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Study of non steady convective regimes using Poincaré sections

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Résumé. — Une méthode expérimentale permettant de déterminer la section de Poincaré du diagramme de phase relatif à la convection de Rayleigh-Bénard est décrite. On illustre l'intérêt de cette méthode par l'étude de l'approche de l'accrochage en fréquence de deux oscillateurs thermoconvectifs.

Abstract. — An experimental method allowing the determination of the Poincaré section of the phase diagram related to the Rayleigh-Bénard convection is described. One points out the interest of this method through the study of the approach of the frequency locking of two thermoconvective oscillators.

Dynamical systems are well known to be able to reach a chaotic state even if they have a small number of degrees of freedom. Among those systems, the best known experimental examples are the hydrodynamic instabilities, such as the Rayleigh-Bénard instability in a confined geometry.

Until now, the experimental diagnostic consists generally in the Fast Fourier Transformation (F.F.T.) of one of the system variables (heat flux [1], convective velocity [2, 3], thermal perturbation [4]). It is then possible to determine if the state of the system is periodic, biperiodic or chaotic.

However powerful this technique is, it does not give all the informations necessary to understand the behaviour of a dynamical system. Further very worthwhile information is contained in the representative trajectories of the system in the phase space which may be obtained, for example, by plotting the time derivative of a variable against this variable. Experimental phase diagrams have been previously reported [5, 6, 7] but their interest is limited, since, as they are plotted in two dimensions, they become incredibly complicated when the dimension of the phase space is greater than two. For example, in a quasiperiodic regime with two incommensurate frequencies, often encountered just before a chaotic regime, such a diagram is formed by an infinity of loops corresponding to the projection of the trajectories in the phase space whose

dimension is three (or four) see for example figure 1a. A quantitative study is then impossible to achieve. Nevertheless, when a time dependence contains a periodic component (of period τ) the dimension of the phase space may be reduced by doing a Poincaré transformation. This transformation consists in sampling the trajectories in the phase space at the times : $t_0, t_0 + \tau, t_0 + 2\tau \dots$ etc.

In this manner, a unique oscillator, the trajectory of which is a closed loop (or limit cycle) in the phase space of dimension two has its Poincaré section constituted by a single dot.

We describe here an experimental tool leading directly, and thus without intermediate calculations, to the Poincaré sections of the dynamical states of the convection of Rayleigh-Bénard.

Fig. 1. — a) 13 successive phase loops $\dot{T} = f(T)$, $Ra/Ra_c = 569$, $f_1 = 26.2 \times 10^{-3}$ Hz, $f_2 = 9 \times 10^{-3}$ Hz. b) Poincaré section corresponding to the preceding loops, sampled at the frequency f_1 . The higher density regions are localized in A, B and C. The Poincaré section has 200 points.

1. Rayleigh-Bénard convection in a confined geometry, Poincaré diagrams. — A great number of studies have shown that in a confined geometry (the horizontal extension being twice the height of the layer), the occurrence of the turbulent state by increasing the Rayleigh number was preceded by a limited number of transitions, or bifurcations [9].

From a stationary state, the first transition is always related to the appearance of a monoperiodic state where the velocity (and also all the other representative variables) becomes modulated at a frequency f_1 . By further increasing the Rayleigh number, a bi-periodic regime is often observed. A new frequency f_2 appears in the time evolution of the system, most often incommen-

surate with f_1 . The thermal oscillators responsible for f_1 and f_2 are generally well localized in the convective layer (at least in the case of large Prandtl number considered here). By performing local measurements, such as that of the velocity, and by selecting the point of measurement, the effect of only one of the oscillators may be detected, corresponding to the frequency f_1 for example.

Additional filtering of the signal so obtained, performed by a tuned filter, eliminates all the remaining contributions of the frequencies different from f_1 . This modulated signal, at the frequency f_1 alone, will be used as the timing reference signal needed for the Poincaré section.

The phase diagram is then obtained by the measure of another dynamical variable, here the localized thermal gradient. This one is measured by the deflection of a parallel light-beam crossing horizontally the fluid layer. At one point of the transmitted image, the intensity fluctuations measured by a photodiode contain the dynamical properties of the system. In contrast to what is done in the velocity measurement, the point of measurement of the light intensity is chosen so that it corresponds to a region of the fluid layer where the two oscillators are active. The signal, let us say T , is connected to the X axis of a XY plotter, its derivative $\dot{T}$ to the Y axis. The Poincaré section of the phase diagram $(\dot{T}, T)$ is obtained by pointing (by driving the pen down on the XY plotter) at a precise (and tunable) phase of the reference oscillation at frequency f_1 .

2. Results. — The biperiodic regime that we have studied by drawing the Poincaré section in the phase space has been observed at high Rayleigh number in a cell of dimensions $L_x = 2 d$, $L_y = 1.2 d$ where d , the thickness of the layer, equals 2 cm. The fluid in convection is a silicone oil, with Prandtl number 130. A Fourier spectrum shows the two fundamental frequencies f_1 and f_2 and their combinations $nf_1 + mf_2$. The width of the peaks is nearly the instrumental one and no noise may be found.

The loops in the phase space (Fig. 1a) show a very complex picture. On the contrary, the associated Poincaré section (200 points) (Fig. 1b) is of a great simplicity. The general shape reminds an ellipse and the thickness of the drawing lets us state that the phase space of the system is a three-dimensional one. Further, we may detect a tendency to a locking of order 3, according to the highest density of points in the regions A, B, C. This tendency to a dynamic locking is confirmed when the Rayleigh number is increased. The regions A, B and C become more and

Fig. 2. — Poincaré section obtained under the same conditions as that of the figure 1b, but at $Ra/Ra_c = 590$ (1 500 points); $f_1 = 26.7 \times 10^{-3}$ Hz, $f_2 = 8.9 \times 10^{-3}$ Hz.

more dense at the expense of the remainder of the curve whereas the spectrum indicates that if the frequency f_2 is almost constant, the frequency f_1 increases gradually with Ra and finally locks on $3 f_2$. On the other hand, a recording of the successive values of $\dot{T}$, sampled like the Poincaré section with the frequency f_1 , clearly demonstrates that when the phaseshift between the two oscillators is nearly zero, this phaseshift varies very slowly; the unlocking then occurs very quickly, the phaseshift varying abruptly by $2\pi/3$ (Fig. 3). Those phases of pseudo-locking become longer and longer when Ra is increased until a real locking occurs, in the case of the structure under study, at $Ra/Ra_c = 593$. The Poincaré section is then represented by three points. It should be noticed that for $Ra/Ra_c = 590$, a quick study of the Fourier spectrum could have indicated a locking between the two oscillators, whereas the Poincaré section (Fig. 2) clearly shows that the locking was in latency but not real. It is clear that all this information on the dynamical behaviour of two coupled oscillators (here two thermoconvective oscillators) could not have been obtained from the simple Fourier spectra.

Further essential information may be deduced from experimental Poincaré diagrams concerning the mechanism of the occurrence of weak turbulence and can allow useful comparisons with theoretical models.

Fig. 3. — Ordinates $\dot{T}$ of the figure 2 plotted versus time. Note the long period during which the relative phases are almost constant compared to the shortness of the phase unlocking.

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