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A model for exchange in liquid ^3He (*)

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Résumé. — Dans cette note, on remarque que l'on peut décrire le ^3He liquide normal en attribuant la dépendance linéaire avec la température T de la chaleur spécifique C_v , à la présence des systèmes à deux niveaux que sont les spins des atomes (on suppose donc que les polarisations $+$ et $-$ de chaque spin ont, pour une raison non encore éclaircie, des énergies différentes). On obtient alors pour le rapport $C_v/\chi T$ (χ : susceptibilité) une valeur proche de la valeur expérimentale. Plus encore, pour corriger la susceptibilité légèrement trop forte ainsi obtenue, on peut introduire un *champ moléculaire* anti-ferromagnétique. Celui-ci est alors l'extrapolation presque parfaite de sa valeur dans le solide.

Abstract. — In this letter, I remark that normal liquid ^3He can be described as a set of two levels systems, which are the spins of atoms (thus I assume that, for an unknown reason, $+$ and $-$ polarization of each spin have different energies). The $C_v/\chi T$ ratio thus obtained is close to the experimental value (χ : susceptibility, C_v : specific heat, proportional to the temperature T). Moreover, the antiferromagnetic exchange field one can introduce to fit this ratio is almost exactly the extrapolation of the solid one.

1. Introduction. — Normal liquid ^3He is a Fermi liquid. It means, following Landau's theory [1], that its macroscopic and low frequency behaviour is that of an imperfect gas of quasi-particles. These quasi-particles are spin $1/2$ fermions as ^3He atoms. They have fictitious interactions, induced by the strong interaction between ^3He atoms. It results in a modified density of states at the Fermi level, which renormalizes the specific heat C_v (increasing linearly with the temperature T). The susceptibility χ is further enhanced, because of the spin dependence of these fictitious interactions. Increasing the pressure, the $C_v/\chi T$ ratio goes from 3 to 4 times its perfect gas value.

This strong susceptibility is generally expected to be due to the proximity of a ferromagnetic instability [2]. However, the strong localization of the atoms (proximity of solidification) can give the same result, by decoupling the spins [3]. In this letter this idea is developed, in a glass-like approach [4]; the classical two level systems are in this model the spin states of each atom. It may appear, at first sight, that the only result is to replace two Landau parameters by two

energies : $k_B T_0$ and $k_B \theta$. The interesting point is that they correspond to characteristic energies of the solid phase. Near freezing point, θ is close to the Néel temperature of the solid and $k_B T_0$ to the vacancy band width. In this context one can say that, without any adjustable parameter, this model predicts a $C_v/\chi T$ ratio, in good agreement with its experimental value. Another advantage is to put a new light on the solidification tendency in liquid ^3He .

2. The model. — Let us start from the following assumptions :

a) The $+$ and $-$ spin states of each atom gave different energies $+E$ and $-E$.

b) If $f(\varepsilon) d\varepsilon$ is the number of atoms for which E is between ε and $\varepsilon + d\varepsilon$, $f(0)$ is different from zero.

Without the second hypothesis, as we will see, the specific heat would not be proportional to T [4]. We do not consider any other excitation of the liquid, assuming that its entropy at $T = T_F$ is entirely due to spin disorder. This is probable, the phonon characteristic Debye temperature being much higher than T_F .

A similar model has been introduced by Blandin and Friedel [5], in order to explain the properties of magnetic alloys (spin glasses).

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In such a mean field theory, the spin correlations are neglected. The entropy is thus the sum of the individual spin entropies in their local *field* E . Thus the specific heat and the susceptibility are also sums of the individual corresponding terms. At low temperature :

$$C_v = \int_{-\infty}^{+\infty} \frac{\varepsilon^2}{k_B^2 T^2} f(\varepsilon) \frac{1}{\cosh^2(\varepsilon/k_B T)} d\varepsilon$$

$$\simeq f(0) k_B^2 T \int_{-\infty}^{+\infty} \frac{x^2 dx}{\cosh^2 x} = \frac{\pi^2}{6} f(0) k_B^2 T.$$

An antiferromagnetic molecular field is present in the solid state, due to exchange between ^3He atoms. Calculating the susceptibility of the liquid, we introduce *a priori* such a field, translating the change in the distribution $f(\varepsilon)$ due to the polarization. The magnetization due to an external field H is :

$$M = -\mu \int_{-\infty}^{+\infty} f(\varepsilon) d\varepsilon \tanh\left(\frac{\varepsilon - (\mu H - k_B \theta m)}{k_B T}\right)$$

$M = Nm\mu$ (N atoms of magnetic moment μ).

The susceptibility χ is thus given by :

$$\chi = \frac{\mu}{k_B T} \left(\mu - \frac{k_B \theta}{N\mu} \chi \right) \int_{-\infty}^{+\infty} f(\varepsilon) \frac{d\varepsilon}{\cosh^2(\varepsilon/k_B T)}$$

$$\simeq 2\mu^2 f(0) - \frac{2k_B \theta}{N} f(0) \chi.$$

Putting $N/2 k_B f(0) = T_0$ we obtain :

$$\chi = \frac{N\mu^2}{k_B(T_0 + \theta)}.$$

The corresponding formulas in Landau's [1] theory are :

$$C_v = \frac{\pi^2}{3} k_B^2 T N(E_F)$$

$$\chi = \mu^2 N(E_F)/(1 + F_0^a)$$

where $N(E_F)$ is the density of states at the Fermi level.

By identification :

$$1 + F_0^a = \frac{1}{4} \left(1 + \frac{\theta}{T_0} \right).$$

In the experimental tables, one finds the value of :

$$T_F = \frac{N\mu^2}{k_B \chi} = T_0 + \theta.$$

Thus :

$$\theta = \frac{3 + 4 F_0^a}{4(1 + F_0^a)} T_F.$$

On figure 1 we represent θ versus the molar volume V . We take two series [6, 7] of experimental values for

Fig. 1. — θ dependence versus molar volume (logarithmic scales). These values are computed from the data of Refs. [6] : ● and [7] : Δ. The Néel temperature of the solid is given by the full line and its extrapolation by the dashed line.

T_F and F_0^a . The main difference is for F_0^a due to the small number of published measurements of liquid ^3He specific heat versus temperature and pressure. Anyway, they are in agreement on the main point, which is that the θ value at high pressure goes towards the Néel temperature of the solid. This temperature being much smaller than T_F , this model explains well why F_0^a goes towards -0.75 at high pressure.

3. Discussion. — A theory exists which is very useful for normal liquid ^3He : it is the paramagnon theory. In its framework one can predict [2] the temperature dependence of the susceptibility, without adjustable parameter and in good agreement with the experiments.

The specific heat, however, depends on an adjustable parameter, independent of the susceptibility.

It would thus be very interesting to know whether or not our model is compatible with this theory. It is difficult to answer, the point of view being so different. In the paramagnon theory, liquid ^3He is a gas of ^3He atoms, interacting mainly *via* spin fluctuations. In our model, the atoms are nearly localized. The nature of the quasi-particles is not specified ; they are probably similar to the vacancies in the solid. However in both models, while indirectly in the paramagnon case, the strong specific heat is mainly due to magnetic excitations.

Let us remark an experimental result, qualitatively in agreement with our model, but probably also with the paramagnon theory : the Prandtl number of liquid ^3He , which is close to $P \simeq 0.4$ at low pressure and high temperature (3 K), becomes $P \simeq 12$ [7] at low temperature, near the solidification. The Prandtl number is the ratio between the kinematic

viscosity (the momentum diffusivity) and the heat diffusivity. This seems coherent with a model where the energy carrying excitations are different from those carrying momentum. Liquid mercury is an opposite example, where the energy is carried by the electrons and the momentum by the phonons ($P \simeq 0.04$). In a gas, the molecules carry both energy and momentum, and P is always of order 1.

4. Conclusion. — This model first assumes that the liquid ^3He entropy is mainly due to the spin disorder,

secondly, that there is a sufficiently small correlation between spins to use a mean field approximation. Its main justification lies in its results.

First it explains, without adjustable parameter, why the Landau parameter F_0^a goes toward -0.75 when one approaches solidification; secondly, the molecular field one has to introduce is the extrapolation of the solid one. The magnetic properties of the liquid thus appear to be the extrapolation of the solid ones, and that was not clear in the preceding theories [8].

References

- [1] See for instance :
NOZIERES, P., *Le problème à N corps* (Dunod) 1963.
- [2] BEAL-MONOD, M. T., SHANG-KENG MA, FREDKIN, D. R.,
Phys. Rev. Lett. **20** (1968) 929.
- [3] CASTAING, B. and NOZIERES, P., *J. Physique* **40** (1979) 257.
- [4] See for instance :
DOUSSINEAU, F., *Revue Phys. Appl.* **12** (1977) 809.
- [5] BLANDIN, A. and FRIEDEL, J., *J. Physique Radium* **20** (1959) 160.
- [6] RAMM, H., PEDRONI, P., THOMPSON, J. R., MEYER, H., *J. Low Temp. Phys.* **2** (1970) 539.
- [7] WHEATLEY, J. C., *The Helium Liquids* (Academic Press) 1975, p. 310.
- [8] In his liquid ^3He theory, GOLDSTEIN, L. (*Phys. Rev.* **96** (1954) 1455) introduces a spin entropy S_σ proportional to the susceptibility : $S_\sigma = k_B^2 \ln 2 \frac{T}{\mu^2} \chi(T)$. The residual entropy $S_{n\sigma}$ would be due to positional disorder. Putting $S_{n\sigma} = 0$, one finds $F_0^a = -0.79$. Thus the adjustable quantity $S_{n\sigma}$ cannot be neglected at any pressure.