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Tome 42 Nº 9 1er MAI 1981

LE JOURNAL DE PHYSIQUE - LETTRES

J. Physique — LETTRES **42** (1981) L-181 - L-184 1er MAI **1981,** PAGE L-181

Classification
Physics Abstracts
21.10F — 23.60

The nuclear radius from alpha decay (*)

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(Reçu le 29 janvier 1981, révisé le 5 mars, accepté le 20 mars 1981)

Résumé. — Par l'étude de l'émission alpha entre les états fondamentaux de noyaux pair-pair, nous avons calculé les rayons nucléaires de 121 de ces noyaux dont les masses sont comprises entre 140 et 252. Pour ce faire nous utilisons la solution du problème à deux corps avec un puits de potentiel carré que nous avons donné récemment. Les rayons calculés obéissent à la loi $A^{1/3}$ et ils sont compatibles avec les rayons très précis obtenus par diffusion d'électrons. Ces deux ensembles de rayons présentent des structures qui sont en corrélation.

Abstract. — Nuclear radii for mass numbers between $A = 140$ and 252 have been computed from 121 alpha decays between ground states of even-even nuclei. We use for that the two-body solution for the square well potential that we gave recently. The computed radii obey the $A^{1/3}$ law and they are consistent with the precise radii obtained from electron scattering. These two sets of radii show structures which are correlated.

1. **Introduction.** — The theory of alpha decay was the first successful application of quantum mechanics to the nucleus. There have been many versions of the two-body theory and we will only mention five of them that had an important impact on the field. The first two papers were those of Gamow [1] and of Condon and Gurney [2] in 1928. Then came Born [3] in 1929, Bethe [4] in 1937 and Preston [5] in 1947.

All these papers have common aspects but arrive at final expressions which are different, and give different results. It seemed to us that such a simple two-body problem should have one and only one exact solution.

Pierronne and Marquez [6] made an attempt to give the exact two-body solution. In this paper the energy given by the solution of the equation is a complex eigenvalue of the form $E = E_a - i\Gamma/2$ where E_a is the decay energy and Γ the width of the state. The new feature in our treatment is that the eigenstates depend on the radial quantum number Q, which also determines the depth of the potential well. We computed the alpha decay width for 102 transitions between even-even nuclei with a radius of the form $R = r_0(A_1^{1/3} + A_2^{1/3})$ and found a reasonable agreement with experimental widths if r_0 has a value between 1.1f and 1.2f.

Marquez [7] made similar calculations for a sample of 32 decays between ground state even-even nuclei heavier than ^{208}Pb with fair agreement if $r_0 = 1.1$f. In this latter paper there is a critical discussion of the five previously mentioned treatments.

It is well known that the nuclear radius has a decisive role in alpha decay. Recent papers by Heilig *et al.* [8] and by Jacquinot and Klapisch [9, 10] have shown that the nuclear radius is not so simple and the law $A^{1/3}$ is only an approximation.

This lead us to think that perhaps a better test of the theory would be to compute the nuclear radii and see if they are consistent with the very reliable radii available from electron scattering [11]. We assume that the theoretical computed width is equal to the experimental width.

2. **The formalism and the calculations. The alpha decay radius.** — We treat the case of the square well potential with a pure Coulomb potential outside

(*) La version française de cet article a été proposée aux Comptes Rendus de l'Académie des Sciences.

the well. We call Y/r the radial wave function inside; outside it is given by $(G + iF)/r$. We impose the continuity of the logarithmic derivative at the well radius and for $E = E_a - i\Gamma/2$. We make Taylor expansions in powers of $-i\Gamma/2$ and drop the negligible terms.

The real part gives,

$$\partial Y/\partial r/Y = \partial G/\partial r/G$$

and from the imaginary part we get,

$$\Gamma = \gamma/G^2 \qquad \gamma = k_e/(T_i - T_e)$$
$$T_i = (\partial Y/\partial r . \partial Y/\partial E - Y . \partial^2 Y/\partial r\, \partial E)/(2\, Y^2)$$
$$T_e = (\partial G/\partial r . \partial G/\partial E - G . \partial^2 G/\partial r\, \partial E)/(2\, G^2) .$$

As the well depth increases different solutions are found with increasing radial quantum number $Q = 1$, 2, 3, etc... We pick the right Q from the rule [12] :

$$2(Q - 1) + L = \Sigma_i(2(q_i - 1) + l_i) .$$

In this expression L is the orbital angular momentum of the emitted alpha, q_i and l_i are the corresponding quantum numbers of the nucleons forming the alpha particle.

The value of Q found by this rule is 10, 11 or 12 for known alpha emitters and this corresponds to a well depth of about 100 MeV. This depth is in agreement with the value usually found in alpha elastic scattering.

We compute the theoretical width for different radii and we get the alpha decay radius R_a by interpolation. This procedure was applied to all the ground state even-even alpha decays given in the table of isotopes by Lederer *et al.* [13]. This gave a set of 121 alpha decay radii.

3. **Comparison of alpha decay and electron scattering radii.** — The electron scattering radii were taken from the table by de Jager *et al.* [11]. They give 183 root mean square (r.m.s.) radii which correspond to 87 nuclides having $A \geqslant 12$. For each nuclide the average of the r.m.s. radii was taken and called R_e. We also use the radius of the equivalent uniform distribution R_u given by $R_u = \sqrt{5/3}\, R_e$.

The first thing to verify is the $A^{1/3}$ law. For each value of R_a and R_e there is a corresponding value of r_a and r_e given by $r_a = R_a/A^{1/3}$ and $r_e = R_e/A^{1/3}$. We call $\bar{r}_a$ and $\bar{r}_e$ the mean values of r_a and r_e taken for $A \geqslant 140$. We compute the quantities $R_a/(\bar{r}_a\, A^{1/3})$ and $R_e/(\bar{r}_e\, A^{1/3})$ and plot them as functions of A in figure 1. The electron scattering ratios show first a decrease, usually explained by the lack of saturation and then it approaches a constant value, but all throughout there is a spread of the points around the mean value. The alpha decay ratios are consistent with a constant value but the spread is greater than for the electron scattering. The value found for the parameters are $\bar{r}_a = 1.409$f, $\bar{r}_e = 0.943$f.

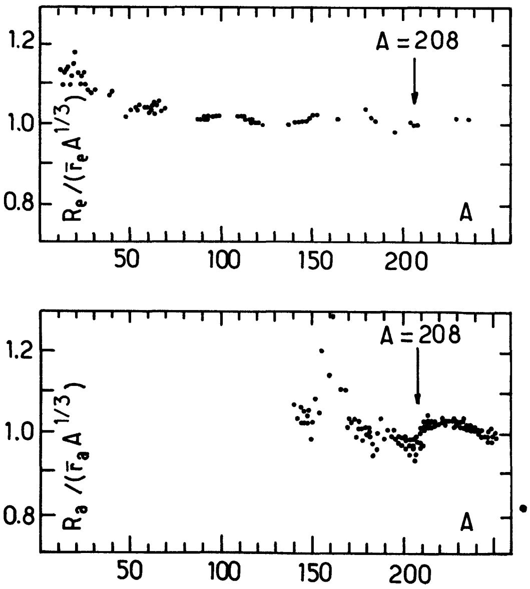

Fig. 1. — Upper part, plot of the ratios $R_e/(\bar{r}_e\, A^{1/3})$. R_e is the root mean square electron scattering radius. Lower part, plot of the ratios $R_a/(\bar{r}_a\, A^{1/3})$. R_a is the alpha decay radius. Both sets of data are shown against the mass number A.

We make now a test of consistency between these two sets of radii by a very general argument. R_a should be the sum of two parts, one part corresponds to the emitted alpha and the other part corresponds to the residual nucleus.

We should compare R_a with the sum of the radii of the alpha and the residual nucleus given by electron scattering. The value given by the sum of the two R_e corresponds to a very deep interpenetration of the two spheres. The value given by the sum of the two R_u corresponds to spheres barely touching each other. One expects that R_a should be in between and this is the case as shown in figure 2.

4. **Correlation between alpha decay and electron scattering radii.** — We see evidence in figure 1 for a shell effect at $A = 208$. This is made more striking if we plot the same data as function of N, and this is done in figure 3. We can see in the electron scattering data that there is a smooth decrease as one approaches $N = 126$. This trend is even more pronounced with the alpha decay data. There is clear evidence of a shell effect at $N = 126$ in the two sets of data.

Figure 4 shows the same quantities as a function of Z. The result is less clear than in the previous plot.

The similarities seen in figures 1 and 3 are interesting, but there exists a decisive test, namely, if there is a correlation between the two radii for identical nuclei. We pick the electron scattering r_e for a given (N, Z) and the alpha r_a for the same (N, Z). There are $n = 7$ nuclei for which we have both the alpha decay and the electron scattering radii. The quantities to compare are r_e and r_a to remove the $A^{1/3}$ depen-

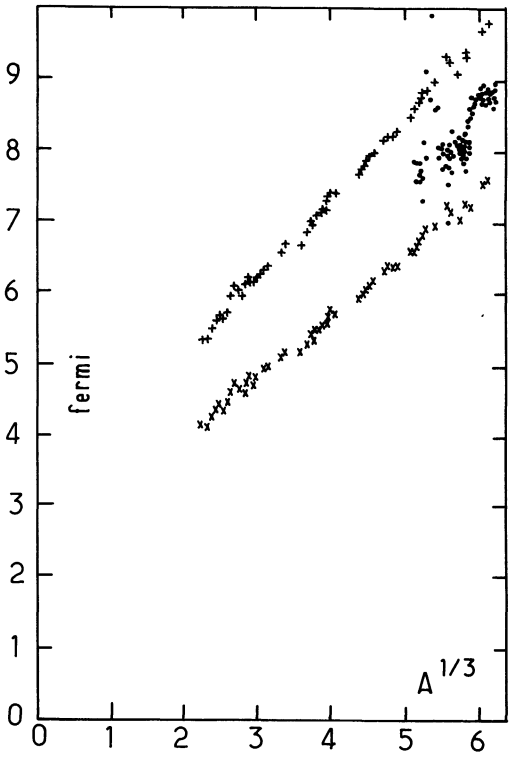

Fig. 2. — The + represent the sum of the uniform distribution radii from electron scattering for the alpha particle and the residual nucleus. The dots represent the alpha decay radii. The × represent the sum of the root mean square radii from electron scattering for the alpha particle and the residual nucleus. The points are plotted against $A^{1/3}$.

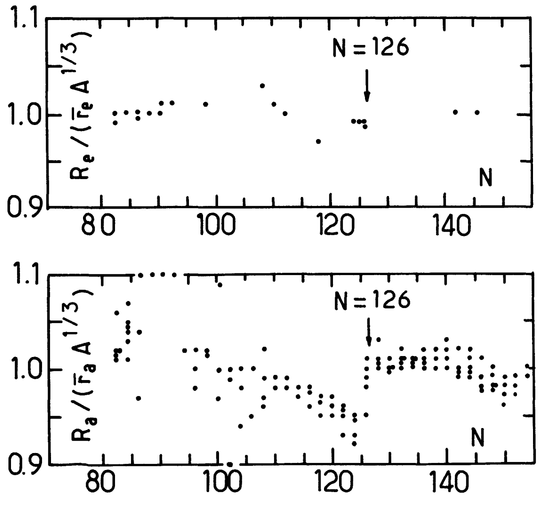

Fig. 3. — Upper part, plot of $R_e/(\bar{r}_e A^{1/3})$. Lower part, plot of $R_a/(\bar{r}_a A^{1/3})$. The points are shown against the neutron number N.

dence. The data are shown in table I. The correlation coefficient defined as [14],

$$r = \Sigma_i(x_i - \bar{x})(y_i - \bar{y})/\sqrt{\Sigma_i(x_i - \bar{x})^2 . \Sigma_i(y_i - \bar{y})^2}$$

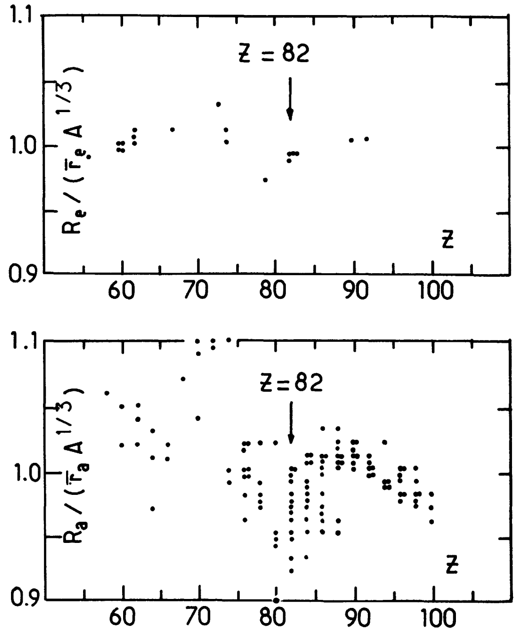

Fig. 4. — Upper part, plot of $R_e/(\bar{r}_e A^{1/3})$. Lower part, plot of $R_a/(\bar{r}_a A^{1/3})$. The points are shown against the proton number Z.

Table I. — *Parameters of the nuclides for which we have both r_e and r_a.*

N	Z	r_e	r_a
82	60	0.942 2	1.442
84	60	0.939 8	1.481
86	62	0.943 2	1.471
124	82	0.932 8	1.296
126	82	0.928 6	1.382
142	90	0.939 5	1.434
146	92	0.942 9	1.419

has the value of $r = 0.71$ and this corresponds to a probability $P = 0.96$. This means that the hypothesis that there is no correlation may be rejected with only 0.04 chance of being wrong.

We then looked how quickly the correlation fades away when we make a relative displacement in the (N, Z) plane. We looked for the two radii given by (N, Z) for the electron scattering and $(N - \Delta N, Z - \Delta Z)$ for the alpha decay. ΔN and ΔZ take negative, zero or positive even values. The number of cases n for which we have the pairs (r_e, r_a) varies with ΔN and ΔZ. We compute the correlation coefficient r, then $t = r\sqrt{(n-2)/(1-r^2)}$ and finally the probability P that there is a correlation between r_e and r_a. P is given by the t-distribution [14]. The results are shown in figure 5. We see that the probability of having a correlation spans over a wide range of ΔN but it is lower immediately when $\Delta Z \neq 0$.

One should remark that there is correlation with the Z of the residual nucleus and not with the Z of the parent nucleus.

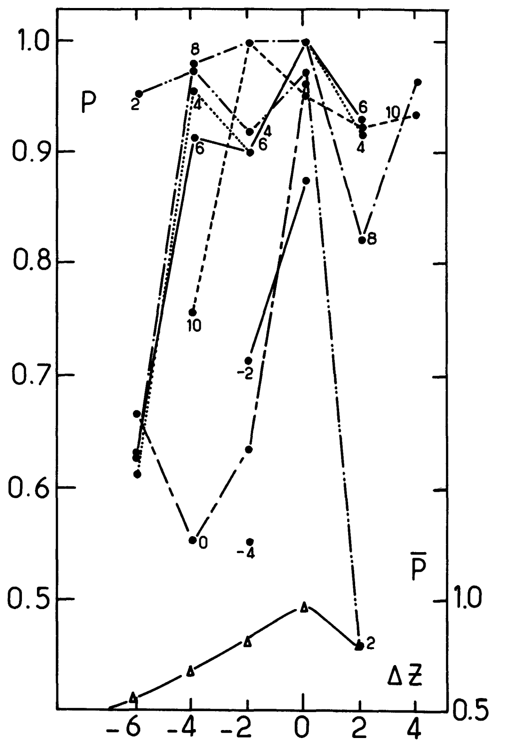

Fig. 5. — The dots represent the probability P that there is a correlation between r_e and r_a. They are shown as functions of ΔZ. The lines join the points with constant ΔN and the numbers on the lines are ΔN. On the lower part, the triangles show the average value of P for a given ΔZ as function of ΔZ.

5. **Discussion.** — We have seen that there are similar structures in R_e and R_a, the break at $N = 126$ being the most striking one. We can estimate from figure 3 the change in radius associated with this break and we find 0.24f and 0.48f for R_e and R_a.

In order to assess the significance of these numbers, we should compare them with the corresponding errors.

The errors in R_e are given in [11] or we can calculate them from the tabulated values. It is on the average 0.03f for the region of the alpha emitters. If we draw the error bars in figure 3, the whole distance between the bars will be twice the size of the dot inside the bars.

We can compute the errors in R_a coming from experimental errors. There are three sources; the decay energy, the half-life and the branching ratio. From the data [13] we can safely estimate that the error in the energy is 4 keV; the relative errors in the half-life and the branching ratio are 0.02. We computed the errors for the 121 alpha decays and we found that on the average the computed error of R_a is 0.014f. If we draw the error bars in figure 3, the bars will be inside the dots.

In conclusion, the electron scattering and the alpha decay radii obey approximately the $A^{1/3}$ law. The two sets of radii have structures and there is a high degree of correlation between them. This latter fact is very satisfying since both methods are completely independent.

Acknowledgments. — I am indebted to R. Foucher and B. Giraud for useful discussions concerning some aspects of this work.

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