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Semiempirical calculations of shell model states in ^{212}Rn

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Résumé. — En utilisant l'algèbre standard des moments angulaires et les énergies d'interaction empiriques, c'est-à-dire les énergies d'interaction à 2 corps extraites des niveaux d'énergie expérimentaux connus du ^{210}Po , on étudie les propriétés des états du ^{212}Rn . Les énergies d'excitation, les périodes et moments magnétiques sont donnés. L'effet de la troncation dans l'espace des protons est examiné. On trouve que les données sont bien vérifiées par l'expérience.

Abstract. — Using standard angular momentum algebra and empirical interaction energies, i.e. two-nucleon interaction energies extracted from experimentally determined levels in ^{210}Po , properties of valence nucleon states in ^{212}Rn are calculated. Excitation energies and some half-lives and magnetic moments are given. The effect of the truncation of the configuration space is discussed. Comparison with experimentally determined properties gives good agreement.

1. Introduction. — The shell model predictions of nuclear structure have met with the greatest success for nuclei in the regions where N or $Z = 20, 28, 50, (64), 82$ or 126 . Here the states can well be depicted as few-nucleon configuration plus a *closed* core. Such core nuclei are e.g. ^{16}O , ^{40}Ca and ^{208}Pb . The *closure* mentioned above is a relative concept and retains some meaning only for cases where the energy of the first excited state of a core-nucleus is well above the single nucleon orbital spacing. Such is the case for ^{208}Pb where its first excited state, the collective octupole vibration (3^-) is located at 2.6 MeV whereas e.g. the first proton orbitals in ^{209}Bi are situated at 0.9 and 1.6 MeV, respectively. Thus ^{209}Bi can be depicted as proton orbitals outside a *closed* core. The problem of 209 particles is thus reduced to $208 + 1$, where the 208 first ones represent a zero-energy, $J^\pi = 0^+$, vacuum state.

When e.g. protons are added we get the $N = 126$ isotones ^{209}Bi , ^{210}Po , ^{211}At , ^{212}Rn , etc. In all these nuclei it has been experimentally verified [1] that the lower-lying states are formed from valence nucleons.

In order to study the structure of many proton states and to check the limitations of configurational purity, we have undertaken a calculation of valence proton states in ^{212}Rn . The lower-lying states in this nucleus are thus supposed to be made up from four-

proton configurations. As is seen in the discussion, this assumption is well verified. It is also seen that the yrast states agree with experiments to an astonishingly high degree.

2. Computational procedure. — For the proton orbitals outside the $Z = 82$ shell closure, the level ordering is established [2]. We retain the three first proton orbitals, partly to reduce the configuration space and partly because we are mostly interested in yrast states (an yrast level is that level of a given J that has the lowest excitation energy). We thus calculate four-proton configurations of the following types : $h_{9/2}^4$, $h_{9/2}^3 f_{7/2}$ and $h_{9/2}^3 i_{13/2}$. Configurations of e.g. the type $h_{9/2}^2 f_{7/2} i_{13/2}$ would meet with the problem that the two-proton interaction $f_{7/2} i_{13/2}$ is not empirically known.

The calculational procedure closely follows that introduced by I. Talmi for light nuclei [3] and applied to the lead region by J. Blomqvist [4].

We have a Hamiltonian of the form

$$H = \sum_{i=1}^4 H_0(i) + \sum_{i<j} H_0(i,j) + \sum_{i<j<k} H_0(i,j,k) +$$

$$+ (\text{higher order correlations})_0$$

$$+ \sum_{i=1}^4 H_{\text{pol}}(i) + \sum_{i<j} H_{\text{pol}}(i,j)$$

$$+ \sum_{i<j<k} H_{\text{pol}}(i,j,k) + (\text{hoc})_{\text{pol}} \quad (1)$$

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where i, j, k run over available proton orbitals. The terms H_0 are the *bare* interaction energies whereas the H_{pol} are energies resulting from the polarization of the ^{208}Pb core.

Taking the terms pair-wise, e.g.

$$H_0(i) + H_{\text{pol}}(i)$$

we get energies which allow for core-correlations. These are represented by the empirical single-nucleon energies (proton energies in ^{209}Bi) and empirical two-nucleon energies (two-proton energies in ^{210}Po). The three-body effective energies are shown to be small [5], and we tacitly assume the higher-order correlations to be even smaller. Our Hamiltonian thus reduces to the expression

$$H = \sum_{i=1}^4 H(i) + \sum_{i<j} H(i, j). \quad (2)$$

Since the calculation is based on the assumption that the protons move in the average field of ^{208}Pb , this core thus represents the vacuum and its energy is set to zero. We have the following relation

$$E(j_i, ^{209}\text{Bi}) = -S_p(^{209}\text{Bi}) + \varepsilon_{j_i} \quad (3)$$

where $S_p(^{209}\text{Bi})$ is the proton separation energy in ^{209}Bi and ε_{j_i} is the excitation energy of orbital j_i in ^{209}Bi [2]. The two-proton interaction energy is extracted from ^{210}Po (Ref. [1b]) using the expression

$$\Delta_J(i, j) = E((j_i j_j)^J, ^{210}\text{Po}) - S_{2p}(^{210}\text{Po}) - 2 E(j_i, ^{209}\text{Bi}).$$

We thus get the excitation energy of levels in ^{212}Rn by adding all interaction energies Δ_J to the zero-order proton energies $E(j_i, ^{209}\text{Bi})$ and subtracting the ground state energy of ^{212}Rn relative to ^{208}Pb , viz.

$$E_{\text{exc}}^J(^{212}\text{Rn}) = \sum_{i=1}^4 E(j_i, ^{209}\text{Bi}) + \sum_{i<j} \Delta_J(i, j) - E_g(^{212}\text{Rn}). \quad (4)$$

The ground state energy of ^{212}Rn can be calculated as

$$E_g(^{212}\text{Rn}) = -S_{2p}(^{212}\text{Rn}) - S_{2p}(^{210}\text{Po}).$$

We use the proton separation energies, since they are found in tables [6].

The reduction of the $h_{9/2}^4$ configurations to $h_{9/2}^2$ -pairs gives coefficients of fractional parentage (*cfp*) and the recoupling gives 6j-symbols. The *cfp*'s are given in the Appendix of ref. [3] and 6j-symbols have been calculated for each case.

Using the wavefunctions calculated above we also calculate some allowed transition probabilities, i.e.

for E2 and E3 multipoles. The reduced transition probability can be written as [3]

$$B(EL\downarrow) = C_i C_j (2j_f + 1)(2l_f + 1)(2l_i + 1) \frac{(2L + 1)}{4\pi} \times \left\{ \begin{matrix} l_f & j_f & 1/2 \\ j_i & l_i & L \end{matrix} \right\}^2 \left(\begin{matrix} l_f & L & l_i \\ 0 & 0 & 0 \end{matrix} \right)^2 \langle r^L \rangle^2 \quad (5)$$

where C_i and C_j are geometrical factors arising from the decoupling of the four-proton configurations and the radial matrix elements $\langle r^L \rangle$ are integrated from the numerical tables of reference [7].

The *g*-factors of some states with measurable half-lives were also calculated, mostly with the aid of Lande's formula.

3. Results. — Using the method outlined in section 2 the following levels were calculated (v indicates seniority within a multiplet) :

$h_{9/2}^4$	$v = 0$	$J^\pi = 0^+$
	$v = 2$	$J^\pi = 0^+ - 8^+$
	$v = 4$	$J^\pi = 0^+ - 12^+$
	(i.e. all levels)	
$h_{9/2}^3 f_{7/2}$	$v(h_{9/2}) = 1$	$J^\pi = 8^+$
	$v(h_{9/2}) = 3$	$J^\pi = 11^+(3) - 14^+$
$h_{9/2}^3 i_{13/2}$	$v(h_{9/2}) = 1$	$J^\pi = 11^-$
	$v(h_{9/2}) = 3$	$J^\pi = 14^-(3) - 17^-$

The low-spin levels of all configurations $h_{9/2}^3 j$ were not calculated, since they are not likely to be populated in a (HI, xn) reaction, and we wanted to give an explicit comparison with experiments. On the other hand, with the levels above, one can make a check of the linearity of the mean excitation energy of a multi-

Table I. — Calculated excitation energies in ^{212}Rn .

Configuration	J^π	E_{exc} (± 15)	Configuration	J^π	E_{exc} (± 15)
$h_{9/2}^4$ $v = 4$	12^+	2 916 keV	$h_{9/2}^3 f_{7/2}$ $v = 3$	14^+	3 430
	10^+	2 655		13^+	3 499
	9^+	2 712		12^+_a	3 671
	8^+	2 618		12^+_b	3 363
	7^+	2 414		11^+_a	3 799
	6^+_a	2 561		11^+_b	3 556
	6^+_b	2 276		11^+_c	3 288
	5^+	2 391	$v = 1$	8^+	2 167
	4^+_a	2 149			
	4^+_b	2 471	$h_{9/2}^3 i_{13/2}$ $v = 3$	17^-	4 138
	3^+	2 470		16^-	4 167
	2^+	2 259		15^-_a	4 166
	0^+	2 475		15^-_b	4 056
$v = 2$	8^+	1 766		14^-_a	4 557
	6^+	1 735		14^-_b	4 084
	4^+	1 598		14^-_c	3 498
	2^+	1 466			
$v = 0$	0^+	216	$v = 1$	11^-	2 739

plet as a function of seniority quantum number. The results are given in the table above, and in the level scheme in figure 1 we present a comparison of our calculation with experimentally determined levels.

As is seen in the figure, the agreement between theoretical and calculations and experimental results

is strikingly good, except for low-spin levels. This discrepancy is readily accounted for as a consequence of the truncation of the configuration space.

4. Conclusion. — We find that the rather simple-minded calculation presented above gives remarkably good results. However, the most striking disagreement, i.e. the large discrepancies in excitation energies for low-spin states, can well be accounted for if one includes configuration mixing. This amounts to enlarging the configuration space beyond $1h_{9/2}$, $2f_{7/2}$ and $1i_{13/2}$. Thus e.g. the $|0^+\rangle$ in ^{210}Po , is according to Kuo-Herling [9]

$$|0^+, ^{210}\text{Po}_{\text{g.s.}}\rangle = 0.821 |h_{9/2}^2\rangle + 0.367 |f_{7/2}^2\rangle + 0.150 |f_{5/2}^2\rangle + 0.095 |p_{3/2}^2\rangle + 0.053 |p_{1/2}^2\rangle - 0.397 |i_{13/2}^2\rangle.$$

Even if one truncates as above, one is faced with the problem that the empirical excitation energies of the states $(f_{7/2}^2)_{0^+}$ and $(i_{13/2}^2)_{0^+}$ are not known.

In contrast to the low-spin states, the high-spin configurations of [9] are not mixed and this is used as the basic argument for doing calculations assuming pure configurations.

As is also seen in figure 1, there is agreement between theory and experiment for the quoted half-lives and g -factors. In these the configuration mixing enters linearly and thus larger deviations might be expected.

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Fig. 1. — Theoretical level scheme of ^{212}Rn . The experimentally [8] found levels are given on the left.