LE JOURNAL DE PHYSIQUE-LETTRES

J. Physique — *LETTRES* 41 (1980) L-341 - L-345

1^{er} AOÛT 1980, PAGE L-341

Classification
Physics Abstracts
 47.25C — 47.25Q

Intermittency in Rayleigh-Bénard convection

P. Bergé, M. Dubois, P. Manneville and Y. Pomeau (*)

DPh-G/PSRM, CEN-Saclay, Boîte Postale 2, 91190 Gif-s/Yvette, France

(Reçu le 13 mars 1980, accepté le 13 juin 1980)

Résumé. — Nous présentons la première évidence expérimentale d'une transition vers la turbulence par intermittence dans une expérience de convection à nombre de Prandtl élevé et en géométrie confinée. Il est possible d'interpréter qualitativement cette transition à partir d'un modèle qui se déduit de résultats récents obtenus sur le système de Lorenz.

Abstract. — Experimental evidence of a transition to turbulence *via* intermittency is reported for a convection experiment of a high Prandtl number fluid in confined geometry. This transition is qualitatively understood in terms of a model derived from recent results on the Lorenz system.

1. Introduction. — Turbulence can appear in a system after a small number of bifurcations. This is now a well established fact both from experimental evidences [1] and from theoretical considerations [2]. A typical example is given by the Rayleigh-Bénard convective instability of a fluid heated from below, particularly in the case of confined geometries i.e. when the ratio of the horizontal extension to the height of the fluid layer, the so-called *aspect ratio*, is small enough. Increasing the Rayleigh number R_a which is the relevant control parameter, one observes a cascade of instabilities, each step of which adds a new complication to the convective behaviour. After the onset of the stationary convection at R_a^c , the first manifestation of time dependence generally occurs in the form of an oscillatory regime characterized by a remarkable time periodic behaviour of all the convective parameters [3, 4], velocity components [5] and temperature fluctuations [6]. Fourier analysis reveals a unique frequency f_1 (and its harmonics). From this point on, the different routes to turbulence begin to diverge according to the convective pattern and to the Prandtl number essentially. One possible way involves the appearance of a second frequency f_2 in the spectrum [1c, 7, 13]. In this paper we shall report on another route which involves only one fundamental frequency f_1 and which has been observed in a high Prandtl number fluid. Temporal chaos sets in continuously and manifests itself under the form of (apparently) random *bursts* interrupting quite regular *lami-*

nar phases. A similar *intermittent* phenomenon has also been observed in low Prandtl number fluids [6b, 7].

Since the work of Lorenz [8] and Ruelle [2], theories of the transition to stochasticity in dynamical systems used to relate chaotic temporal behaviour with global unpredictability of otherwise deterministic systems (i.e. systems governed by ordinary differential equations or iterative transformations which are locally deterministic), this unpredictability being the result of the intrinsic instability of trajectories which belong to *strange attractors* [9]. The formal framework of these theories based on genericity arguments is in fact sufficiently versatile to allow for different possible transitions. One of these is the transition *via intermittency* just sketched above. In the following we shall present an interpretation of experimental findings which relates them directly to our present knowledge of the subject gained mainly by studying the Lorenz [10] system and other even simpler models [11].

2. Experimental results. — The convective fluid is silicone oil with a viscosity $\nu = 0.1$ St at 25 °C and Prandtl number 130. The cell is rectangular with height $d = 2$ cm, horizontal aspect ratios $\Gamma_x = L_x/d = 2$ and $\Gamma_y = L_y/d = 1.2$; for details on the set-up see reference [13]. Two types of measurements have been performed simultaneously : i) laser Doppler anemometry gives us the local vertical velocity component and ii) a differential interferometric method provides us vertical temperature gradients $(\nabla_z T)$ projected on the xz and yz planes ; in this latter case the information being no longer local but integrated along a

(*) DPh-T, Orme des Merisiers.

direction gives an overall picture of the structure at any time.

Even in small cells such as the one used here, several different convective structures are possible at high Rayleigh number. In particular, we have observed two types of structure, which can be schematically described as the superposition of two rolls along the X -direction with either one or with two rolls along the Y -direction. In the first type of structure (one Y roll), the time-dependent phenomena are related to periodic instabilities in the cold boundary layer leading to the presence of one, then two frequencies in the corresponding spectra of the convective behaviour [12]. Results we present in the following are all related to the second structure which, though not exactly symmetrical, can be schematically described by two rolls along the X -direction and two rolls along the Y -direction. For that kind of structure the mechanism responsible for the time dependent regime (Fig. 1A) which sets in at $R_a/R_a^c = 250$, is the periodic formation, advection and thermal relaxation of a hot (or cold) droplet in a corner of the box [5]. The corresponding spectrum consists of the fundamental frequency f_1 and its harmonics only. When the Rayleigh number increases, the subharmonic $f_1/2$ appears in the spectrum, which corresponds to the fact that one droplet over two contains a greater thermal perturbation. We have to note that, all our experimental data show that, whatever the observed route to turbulence, there is a one to one correspondence between the particular route followed and the corresponding structure.

Increasing further the Rayleigh number, one observes a qualitatively new behaviour : from time to time, the amplitude of the velocity oscillations — see figure 1B where the velocity is measured near the centre of the cell — is strongly perturbed and then relaxes toward the *normal* oscillatory regime (that

we call *laminar phase* for convenience). The time lag between two perturbations seems to be random but, when the Rayleigh number increases beyond the threshold R_a^I for this intermittent behaviour at $R_a/R_a^c \sim 290$, i) the mean duration of laminar phases shortens and ii) the statistical dispersion of these durations increases. Looking at the interferometric images one does not see any fundamental deviation from the basic picture sketched above, except perhaps small fluctuations of the overall circuit followed by a *normal* droplet during its growth and advection. Strong perturbations of the amplitude of the oscillations are associated with the occurrence of *giant* droplets which have roughly the same fate as *normal* droplets. The structure being more strongly distorted then relaxes to the regular oscillatory regime. That the giant droplet may result from a cumulative effect of fluctuations on normal droplets will be analysed further. At even larger Rayleigh number ($R_a/R_a^c > 310$) the local intermittent perturbations show a more chaotic nature, resembling bursts of turbulence (Fig. 1C) and the corresponding spectrum of the velocity fluctuations shows a *noise* at low frequencies superimposed to the somewhat enlarged peaks given by the oscillations.

3. Theoretical interpretation. — A detailed quantitative interpretation is clearly out of reach, even from the simplified point of view of dynamical systems with few degrees of freedom which are believed to contain most of the mechanics of the transition to turbulence in confined geometry. Indeed the drastic simplifications concerning the geometry, the boundary conditions, the order of truncation, etc... impede the construction of a realistic differential system relevant to the experiment reported above. Anyway, this unknown realistic dynamical systems should share some generic features with already well studied models and it has

Fig. 1. — Time dependence of the vertical velocity component v_z measured at the centre of the cell ; A : $R_a/R_a^c = 270$; B : $R_a/R_a^c = 300$; C : $R_a/R_a^c = 335$ where $R_a^c = 1\,700$ is the critical Rayleigh number at the onset of convection in infinite geometry.

seemed more promising, i) to gain as much information as possible from these models and more especially from the Lorenz system [8, 10] and ii) to build abstract models displaying these generic features most susceptible of *explaining* the experimental observations [11].

In the Lorenz system [10], as well as in the present experimental case, one follows the destabilization of a limit cycle and the Poincaré map [14] is the best tool to understand the phenomenon. It consists of a series of successive intersections of the trajectories in phase space with a well chosen (hyper)-surface, which gives a kind of stroboscopic vision of the motion. From a general point of view, the structure of a strange attractor corresponding to an aperiodic regime is quite complicated. Known examples are the product of a manifold by a Cantor-like set [16]. If one can forget the transverse cantor-like fine structure, then one can restrict oneself to consider a *reduced* Poincaré map which takes the form of a simple iteration on the coordinates which parametrize the manifold. Such a circumstance precisely occurs in the Lorenz model and allows for the quite simple picture of the transition to turbulence [10] summarized in figure 2.

Fig. 2. — A stable limit cycle is associated with a fixed point of the Poincaré map. For the Lorenz model [11] it turns out that in the vicinity of this fixed point the graph of the reduced map is roughly parabolic and that increasing the control parameter r comes to translating the curve upwards (Fig. 2A). Then below the intermittency threshold $R_I = 166.07$, one has two fixed points one stable the other unstable. They collapse at $R = R_I$ and disappear completely above R_I . In the intermittent regime $R > R_I$, the system is allowed to visit from time to time the ghost of the limit cycle; to this *laminar phase* of the motion corresponds a slow drift of the successive iterates in a kind of channel which opens between the first bisetrix and the parabolic curve (Fig. 2B). Then the iterates are expelled from the vicinity of the ghost of the fixed point of the *channel* (the *turbulent burst*) and further recycled toward the entrance.

The Poincaré map is a theoretician's tool, the disposal of which we do not have in a concrete experiment. However we can get a similar stroboscopic vision of the motion (capable of pointing out a trend in the evolution of fluctuations affecting the periodic motion of the droplets) in examining a clearly identifiable feature of the oscillatory behaviour. Such a feature can be for example the minimum M of the velocity variations at a point x_0 recorded in figure 1,

which would correspond to a projection of the Poincaré map with the surface of section given by the condition $\frac{d}{dt} v_z(x_0, t) = 0$. The procedure then consists in plotting a given minimum M_n against the precedent one M_{n-1} or, more judiciously since a subharmonic bifurcation has already taken place, against the minimum M_{n-2} . Figure 3 displays two typical *laminar*

Fig. 3. — *Reduced Poincaré map* extracted from experimental results of figure 1. A minimum M_n of v_z measured at the centre of the cell is plotted against M_{n-2} . Two typical sequences beginning at the end of a *turbulent burst* and ending at the beginning of the next one are plotted, one long (+) and the other short (.); a stable limit cycle would correspond to a clustering of points at a given place on the first bisetrix. Here the upper branches of the cusp shaped graphs correspond to the relaxation of the structure toward what resembles a limit cycle while the lower branches describe the escape phase.

phases. It confirms the view of a behaviour which is deterministic in the short term, since the points organize themselves along a definite pattern hardly disturbed by experimental noise. Now let us assume that intermittency here is of the simplest conceivable type, namely that which occurs in the Lorenz model. The point to understand is then the cusp shape of the *Poincaré map* when compared to figure 2. Noticing that in the Lorenz model the convergence towards the ghost of the limit cycle is rather rapid (see Fig. 2 of Ref. [10]) while here it is much slower (see Fig. 1), gives a hint to construct a model which allows for a qualitative understanding of the behaviour depicted in figure 3 and of its generic meaning. The idea is then to add explicitly a weakly stable direction to the unstable one. The starting point will be the well known *Baker's Transform* of the square [16] $[0,1] \times [0,1]$ defined by $x \rightarrow 2x$, $y \rightarrow y/2$ for $x < 1/2$ and $x \rightarrow 2x - 1$, $y \rightarrow (y + 1)/2$ for $x > 1/2$. The divergence of nearby trajectories and the sensitivity to initial conditions, typical of a *mixing* system and required for unpredictability, are contained in the

expanding transformation on x (which then parametrizes the unstable manifold) while the transformation on y , which insured the area preserving property, has now to be modified to account for the contraction in the stable manifold direction [15]. The transition from the stable limit cycle (fixed point of the Poincaré map) to the aperiodic trajectories is then obtained by a continuous deformation of the expanding map $x \rightarrow 2x$. Finally we retain :

$$x \rightarrow 2x, y \rightarrow ky \quad \text{for } x < 1/2$$

$$x \rightarrow 2x - 1 - \alpha \sin 2\pi x + \beta \sin 4\pi x, \quad y = ky + 1 - k$$

for

$$x > 1/2$$

with $k < 1/2$ to insure contraction, $\beta = 0.1$ and α variable around 0.25 ; the critical value is $\alpha_c \simeq 0.247$, the fixed points in the map disappear for $\alpha > \alpha_c$. A slightly more complicated (though equally generic) transformation on x would even allow to reproduce the sequence, subharmonic bifurcation $\rightarrow$ intermittency, which then gets a general meaning. Two typical trajectories in phase space are presented in figure 4. If the experiment could give a quantity directly linked to the x -coordinate (unstable direction) one would obtain the picture of figure 2, but this is not the case and one rather follows a projection of the motion along another direction. One easily sees from figure 4 that the projection in a direction of the form $u = y + \varepsilon x$ ($\varepsilon > 0$ and small) will lead to an iteration, the graph of which presents a striking similarity with experimental results, more specifically the existence of the cusp and of abortive visits to the ghost of the fixed point (see Fig. 5).

4. Conclusion. — Of course a complete quantitative account is out of reach but on the basis of quite general arguments, theory touches on several other points such as the mean duration of laminar phases or the decrease of correlation functions. Experiments are underway to further check these predictions. Finally recent abstract theories could appear at first sight only as conjectures on how things happen but contributions such as the work presented in this paper tend to confirm their general validity up to a quasi-quantitative level [17].

Fig. 4. — Two typical trajectories visiting the ghost of the fixed point of the modified *Baker's Transform* at $x \simeq 0.726$, $y = 1$. The first one (+) passes in its immediate vicinity ; we call the second one (.) an abortive visit since it is much shorter.

Fig. 5. — The projection of the mapping in a direction $u = y + \alpha x$ (here $\alpha = 0.2$) for the two trajectories of figure 4 should be compared with the experimental ones (Fig. 3). The general aspect is similar and forgetting the experimental dispersion, one can even distinguish between the long laminar phase (+) and the abortive one (.).

References

- [1] a) AHLERS, G. and BEHRINGER, R. P., *Prog. Theor. Phys. Suppl.* **64** (1978) 186 ;
 b) FENSTERMACHER, P. R., SWINNEY, H. L., BENSON, S. A., GOLLUB, J. P., *Ann. N.Y. Acad. Sci.* **316** (1979) 652.
 c) MAURER, J. and LIBCHABER, A., *J. Physique Lett.* **40** (1979) L-419.
 d) BERGÉ, P., in *Dynamical Critical Phenomena and Related Topics*, C. P. Enz ed. (Springer Verlag, Berlin) 1979.
- [2] RUELLE, D., TAKENS, F., *Commun. Math. Phys.* **20** (1971) 167.
- [3] KRISHNAMURTI, R., *J. Fluid Mech.* **60** (1973) 285.
- [4] BUSSE, F. H., WHITEHEAD, J. A., *J. Fluid Mech.* **66** (1974) 67.
- [5] BERGÉ, P., DUBOIS, M., *J. Physique Lett.* **40** (1979) L-505.
- [6] a) LIBCHABER, A., MAURER, J., *J. Physique Lett.* **39** (1978) L-369 ;
 b) LIBCHABER, A., MAURER, J. (private communication).

- [7] GOLLUB, J. P., BENSON, S. A., *Phys. Rev. Lett.* **41** (1978) 948 and submitted to *J. Fluid Mech.*
 - [8] LORENZ, E. N., *J. Atmos. Sci.* **20** (1963) 130.
 - [9] POMEAU, Y., in *Intrinsic Stochasticity in Plasmas*, G. Laval and D. Gresillon ed. (Editions de Physique, Orsay) 1979.
 - [10] MANNEVILLE, P. and POMEAU, Y., *Phys. Lett.* **75A** (1979) 1 and to be published in *Physica D* (Non Linear Physics).
 - [11] POMEAU, Y. and MANNEVILLE, P., cf. ref. [10] and to be published in *Commun. Math. Phys.*
 - [12] DUBOIS, M. and BERGÉ, P., *Phys. Lett. A* **76** (1980) 53.
 - [13] DUBOIS, M. and BERGÉ, P., *J. Fluid Mech.* **85** (1978) 641.
 - [14] IOOSS, G., *Bifurcation of Maps and Applications*, Math. Studies n° 36 (North Holland, Amsterdam and N.Y.) 1979.
 - [15] MANDELBROT, B., *Fractals Form Chance and Dimension* (Freeman and Co, San Francisco) 1977.
 - [16] ARNOLD, V. I., AVEZ, A., *Problèmes Ergodiques de la Mécanique Classique* (Gauthier-Villars, Paris) 1967.
 - [17] Cf. for example FEIGENBAUM, M. J., *Phys. Lett.* **74A** (1979) 375.
-