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Acoustics of water saturated packed glass spheres

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Résumé. — Nous avons mesuré l'atténuation et la vitesse du son dans un empilement de sphères de verre saturé d'eau. La fréquence et le diamètre des sphères varient respectivement de 200 kHz à 10 MHz et de 50 μm à 500 μm. Les mesures montrent clairement la validité de la théorie développée par Biot pour la propagation du son dans un milieu poreux.

Abstract. — We have measured the velocity and attenuation of sound in a pack of water saturated glass spheres. The frequency range spreads from 200 kHz to 10 MHz and the diameter of the spheres is varied from 50 μm to 500 μm. Our experimental data clearly confirm the validity of Biot's theory of the propagation of sound in porous medium.

1. **Introduction.** — The theory of the propagation of sound in porous material was developed twenty five years ago by Biot [1] and except for some experiments [2, 3] in geophysical materials the theory has not been widely tested. In the last few years there has been a new interest in the study of the acoustics of porous media charged with fluids : attenuation in marine sediments [4], superfluid helium confined in porous Vycor glass [5], fourth sound experiments in ^{4}He [6, 7], observation of a second bulk compressional mode in sintered glass beads [8], ...

The purpose of this letter is to present a test of Biot's theory on a rather simple system : a pack of calibrated glass spheres saturated with water. We have measured the velocity and attenuation of sound of the fast compressional wave in this material at high frequencies. We have varied both the frequency from 200 kHz to 10 MHz and glass sphere diameters from 50 μm to 500 μm. Until now there exists only one series of measurements [9] in such a system at low frequencies (20 to 300 kHz) for only one diameter (180 μm) of glass beads. Therefore our measurements for a wide range of diameters and frequencies provide a good test for the theory.

2. **Theoretical survey.** — Let us first recall the theoretical aspect of the problem. Basically there are two propagation equations, one for the liquid, one for the solid, coupled by a friction, proportional to the difference between the solid and liquid displacement velocities. Among the different new formulations [10-12] of Biot's original work, we choose to write the two propagation equations as [11] :

$$\nabla^2(He - C\xi) = \frac{\partial^2}{\partial t^2}(\rho e - \rho_f \xi) \tag{1}$$

$$\nabla^2(Ce - M\xi) = \frac{\partial^2}{\partial t^2}(\rho_f e - \rho_c \xi) - \frac{\eta}{B_0} F(\kappa) \frac{\partial \xi}{\partial t} \tag{2}$$

e is the dilatation of the bulk material and ξ is the relative dilatation of the fluid ; the elastic coefficients, C, H, M, are related to the porosity β, the bulk moduli K_g and K_f of glass and fluid, and to the bulk K^* and shear μ^* moduli of the porous frame (the pack of spheres) :

$$H = K^* + \tfrac{4}{3}\mu^* + (K_g - K^*)^2/(D - K^*) \tag{3}$$

$$C = K_g(K_g - K^*)/(D - K^*) \tag{4}$$

$$M = K_g^2/(D - K^*) \tag{5}$$

$$D = K_g(1 + \beta[(K_g/K_f) - 1]) . \tag{6}$$

In equation (3), $K^* + \frac{4}{3}\mu^*$ is defined through the measurement of the velocity of sound of the dry porous frame

$$V_s^2 = (\tfrac{4}{3} K^* + \mu^*)/(1 - \beta)\,\rho_s \tag{7}$$

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ρ_f and ρ_s are the fluid and glass densities and

$$\rho = (1 - \beta)\,\rho_s + \beta\rho_f\,.$$

ρ_c is defined as

$$\rho_c = \alpha\rho_f/\beta \tag{8}$$

α is a geometrical factor which is related to the induced mass due to the oscillation of solid particles in the fluid [13] (back flow). B_0 is the permeability of the porous frame. For glass spheres of diameter d, it has been shown [13] that $B_0 = \beta a^2/20$, $\alpha = 1 + (1 - \beta)/2\,\beta$ with

$$a = \beta d/3(1 - \beta)\,. \tag{9}$$

The second term of equation (2) corresponds to the friction between solid and liquid; η is the kinematic viscosity of the fluid and $F(\kappa)$ is a complex dynamic viscosity operator explicitly defined in the original work [1] :

$$\kappa = \left(\frac{\omega}{\omega_c}\right)^{1/2} = \left(\frac{\omega a^2}{\eta}\right)^{1/2}. \tag{10}$$

The frequency $\omega_c = \eta/a^2$ characterizes the viscous skin effect [14]. For $\omega \ll \omega_c$, the flow of fluid in the porous material is of the Poiseuille type and $F(\kappa) \sim 1$. For $\omega \gg \omega_c$, Poiseuille's flow breaks down and $F(\kappa) \sim \dfrac{1 + i}{4\sqrt{2}}\,\kappa$ has a real and imaginary contribution to the friction. For intermediate frequencies ($\omega \sim \omega_c$), $F(\kappa)$ has to be computed numerically.

Resolution of the system (1-2) for compressional wave of amplitude proportional to $\exp i(\omega t - kr)$ gives the dispersion relation $k(\omega)$. k is the complex wave vector, $k = \dfrac{\omega}{V} + ik''$, where V and k'' are respectively the velocity and attenuation of sound. As a result, there are two compressional waves corresponding to a fast (R) and a slow (L) mode. At low frequencies ($\omega \ll \omega_c$) the slow mode is viscoelastic and the fast one propagates with velocity V_0. At high frequencies ($\omega \gg \omega_c$) both modes propagate with velocity $V_{R\infty}$ (fast) and $V_{L\infty}$ (slow). A last comment about results of the calculation : in the two frequency limits ($\omega \ll \omega_c$ and $\omega \gg \omega_c$) the different velocities V_0, $V_{R\infty}$, $V_{L\infty}$, depend only on elastic constants (H, C, M), densities (ρ_f, ρ, ρ_s) and porosity β but do not depend on the friction coefficient $\left(\dfrac{\eta}{B_0}\,F(\kappa)\right)$ i.e. not on the pore size. Thus velocity measurements in these limits are measurements of the parameter of the porous material charged with fluid. Attenuation measurements would then be the direct measurement of the friction between the porous frame and the fluid.

As we will see later on, in our experiment, we are mainly interested, in the high frequency regime ($\omega \gg \omega_c$); in such a case, the attenuation k''_∞ of the fast compressional mode can easily be written as

$$k''_\infty = \frac{\Delta V}{4\sqrt{2}\,V_{R\infty}^2}\,\frac{\eta}{B_0}\left(\frac{\omega}{\omega_c}\right)^{1/2} \tag{11}$$

where $\Delta V = V_{R\infty} - V_0$ is the total velocity dispersion. If we explicit the d dependence of B_0 and ω_c we find that the attenuation of the fast mode in our pack of spheres is expected to vary as

$$k''_\infty \sim \sqrt{\omega}/d\,. \tag{12}$$

This is this behaviour we check in our experiment.

3. **Experimental set up.** — The use of three pairs of wide band transducers, with a central frequency at 500 kHz, 2.25 MHz and 5 MHz, allowed us to cover the frequency range 200 kHz to 10 MHz, with a typical signal to noise ratio of 60 dB. The space between the transducers is varied from 1 to 20 cm. We use a pulse echo technique. Our measurements of the attenuation in our pack of spheres are compared to that for water (at the same room temperature) in order to eliminate any parasitic effects such as a parallelism defect, or diffraction and transducer response. Our glass spheres have a density $\rho_s = 2.9$ g/cm^3 and the porosity of the pack is $\beta = 0.40 \pm 0.02$. The beads were filtered through calibrated sieves and their shapes and diameters were controlled through a microscope. We have used four samples of different diameters : 40-50 μm, 80-100 μm, 200-250 μm, 400-500 μm. The glass spheres, confined in a tank, are water saturated with a vacuum immersion technique.

4. **Experimental results and discussion.** — A first result concerns velocity of sound measurements. For the dry pack of glass spheres the velocity of sound V_s we observe is of the order of 400 m/s at 200 kHz (this is the highest frequency at which we observe this mode). From equation (7) we get $K^* + \frac{4}{3}\mu^* \simeq 2.8 \times 10^9$ c.g.s. We do not know the ratio between K^* and μ^*, but we do have the order of magnitude of K^* ($\sim 10^9$ c.g.s.) which is much smaller than either the bulk modulus $K_g \sim 750 \times 10^9$ of the glass or the bulk modulus of the fluid, $K_f \sim 22.5 \times 10^9$. From equations (3) to (7) we compute $H \sim C \sim M \sim 54 \times 10^9$ c.g.s. For the water saturated pack of glass spheres we have measured $V_{exp} = (1\,710 \pm 30)$ m/s whatever the frequency or the sphere diameter (in pure water the velocity is 1 500 m/s). This value V_{exp} is in reasonable agreement with the value $V_{R\infty}$ we compute from the set of equations (1-7) : $V_{R\infty} = 1\,725$ m/s.

Our experimental results on the attenuation of sound k''_{exp} are given in figure 1, in a log-log plot. It is easily seen that after a smooth increase with the frequency (between $\sqrt{\omega}$ and ω) the attenuation increases drastically at high frequencies. Moreover if for small ω, as expected from equation (12) k'' decreases as d increases, at higher frequencies the

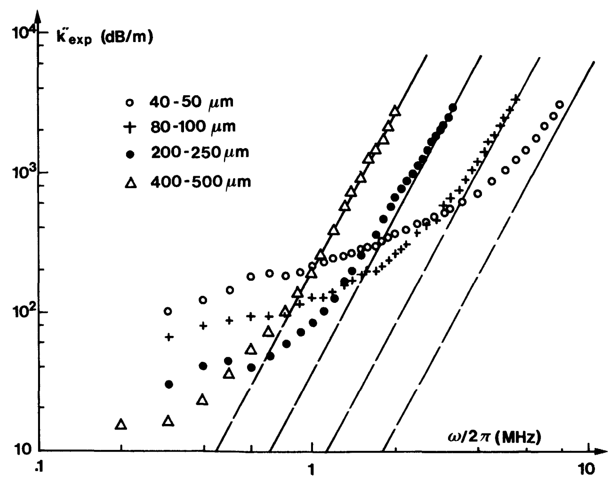

Fig. 1. — Log-log plot of attenuation of sound measurements k''_{exp} *versus* frequency $\omega/2\pi$ for different sphere diameters. Full and dashed lines correspond to the extrapolation of the unexpected high frequency diffusion k''_B from high (full) to low (dashed) frequencies.

attenuation increases as d increases : the curves for different d intersect. We have then first to discuss this unexpected attenuation. As ω increases our log-log plot shows that the different curves tend to straight lines of rather the same slope (3.6 ± 0.1). Such a frequency dependence suggests that this extra attenuation may be due to Rayleigh scattering (slope 4). But this type of scattering is only well defined when the wavelength λ is much bigger than the size of the diffusing centre (d or $a \sim d/5$). In the same way, Biot's theory assumes continuous medium on the scale of λ. For the fast compressional mode λ varies from 5 mm at 30 kHz to 200 μm at 8 MHz. d varies from 50 to 500 μm and a from 10 to 100 μm. Therefore the unexpected behaviour of the attenuation at high frequencies may be due to a transition from the continuous medium description ($\lambda \gg d$) to a diffraction description ($\lambda \sim d$). This unexpected high frequency behaviour of k''_{exp} gives the upper limit of our range of investigation of Biot's theory. However, as high frequency measurements have a quite well defined slope (3.6), we can extrapolate the high frequencies behaviour k''_B of the attenuation from high to low frequencies (full line to dashed line in figure 1). Then we subtract k''_B from our data to get the attenuation k''_∞ in which we are chiefly interested. The remaining attenuation, $k''_\infty = k''_{exp} - k''_B$ is plotted in figure 2, for our four spheres diameters, as a function of the frequency. From expression (12) we expect the experimental points to lie on a straight line of slope 0.5. This is not the case in figure 2 and especially for small d at low frequencies. This would mean that ω is not high enough compared to ω_c.

$$\omega_c/2\pi\,(= \eta/2\pi a^2 = 9\,\eta(1-\beta)^2/2\pi d^2\beta^2)$$

varies from 2 kHz to 20 Hz, as d varies from 50 to 500 μm, and ω/ω_c from 10^2 to 10^6 in our frequency range. The condition $\kappa = \sqrt{\omega/\omega_c} \gg 1$ is not always fulfilled and the dispersion relation has been solved numerically with the explicit form of $F(\kappa)$ [13]. The resulting curves are drawn in figure 2 as solid lines. The values used in the computation are given along the paper and $\eta = 10^{-2}$ c.g.s. (water). About the diameter dispersion ($\sim 10\%$), the curves drawn in figure 2 correspond to the mean value between the two extremal diameter curves of each samples (for instance 40 and 50 μm for the 40-50 μm samples). According to this calculation the velocity of sound $V(\omega)$ varies between 1 705 and 1 725 m/s for the extremal value of κ (300 kHz, d = 40 μm to 2 MHz, d = 250 μm) in agreement with the dispersion of our measured values $V_{exp} = (1\,710 \pm 30)$ m/s.

The agreement between theory and experiment is reasonably good (Fig. 2). We must notice that we fit our experimental data over a wide range of both frequencies and glass sphere diameters, i.e. from a regime ω close to ω_c ($\omega \sim 100\,\omega_c$) to a high frequency regime ($\omega \sim 10^6\,\omega_c$).

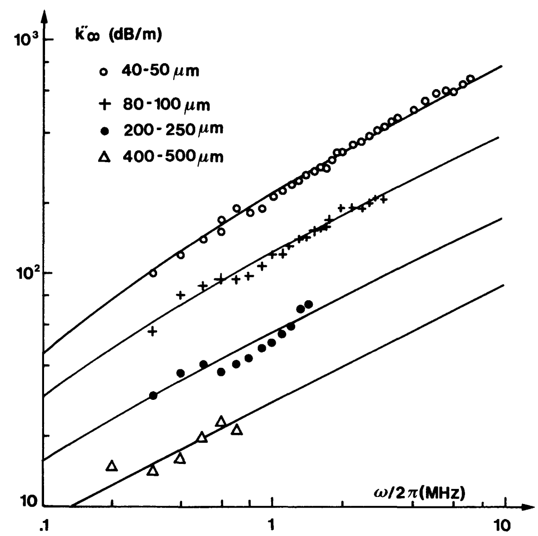

Fig. 2. — Log-log plot of the remaining attenuation of sound $k''_\infty = k''_{exp} - k''_B$, *versus* frequency. The different lines are theoretical curves (see text).

5. **Conclusion.** — We have measured the velocity and attenuation of the fast compressional wave of packed glass spheres saturated with water. Our frequency range of investigation is limited by a strong unexpected diffusion. However velocity measurements and the frequency and glass bead diameter dependences confirm the applicability of the theory of propagation of sound in porous media developed by Biot.

References

[1] BIOT, M. A., *J. Acous. Soc. Am.* **28** (1956) 168.
[2] NOLLE, A. W., HOYER, W. A., MIFSUD, J. F., RUNYAN, W. R., and WARD, M. A., *J. Acous. Soc. Am.* **35** (1963) 1394.
[3] HAMILTON, E. L., *Geophys.* **37** (1972) 620.
[4] HAMILTON, E. L., *J. Geophys. Res.* **76** (1971) 579.
[5] LAMBERT, B. and SALIN, D., *J. Physique Lett.* **41** (1980) L-487.
[6] SHAPIRO, K. A. and RUDNICK, I., *Phys. Rev. A* **137** (1965) 1383.
[7] JOHNSON, D. L., *Appl. Phys. Lett.*, to be published.
[8] PLONA, T. J., *Appl. Phys. Lett.* **36** (1980) 259.
[9] HOVEM, J. M. and INGRAM, G. D., *J. Acous. Soc. Am.* **66** (1974) 1807.
[10] STOLL, R. D., in *Physics of sound in marine sediments*, edited by L. Hampton (Plenum, New York) 1974, p. 19-39.
[11] STOLL, R. D., *J. Acous. Soc. Am.* **68** (1980) 1341.
[12] GEERTSMA, T. and SMIT, D. C., *Geophys.* **26** (1961) 169.
[13] BERRYMAN, J. G., *Appl. Phys. Lett.* **37** (1980) 382.
[14] LANDAU, L. and LIFCHITZ, E., *Fluid Mechanics* (Pergamon Press), chap. II.