK for 10^{19} cm⁻³). A fundamental limitation to achieving lower temperatures, which we discuss in this letter, is the Kapitza thermal resistance R_K between $H\downarrow$ gas and walls, which fastly increases at lower T , thus building up a larger and larger temperature difference ΔT between cold walls and a hot gas — due to the continued release of recombination heat (even though it is relaxationally bottlenecked) : about 5 eV per elementary reaction.

One is led, therefore, to think of compressing the gas — but this brings in a related problem. Even with a small ΔT , it takes a finite time, τ_K , to thermally equilibrate the system. τ_K is given by the product of R_K and the gas heat capacity :

$$\tau_K = R_K C_H. \quad (1)$$

And for the compression to be roughly isothermal, it should proceed on a time scale τ_{comp} long compared with τ_K . The compression time, which of course must remain short on the scale of relaxation time T_1 , must therefore obey the double inequality

$$\tau_K < \tau_{\text{comp}} < T_1. \quad (2)$$

2. One can think of various channels for the heat transfer from gas to helium film ($^3\text{He}/^4\text{He}$ liquid mixture ; see Ref. 5). One is the gas-phonon channel (phonon in liquid helium). We write :

$$\dot{Q} = R_K^{-1} \Delta T . \quad (3)$$

here $\dot{Q}$ is the heat flux, and ΔT is assumed to be much smaller than T (thereby allowing a linearization procedure).

R_K^{-1} is proportional to $\bar{v}_{\text{th}} = \sqrt{\frac{2 k_B T}{\pi m_H}}$ an average impact thermal velocity ⁽¹⁾ and to a statistical integral proportional to T^3 (3-dimensional integration over phonon wave-vector). So, one expects : $R_K \sim \tau_K \sim T^{-7/2}$. This result was obtained long ago by Landau [6] (his « accommodation coefficient » is defined without $\bar{v}_{\text{th}}$ and is therefore proportional to T^3). It would yield rather long thermal times below 100 mK.

A more efficient channel, which we now investigate is the gas-to-ripplon transfer. The principle is the same as for phonons, but the integration is now two-dimensional and the dispersion law is different. Let T_G and T_L be the gas and liquid helium temperatures, $\Delta T = T_G - T_L \ll T$, n_H the atomic density of the hydrogen gas. Then $\frac{1}{2} n_H \bar{v}_{\text{th}}$ is the mean number of atoms which hit the helium surface per unit time and unit area. The energy transfer rate $\dot{Q}$ between gas and liquid is given by

$$\dot{Q} = \frac{1}{2} n_H \bar{v}_{\text{th}} A \int_0^{q_{\text{Max}}} A \frac{q \, dq}{2 \pi} \hbar \omega_q \left[(n_q + 1) \exp\left(-\frac{\hbar \omega_q}{k_B T_G}\right) - n_q \right] K_q , \quad (4)$$

where q and ω_q are the ripplon wave-vector and frequency, $n_q = [\exp \hbar \omega_q / k_B T_L - 1]^{-1}$ is the ripplon occupation number, if assumed at the liquid He temperature T_L , and K_q is the intrinsic probability for an H atom to be inelastically scattered off a ripplon mode q . A is the total interfacial area. The term $\frac{1}{2} n_H \bar{v}_{\text{th}} K_q \hbar \omega_q n_q$ thus represents the energy transfer rate from mode q toward to gas. In the limit where the H atom is treated as a classical particle, this rate can be written

$$\frac{1}{2} n_H \bar{v}_{\text{th}} \left(\frac{1}{2} m_H \overline{(2u)^2} \right) ,$$

where $\overline{u^2}$ is the mean-squared normal velocity of the helium surface. It is then easy to obtain the expression of K_q by considering the high-temperature case (where $n_q = k_B T / \hbar \omega_q$) :

$$K_q = \frac{2 m_H \overline{u^2}}{k_B T} .$$

On the other hand, in the same high- T regime, the total energy of a ripplon mode is

$$A \frac{\sigma q^2}{\omega_q^2} \overline{u^2} = A \frac{\rho}{q} \overline{u^2} = k_B T ,$$

where σ and ρ are the helium surface tension ($\sigma = 0.15 \text{ erg cm}^{-2}$ for liquid ^3He) and mass density leading to : $\omega_q^2 = \frac{\sigma q^3}{\rho}$, the capillary dispersion. Thus

$$K_q = \frac{2 m_H q}{\rho A} . \quad (5)$$

⁽¹⁾ See, for example, Landau and Lifshitz, Statistical Physics.

Injecting this classical expression into equations 4 and 3, we get in the low-temperature limit

$$\left(x_{\text{Max}} = \frac{\hbar \omega_q}{k_B T} \text{Max} \gg 1 \right)$$

$$R_K^{-1} = \frac{\pi^3}{45} A \frac{k_B^3}{\hbar^2} \frac{m_H}{\sigma} n_H \bar{v}_{\text{th}} T^2. \quad (6)$$

Note that the classical-trajectory approximation we made for H atoms, amounts to neglecting any horizontal momentum transfer. Put another way : to neglecting the hydrogen de Broglie wavelength, λ_B , as compared with that of the ripplon, λ_r — since the only efficient ripplons are those with $\lambda_r \gtrsim \lambda_B$. These two lengths turn out to be nearly equal at temperatures ~ 100 mK, and their ratio is slowly varying with T . Therefore, although an approximate one, our calculation yields the right order of magnitude for R_K .

For a perfect classical gas in volume V , the heat capacity is $C_H = V \frac{3}{2} k_B n_H$, and the characteristic thermal equilibration time, defined by equation 1, can be written

$$\tau_K = 6.5 T^{-2} \left(\frac{2V}{\bar{v}_{\text{th}} A} \right), \quad T \text{ in K}, \quad (7)$$

or, equivalently : $\tau_K \simeq 1.6 \times 10^{-3} T^{-5/2} \frac{V}{A}$, in seconds, if T in K and $\frac{V}{A}$ in cm. So, compared with the gas-to-phonon channel, we gain one power in temperature, and τ_K is reasonably short at moderately low T , allowing the double inequality 2 to be easily fulfilled. For example, for a flat cell of width $h = 2V/A = 1$ mm, τ_K is $\simeq 40$ ms at $T = 80$ mK.

3. Three remarks should be made at this point. First, from expression 6, the Kapitza resistance is lowered by a factor $\frac{\sigma_4}{\sigma_3} = \frac{0.35}{0.15}$ (the ratio of free-surface tensions) for liquid ^3He , as compared with ^4He . Second, it is readily estimated from the capillary dispersion relation, that the penetration depth for the dominant ripplon ($\hbar \omega_q \simeq k_B T$) will become comparable with the thickness of the helium film (~ 300 Å, [1]), only for temperatures below a few mK — the effect being, again, more favourable (slightly) for ^3He . Finally, the Fermi-liquid viscosity of ^3He will ultimately damp off the ripplon modes, but again only for T lower than a few mK. Liquid ^3He - ^4He mixtures [5] (i.e. ^3He floating on top of the 6 % solution) are to be favoured, therefore — the main reason, though, being that the ^3He quasi-particles provide us with an efficient heat transfer channel from ripplons to bulk and walls, as we now discuss.

Were it is not for the quasi-particles (q.p.), the only channel on the liquid side would be the ripplon-to-phonon one. This can be shown to become very rapidly unefficient at low T : as T^7 (Ref. 7). The reason is that, due to momentum mismatch, *two* phonons are required to de-excite one ripplon mode (recall, for example, the analogous $T_1^{-1} \sim T^7$ argument for Raman spin-lattice relaxation in solids).

For the « gas » of q.p. the thermal resistance can be roughly estimated from equation 6 above, replacing m_H by $m_3^* \simeq 6 m_H$, $\bar{v}_{\text{th}}$ by v_F (same order of magnitude, at $T \simeq 50$ mK, since $T_F \simeq 300$ mK), and — most important — n_H by $n_{\text{q.p.}} \simeq n_3 \frac{T}{T_F} \simeq 10^{21} \text{ cm}^{-3}$. R_K is seen to be orders of magnitude lower for ^3He quasi-particles. So, the total thermal resistance between H-gas and bulk helium is essentially that given by equation 6. Similar arguments hold at the ^3He /dilute solution interface (smaller number of q.p.'s, but smaller interfacial tension too). Therefore, the whole thermal transfer problem is largely governed by the results of section 2.

There are other heat transfer channels which we have not discussed, however : for example *via* sticking to the liquid surface. Sticking coefficients α_{st} have been measured by NMR techniques [2]. At low coverage n_s , α_{st} was found to be $\simeq 1.6 \times 10^{-2}$ for H against liquid ^3He . Were this value independent of both T and n_s , this channel would dominate at low T , with an incoming energy flux of order $(n_H \bar{v}_{th} \varepsilon_a) \alpha_{st}$, where ε_a is the adsorption energy. But the coverage-dependence of α_{st} is not an obvious problem ; in a « site-adsorption » model, we expect α_{st} to be very small $\left(\sim 10^{-2} \exp - \frac{\varepsilon_a}{T} \right)$ for a saturated layer. Although such a model is clearly not acceptable for the present problem, we think the question is open-especially with respect to volume Bose condensation which precisely requires completion of the adsorption hydrogen layer.

On the other hand, the H/q.p. transfer by direct H/ ^3He impact without the mediation of the ripplon bath, seems to be a rather poor channel too : due to the Van der Waals interaction, the incoming H atom interacts with ripplons *before* hitting the ^3He quasi-particle.

Using equation 6 and measured values of the relaxation rate G , the Kapitza temperature difference ΔT can be estimated from equation 3 and the expression $Q_{rec} = V(5 \text{ eV}) G n_H^2$ for the steady-state recombination heat, at $n_H = 10^{17} \text{ at./cm}^3$ and $T = 80 \text{ mK}$ for example, we get : $\Delta T \simeq 80 \text{ mK}$! this of course is an overestimate since expression 6 for R_K assumes $\Delta T \ll T$, and it is expected that R_K is reduced in the non-linear regime. Still, this figure shows that the Kapitza problem is a serious one at low temperature (more so for ΔT than for τ_K , as we have seen). A very recent experiment, performed in Amsterdam [8], gives an upper bound of 15 mK upon ΔT , at $T = 310 \text{ mK}$, $B = 5.7 \text{ tesla}$ and $n_H \sim 3 \times 10^{15} \text{ at./cm}^3$.

References

- [1] SILVERA, I. F., Proc. LT 16, *Physica* **109**, **110B** (1982) 1499 and references therein.
- [2] HARDY, W. N., Proc. LT 16, **109**, **110B** (1982) 1964 and references therein.
- [3] STATT, B. W. and BERLINSKY, A. J., *Phys. Rev. Lett.* **45** (1980) 2105.
- [4] CLINE, R. W., GREYAK, T. J. and KLEPPNER, D., *Phys. Rev. Lett.* **47** (1981) 1195.
- [5] SPRIK, R., WALRAVEN, J. T. M., VAN YPEREN, G. H. and SILVERA, I. F., *Phys. Rev. Lett.* **49** (1982) 153.
- [6] LANDAU, L. D., Coll. Papers, Ed. D. ter Haar (Gordon and Breach), 1965, p. 117.
- [7] CASTAING, B., unpublished.
- [8] VAN YPEREN, G. H., SILVERA, I. F., WALRAVEN, J. T. M., BERKHOUT, R. and BRISSEN, J., preprint.