Acoustics of water saturated packed glass spheres
D. Salin, W. Schön

To cite this version:

HAL Id: jpa-00231981
https://hal.archives-ouvertes.fr/jpa-00231981
Submitted on 1 Jan 1981

HAL is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers. L’archive ouverte pluridisciplinaire HAL, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d’enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.
Acoustics of water saturated packed glass spheres

D. Salin and W. Schön

Laboratoire d’Ultrasons (*), Université Pierre-et-Marie-Curie, Tour 13, 4, place Jussieu, 75230 Paris Cedex 05, France

(Reçu le 1er juillet 1981, accepté le 29 septembre 1981)

Résumé. — Nous avons mesuré l’atténuation et la vitesse du son dans un empièlement de sphères de verre saturé d’eau. La fréquence et le diamètre des sphères varient respectivement de 200 kHz à 10 MHz et de 50 μm à 500 μm. Les mesures montrent clairement la validité de la théorie développée par Biot pour la propagation du son dans un milieu poreux.

Abstract. — We have measured the velocity and attenuation of sound in a pack of water saturated glass spheres. The frequency range spreads from 200 kHz to 10 MHz and the diameter of the spheres is varied from 50 μm to 500 μm. Our experimental data clearly confirm the validity of Biot’s theory of the propagation of sound in porous medium.

1. Introduction. — The theory of the propagation of sound in porous material was developed twenty-five years ago by Biot [1] and except for some experiments [2, 3] in geophysical materials the theory has not been widely tested. In the last few years there has been a new interest in the study of the acoustics of porous media charged with fluids: attenuation in marine sediments [4], superfluid helium confined in porous Vycor glass [5], fourth sound experiments in 3He [6, 7], observation of a second bulk compressional mode in sintered glass beads [8], ...

The purpose of this letter is to present a test of Biot’s theory on a rather simple system: a pack of calibrated glass spheres saturated with water. We have measured the velocity and attenuation of sound of the fast compressional wave in this material at high frequencies. We have varied both the frequency from 200 kHz to 10 MHz and glass sphere diameters from 50 μm to 500 μm. Until now there exists only one series of measurements [9] in such a system at low frequencies (20 to 300 kHz) for only one diameter (180 μm) of glass beads. Therefore our measurements for a wide range of diameters and frequencies provide a good test for the theory.

2. Theoretical survey. — Let us first recall the theoretical aspect of the problem. Basically there are two propagation equations, one for the liquid, one for the solid, coupled by a friction, proportional to the difference between the solid and liquid displacement velocities. Among the different new formulations [10-12] of Biot’s original work, we choose to write the two propagation equations as [11]:

\[ V^2(He - C\xi) = \frac{\partial^2}{\partial t^2} (\rho e - \rho_t \xi) \]  

\[ V^2(Ce - M\xi) = \frac{\partial^2}{\partial t^2} (\rho_t e - \rho_s \xi) - \frac{\eta}{B_0} F(\kappa) \frac{\partial \xi}{\partial t} \]

\[ e \] is the dilatation of the bulk material and \( \xi \) is the relative dilatation of the fluid; the elastic coefficients, \( C, H, M \), are related to the porosity \( \beta \), the bulk moduli \( K_g \) and \( K_f \) of glass and fluid, and to the bulk \( K^* \) and shear \( \mu^* \) moduli of the porous frame (the pack of spheres):

\[ H = K^* + \frac{4}{3} \mu^* + (K_g - K^*)^2/(D - K^*) \]  

\[ C = K_g(K_g - K^*)/(D - K^*) \]  

\[ M = K_g^2/(D - K^*) \]  

\[ D = K_g(1 + \beta[(K_g/K_f) - 1]) \]

In equation (3), \( K^* + \frac{4}{3} \mu^* \) is defined through the measurement of the velocity of sound of the dry porous frame

\[ V_s^2 = (\frac{4}{3} K^* + \mu^*)/(1 - \beta) \rho_s \]
\( \rho_f \) and \( \rho_g \) are the fluid and glass densities and
\[
\rho_c = (1 - \beta) \rho_g + \beta \rho_f.
\]
\( \rho_c \) is defined as
\[
\rho_c = a \rho_f / \beta \tag{8}
\]
x is a geometrical factor which is related to the induced mass due to the oscillation of solid particles in the fluid \([13]\) (back flow). \( B_0 \) is the permeability of the porous frame. For glass spheres of diameter \( d \), it has been shown \([13]\) that \( B_0 = \beta d^2 / 20 \). \( \alpha \) is defined as
\[
\alpha = 1 + (1 - \beta) / 2 \beta
\]
if for small \( \omega \), as expected from equation (12) \( k'' \) decreases as \( d \) increases, at higher frequencies the fast compressional mode can easily be written as
\[
\text{(co ~ we) ; in such a case, the attenuation } k'' \text{ of the fast}
\]
\[
\text{mode is } k'' = \frac{\Delta V}{4 \sqrt{2} V_{R\omega}^2 \eta B_0 (\omega / \omega_c)^{1/2}} \tag{11}
\]
where \( \Delta V = V_{R\omega} - V_o \) is the total velocity dispersion.

If we explicit the \( d \) dependence of \( B_0 \) and \( \omega_c \) we find that the attenuation of the fast mode in our pack of spheres is expected to vary as
\[
k'' \sim \sqrt{\omega / d} \tag{12}
\]
This is this behaviour we check in our experiment.

3. Experimental set up. — The check of three pairs of wide band transducers, with a central frequency at 500 kHz, 2.25 MHz and 5 MHz, allowed us to cover the frequency range 200 kHz to 10 MHz, with a typical signal to noise ratio of 60 dB. The space between the transducers is varied from 1 to 20 cm. We use a pulse echo technique. Our measurements of the attenuation in our pack of spheres are compared to that for water (at the same room temperature) in order to eliminate any parasitic effects such as a parallelism defect, or diffraction and transducer response. Our glass spheres have a density \( \rho_c = 2.9 \text{ g/cm}^3 \) and the porosity of the pack is \( \beta = 0.40 \pm 0.02 \). The beads were filtered through calibrated sieves and their shapes and diameters were controlled through a microscope. We have used four samples of different diameters : 40-50 \text{ gm}, 80-100 \text{ gm}, 200-250 \text{ gm}, 400-500 \text{ gm}. The glass spheres, confined in a tank, are water saturated with a vacuum immersion technique.

4. Experimental results and discussion. — A first result concerns velocity of sound measurements. For the dry pack of glass spheres the velocity of sound \( V_o \) we observe is of the order of 400 m/s at 200 kHz (this is the highest frequency at which we observe this mode). From equation (7) we get
\[
K^* / \mu^* \approx 2.8 \times 10^6 \text{ c.g.s. We do not know the ratio between } K^* \text{ and } \mu^*, \text{ but we do have the order of magnitude of } K^* / \mu^* \approx 10^5 \text{ c.g.s.}
\]
For the dry pack of glass spheres we have measured \( V_{R\omega} \approx 1.710 \pm 30 \text{ m/s} \) whatever the frequency is. This value \( V_{R\omega} \) is in reasonable agreement with the value \( V_{R\omega} \) we compute from the set of equations (1-7) : \( V_{R\omega} = 1725 \text{ m/s} \).

Our experimental results on the attenuation of sound \( k'' \) are given in figure 1, in a log-log plot. It is easily seen that after a smooth increase with the frequency (between \( \sqrt{\omega} \) and \( \omega \)) the attenuation increases drastically at high frequencies. Moreover if for small \( \omega \), as expected from equation (12) \( k'' \) decreases as \( d \) increases, at higher frequencies the
attenuation increases as \( d \) increases: the curves for different \( d \) intersect. We have then first to discuss this unexpected attenuation. As \( \omega \) increases our log-log plot shows that the different curves tend to straight lines of rather the same slope (3.6 \( \pm \) 0.1). Such a frequency dependence suggests that this extra attenuation may be due to Rayleigh scattering (slope 4). But this type of scattering is only well defined when the wavelength \( \lambda \) is much bigger than the size of the diffusing centre (\( d \) or \( a \sim d/5 \)). In the same way, Biot's theory assumes continuous medium on the scale of \( \lambda \). For the fast compressional mode \( \lambda \) varies from 5 mm at 30 kHz to 200 \( \mu \)m at 8 MHz, \( d \) varies from 50 to 500 \( \mu \)m and \( a \) from 10 to 100 \( \mu \)m. Therefore the unexpected behaviour of the attenuation at high frequencies may be due to a transition from the continuous medium description (\( \lambda \gg d \)) to a diffraction description (\( \lambda \sim d \)). This unexpected high frequency behaviour of \( k_{\text{exp}} \) gives the upper limit of our range of investigation of Biot's theory. However, as high frequency measurements have a quite well defined slope (3.6), we can extrapolate the high frequencies behaviour \( k_{\text{exp}} \) of the attenuation from high to low frequencies (full line to dashed line in figure 1). Then we subtract \( k_{\text{exp}} \) from our data to get the attenuation \( k_{\text{res}} \) in which we are chiefly interested. The remaining attenuation, \( k_{\text{res}} = k_{\text{exp}} - k_{\text{ne}} \) is plotted in figure 2, for our four spheres diameters, as a function of the frequency. From expression (12) we expect the experimental points to lie on a straight line of slope 0.5. This is not the case in figure 2 and especially for small \( d \) at low frequencies. This would mean that \( \omega \) is not high enough compared to \( \omega_{c} \).

\[
\omega/2 \pi = \frac{\eta}{2} \pi a^2 = 9 \eta (1 - \beta)^2/2 \pi d^2 \beta^2
\]

varies from 2 kHz to 20 Hz, as \( d \) varies from 50 to 500 \( \mu \)m, and \( \omega/\omega_{c} \) from \( 10^2 \) to \( 10^6 \) in our frequency range. The condition \( \kappa = \sqrt{\omega/\omega_{c}} > 1 \) is not always fulfilled and the dispersion relation has been solved numerically with the explicit form of \( F(\kappa) \) [13]. The resulting curves are drawn in figure 2 as solid lines. The values used in the computation are given along the paper and \( \eta = 10^{-2} \) c.g.s. (water). About the diameter dispersion (\( \sim 10\% \)), the curves drawn in figure 2 correspond to the mean value between the two extradameter curves of each samples (for instance 40 and 50 \( \mu \)m for the 40-50 \( \mu \)m samples). According to this calculation the velocity of sound \( V(\omega) \) varies between 1705 and 1725 m/s for the extremeal value of \( \kappa \) (300 kHz, \( d = 40 \mu \)m to 2 MHz, \( d = 250 \mu \)m) in agreement with the dispersion of our measured values \( V_{\text{exp}} = (1710 \pm 30) \) m/s.

The agreement between theory and experiment is reasonably good (Fig. 2). We must notice that we fit our experimental data over a wide range of both frequencies and glass sphere diameters, i.e. from a regime \( \omega \) close to \( \omega_{c} (\omega \sim 100 \omega_{c}) \) to a high frequency regime (\( \omega \sim 10^6 \omega_{c} \)).

5. Conclusion. We have measured the velocity and attenuation of the fast compressional wave of packed glass spheres saturated with water. Our frequency range of investigation is limited by a strong unexpected diffusion. However velocity measurements and the frequency and glass bead diameter dependences confirm the applicability of the theory of propagation of sound in porous media developed by Biot.
References