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► To cite this version:

M. Dahan. Asymmetrical loading of a penny-shaped crack in an infinite transversely isotropic medium. Journal de Physique Lettres, 1980, 41 (21), pp.495-497. 10.1051/jphyslet:019800041021049500 . jpa-00231830

HAL Id: jpa-00231830

<https://hal.science/jpa-00231830>

Submitted on 1 Jan 1980

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LE JOURNAL DE PHYSIQUE-LETTRES

J. Physique — LETTRES 41 (1980) L-495 - L-497

1^{er} NOVEMBRE 1980, PAGE L-495

Classification

Physics Abstracts

03.40D — 46.30N

Asymmetrical loading of a penny-shaped crack in an infinite transversely isotropic medium (*)

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(Reçu le 15 juillet 1980, accepté le 12 septembre 1980)

Résumé. — La distribution des contraintes et des déplacements dans un massif infini transversalement isotrope, contenant une fissure plane circulaire, est déterminée quand des forces intérieures, concentrées sur une surface, sont appliquées d'un seul côté de la fissure, à une distance finie.

Abstract. — The problem of stress and displacement fields in an infinite transversely isotropic medium is solved when a penny-shaped crack is opened by an asymmetrical system of body forces; the body forces are concentrated on one interior surface situated at a finite distance from the crack.

1. Introduction. — The problem of stress and displacement fields in an infinite medium in the presence of a symmetrical system of body forces has been solved by Sneddon and Tweed [1] for an isotropic material and by Dahan [2] for a transversely isotropic material. The problem of crack opening by an asymmetrical system of body forces, acting on one side of the crack, has been considered by Collins [3] in the isotropic case. The purpose of this paper is to give the complete solution of this problem when the medium is characterized by transverse isotropy, having the z -axis as the elastic symmetric axis. We shall assume that the loading is concentrated on a plane internal disc, around the z -axis, situated at a finite distance h from the crack.

By using the cylindrical coordinates (r, θ, z) , we shall note $(u_r, 0, u_z)$ the components of the displacement field $(\sigma_{rr}, \sigma_{\theta\theta}, \sigma_{zz}, \sigma_{rz})$ the non-zero components of the stress tensor. The penny-shaped crack is defined by its radius r_0 and it is situated in the plane $z = 0$ (cf. Fig. 1). If $p(r)$ is the loading on the plane $z = h$, the presence of body forces implies the continuity of displacements u_r, u_z and shear stress σ_{rz} and the discontinuity of normal stress σ_{zz} following the condition :

$$\lim_{\varepsilon \rightarrow 0^+} [\sigma_{zz}(r, h - \varepsilon) - \sigma_{zz}(r, h + \varepsilon)] = p(r), \quad (1)$$

where p is an arbitrary function so defined for $r \geq 0$ that the Hankel transform of order zero p^H exists.

Fig. 1. — Diagram of the problem.

Over the plane $z = 0$, the stresses and displacements are continuous in the exterior of the crack ($r > r_0$); the boundary of which is stress-free, i.e. the stresses σ_{zz} and σ_{rz} take prescribed values over the surface of the crack, so that :

$$\begin{aligned} \sigma_{zz}(r, 0) &= 0, \\ \sigma_{rz}(r, 0) &= 0, \end{aligned} \quad 0 \leq r < r_0. \quad (2)$$

The remaining boundary conditions assume that

(*) La traduction en français de cet article est soumise pour publication aux Comptes Rendus de l'Académie des Sciences.

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the components of stress and of displacement vanish as $(r^2 + z^2)^{1/2} \rightarrow \infty$.

2. **Resolution.** — In order to solve this problem, we have to divide the space in three parts Ω_i :

$$\begin{aligned}\Omega_1 &= \{ (r, z) : r \in \mathbb{R}_+, h < z < +\infty \}, \\ \Omega_2 &= \{ (r, z) : r \in \mathbb{R}_+, 0 < z < h \}, \\ \Omega_3 &= \{ (r, z) : r \in \mathbb{R}_+, -\infty < z < 0 \}.\end{aligned}\quad (3)$$

Using the results given in a previous paper [4] and the notations of a precedent Letter [2], we know that there exists a potential function of the Love type φ , defined by the components of stress tensor, such that in each part Ω_i , we have :

$$\begin{aligned}\varphi(r, z) &= \int_0^\infty [A_i(m) e^{+s_1 m z} + B_i(m) e^{+s_2 m z} + C_i(m) e^{-s_1 m z} + D_i(m) e^{-s_2 m z}] J_0(mr) m \, dm, \\ (r, z) &\in \Omega_i \quad i = 1, 2, 3.\end{aligned}\quad (4)$$

If we take into account the boundary conditions on the plane $z = h$, the condition at infinity and the continuity of the stresses σ_{zz} and σ_{rz} over the plane $z = 0$, we have the following relations between the twelve functions A_i, B_i, C_i, D_i :

$$\begin{aligned}A_1(m) &= B_1(m) = C_3(m) = D_3(m) = 0, \\ A_2(m) &= e^{-ms_1 h} p^H(m) / [2 m^3 ds_1(s_1^2 - s_2^2)], \\ B_2(m) &= e^{-ms_2 h} p^H(m) / [2 m^3 ds_2(s_2^2 - s_1^2)], \\ C_1(m) &= e^{+ms_1 h} p^H(m) / [2 m^3 ds_1(s_1^2 - s_2^2)] + C_2(m), \\ D_1(m) &= e^{+ms_2 h} p^H(m) / [2 m^3 ds_2(s_2^2 - s_1^2)] + D_2(m), \\ A_3(m) &= A_2(m) + \frac{s_1 + s_2}{s_1 - s_2} C_2(m) + \frac{2 s_1}{s_1 - s_2} \cdot \frac{p_2}{p_1} D_2(m), \\ B_3(m) &= B_2(m) + \frac{s_2 + s_1}{s_2 - s_1} D_2(m) + \frac{2 s_2}{s_2 - s_1} \cdot \frac{p_1}{p_2} C_2(m).\end{aligned}\quad (5)$$

We have to determine the functions C_2 and D_2 . For this, by using the equations (2) and the continuity of the displacements in the exterior of the crack for $z = 0$, we obtain a system of four integral equations :

$$\begin{aligned}\int_0^\infty \left[s_1 g_1 C_2(m) + s_2 g_2 D_2(m) - (g_1 e^{-ms_1 h} - g_2 e^{-ms_2 h}) \frac{p^H(m)}{2 m^3 d(s_1^2 - s_2^2)} \right] m^4 J_0(mr) \, dm &= 0, \quad 0 < r < r_0, \\ \int_0^\infty [s_2 p_1 C_2(m) + s_1 p_2 D_2(m)] m^3 J_0(mr) \, dm &= 0, \quad r_0 \leq r < \infty, \\ \int_0^\infty \left[p_1 C_2(m) + p_2 D_2(m) + (p_1 s_2 e^{-ms_1 h} - p_2 s_1 e^{-ms_2 h}) \frac{p^H(m)}{2 m^3 \sqrt{d}(s_1^2 - s_2^2)} \right] m^4 J_1(mr) \, dm &= 0, \quad 0 < r < r_0, \\ \int_0^\infty [p_1 C_2(m) + p_2 D_2(m)] m^3 J_1(mr) \, dm &= 0, \quad r_0 \leq r < \infty.\end{aligned}\quad (6)$$

This system can be solved if we notice the last two equations are two dual integral equations of the unknown function $p_1 C_2 + p_2 D_2$. Having determined this function, then the first two equations are dual integral equations of the function C_2 or D_2 . Thus, the solution of the system (6) is given by :

$$\begin{aligned}C_2(m) &= [s_1 I(m) - s_1 J(m) - K(m)] / m^3 p_1 (s_1^2 - s_2^2), \\ D_2(m) &= [s_2 I(m) - s_2 J(m) - K(m)] / m^3 p_2 (s_2^2 - s_1^2),\end{aligned}\quad (7)$$

where the functions I, J, K depend on the loading and the geometrical parameters such that :

$$\begin{aligned}I(m) &= \frac{1}{\pi \sqrt{d}(s_1 - s_2)} \cdot \frac{\sin(mr_0)}{mr_0} \int_0^\infty (s_2 p_1 e^{-as_1 h} - s_1 p_2 e^{-as_2 h}) p^H(\alpha) \sin(\alpha r_0) \frac{d\alpha}{\alpha}, \\ J(m) &= \frac{1}{\pi \sqrt{d}(s_1 - s_2)} \int_0^{r_0} \left[\int_0^\infty (s_2 p_1 e^{-as_1 h} - s_1 p_2 e^{-as_2 h}) p^H(\alpha) \cos(\alpha t) \, d\alpha \right] \cos(mt) \, dt,\end{aligned}\quad (8)$$

$$K(m) = \frac{1}{\pi d^{3/2}(s_1 - s_2)} \int_0^{r_0} \left[\int_0^\infty (g_1 e^{-\alpha s_1 h} - g_2 e^{-\alpha s_2 h}) p^H(\alpha) \sin(\alpha t) d\alpha \right] \sin(mt) dt.$$

3. **Stress intensity factors.** — For further discussions interesting the Fracture Mechanics, we can calculate the different stress intensity factors defined by the limits :

$$\begin{aligned} k_1 &= \lim_{r \rightarrow r_0^+} [2 \pi(r - r_0)]^{1/2} \sigma_{zz}(r, 0), \\ k_2 &= \lim_{r \rightarrow r_0^+} [2 \pi(r - r_0)]^{1/2} \sigma_{rz}(r, 0), \\ k_3 &= \lim_{r \rightarrow r_0^+} [2 \pi(r - r_0)]^{1/2} \sigma_{\theta z}(r, 0) = 0. \end{aligned} \quad (9)$$

From these definitions and relations (5) and (7), we deduce :

$$\begin{aligned} k_1 &= \frac{-1}{\sqrt{\pi r_0} d(s_1^2 - s_2^2)} \int_0^\infty (g_1 e^{-ms_1 h} - g_2 e^{-ms_2 h}) p^H(m) \sin(mr_0) dm, \\ k_2 &= \frac{1}{\sqrt{\pi r_0} d(s_1^2 - s_2^2)} \int_0^\infty (s_2 p_1 e^{-ms_1 h} - s_1 p_2 e^{-ms_2 h}) \left[\frac{\sin(mr_0)}{mr_0} - \cos(mr_0) \right] p^H(m) dm. \end{aligned} \quad (10)$$

Closed form solutions can be obtained for usual loadings of the cracked solid (uniform loads over a disc, point concentrated force...). For a point force ($p^H = P/2\pi$), the expressions obtained through a limit operation agree with the results of the only existing isotropic solution given by Kassir and Sih [5].

Moreover, this solution makes possible to calculate the stress intensity factors in the case of two loads p_1 and p_2 applied on each side of the crack at distances h_1 and h_2 . From the principle of superposition, we

have the new factors by using for each k_i the expressions (10) :

$$\begin{aligned} k_1 &= k_1(p_1, h_1) + k_1(p_2, h_2), \\ k_2 &= k_2(p_1, h_1) - k_2(p_2, h_2). \end{aligned} \quad (11)$$

For equal and symmetrically spaced loads, we remark that the factor k_1 is double the value given by (10) and the factor k_2 is zero. These expressions agree with the results obtained in the Letter mentioned above [2].

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