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Superposition of a cylindrical Couette flow and of an electrically induced secondary flow

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Résumé. — L'effet électrovisqueux résultant de la modification de l'écoulement principal par un écoulement secondaire induit par la force de Coulomb est étudié dans le cas d'un écoulement de Couette cylindrique. La loi de variation observée au-dessus de la tension d'instabilité est justifiée par des considérations théoriques.

Abstract. — The electroviscous effect resulting from the perturbation of the main flow, by a Coulomb force induced secondary flow, is considered in the case of a cylindrical Couette flow. A theoretical justification is given for the law of variation observed, above the instability voltage.

1. Introduction. — It has long been known that the external application of an electric field perpendicular to a Poiseuille or Couette flow generally causes the apparent viscosity to increase, sometimes to a very large extent [1, 2]. In the case of isotropic dielectric liquids this phenomenon, often called *electroviscous effect*, may be attributed to the existence of secondary motions disturbing the primary flow.

The mechanism responsible for the secondary motions relates to the build-up of space charges, when these liquids are electrically stressed. Frequently new charge carriers are created at the electrodes, having the same polarity as the electrode on which they are formed (this process is named injection). The electric field E exerts on the space charge Q the Coulomb force QE which can set the liquid in motion. As we are mainly interested by the particular case of unipolar injection in an insulating liquid, let us recall that for the geometry of plane parallel electrodes, there exist two instability thresholds V_c and $V_2 < V_c$ corresponding to linear and non-linear instability mechanisms [3]. At V_c and V_2 there occur jumps in the values of the stationary current and the liquid velocity. Super-imposing a flow should not fundamentally alter the mechanism of instability now inducing a secondary flow.

This qualitative explanation of the electroviscous effect has received full experimental confirmation in the case of Poiseuille flow [4, 5]. Under conditions of space-charge-limited unipolar injection in an insulating liquid and constant flow rate, when slowly increas-

ing the voltage, no change in the pressure drop appears until V reaches V_c . Just above V_c , which is independent of the flow rate, a secondary motion can be observed which looks like longitudinal rolls. Simultaneously the pressure drop is augmented by an amount increasing with the applied voltage. A determination of the laws of variation was however not possible because of edge effects occurring at entrance and exit regions of the channel portion subjected to the electric field. Such perturbations are eliminated by using a cylindrical Couette flow which appears more convenient for the study of the influence of the electrically induced secondary motion on the main flow.

2. Evaluation of the electroviscous effect. — This effect results from the secondary motion through the action of the Reynolds stress which disturbs the distribution of mean velocity and increases the viscous friction on the walls. In order to calculate the shear, let us consider the Navier-Stokes equation :

$$\rho \left(\frac{\partial u_i}{\partial t} + u_j \frac{\partial u_i}{\partial x_j} \right) = - \frac{\partial p}{\partial x_i} + \eta \nabla^2 u_i + QE_i \quad (1)$$

where ρ is the mass density, $u_i = (u, v, w)$ the velocity, p the pressure and η the dynamic viscosity. For the sake of simplicity, only the stationary plane Couette flow is examined here in the case of a viscous hydrodynamic regime. We assume further the invariance of all variables in the x -direction of the main flow. Separating all variables in a mean part (denoted by

a bar) and a perturbation part and retaining only the main contributions of the various disturbances, we obtain :

$$\eta \frac{d^2 \bar{U}}{dz^2} = \rho \frac{d\bar{u}\bar{w}}{dz} \quad (2)$$

$$\eta \nabla^2 u = \rho w \frac{d\bar{U}}{dz} \quad (3)$$

$$\eta \nabla^2 w + \bar{Q}e_z + \bar{E}q - \frac{\partial p}{\partial z} = 0. \quad (4)$$

The viscous friction on the walls expressed by $f = \eta d\bar{U}/dz$ (for $z = 0$ or $z = d$, d being the layer thickness) has a relative increase which is obtained by integration of (2) :

$$\frac{\Delta f}{f_0} = - \frac{\rho}{\eta U_0} \int_0^d \bar{u}\bar{w} dz. \quad (5)$$

Here U_0 is the velocity of the upper wall and $f_0 = \eta U_0/d$. Without specifying the model for electrical conduction we can derive an expression in terms of the z -velocity component of the secondary flow. From (3), u varies as $(\rho V_0 d/\eta) w$ which leads to :

$$\frac{\Delta f}{f_0} \sim \frac{d^2}{v^2} w^2 \quad (6)$$

with $v = \eta/\rho$. A more quantitative law can be obtained by an approximate calculation method which has given good results in several stability problems [6, 7]. Multiplying (3) by u , averaging in an horizontal plane, substituting for $d\bar{U}/dz$ the expression derived from (2) by integration and integrating in the z -direction between 0 and d , we obtain a relation involving only u and w . Then assuming in the y -direction the most simple periodic variation of non-dimensional wave-number k and taking for u and w the following approximate expressions (note that the component v is non-zero) :

$$u(y, z) = U_m \sin(\pi z/d) \sin(ky/d)$$

$$w(y, z) = AKE \frac{1}{2}(1 - \cos(2\pi z/d)) \sin(ky/d)$$

we can calculate U_m as a function of A and k and finally obtain :

$$\frac{\Delta f}{f_0} = \frac{(RA)^2}{(16/3)(\pi^2 + k^2) + (29/35)(RA)^2} \quad (7)$$

where

$$R = T/M^2, \quad T = \varepsilon V/K\eta, \quad M^2 = \varepsilon/\rho K^2,$$

ε being the liquid permittivity, V the applied voltage and K the ionic mobility. Eqs. (6) and (7) show clearly that the effect is independent of the velocity U_0 . The law of variation with V depends on the variation of A with V , i.e. on the coupling between charge and velocity disturbances.

Let us consider more specifically the case of space-charge-limited injection. Above V_c , in the viscous dominated convection regime, the ratio $w_{\max}/KE$ varies proportionally to the applied voltage V [3, 8]. From (6) or (7) we may therefore expect a relative increase of the viscous friction, due to the action of the electric field, varying as V^4 , at least over some interval (RA low enough). The latter result can also be obtained taking into account the fact that above V_c the maximum of the liquid velocity exceeds the mean ionic drift velocity [3]. This implies charge free regions in the liquid and therefore charge disturbances q of the same order as the mean charge density : $q \sim \bar{Q}$ [8]. From (4) we have :

$$w \sim \bar{E}q \sim \bar{E}\bar{Q} \propto V^2$$

and (6) leads to the V^4 dependence for $\Delta f/f_0$.

3. Experimental. — In the constructed viscosimeter, the outer cylinder is rotating in order to produce a stable basic cylindrical Couette flow, in absence of electric field. The inner cylinder, guarded by two immobile cylinders, has its rotation restricted by a torsion wire. Dimensions were chosen so that the friction torques of the various bearings were negligible. The spacing between cylinders ($d = 0.45$ mm) is much lower than their radius ($R = 6$ cm). The curvature is thus very small and the weak deviation from the plane Couette flow should not alter the laws of variation derived above.

The one to one proportion of the nitrobenzene-cyclohexane mixture was determined on the basis of two requirements *a*) working with a liquid of high enough permittivity is desirable in order to obtain injection from a metallic electrode (here $\varepsilon_r \simeq 18$), *b*) the use of a liquid of low enough viscosity is necessary in order to have torque increase of about 1 % when the z -component of the induced velocity perturbation has a maximum value of the order of the ionic velocity KE . The liquid was maintained at a high level of ionic purity by permanent recycling through an electrodialysis cell.

During the experiments the outer cylinder was rotating at a constant angular velocity and the torque Γ exerted by the viscous frictions was determined through the measurement of the rotation θ of the inner cylinder by Poggendorf's method. It would have been desirable to work in conditions of space-charge-limited unipolar injection. Unfortunately the electrodialytic varnish coating on the inner side of the outer cylinder was unable to achieve the expected injection. However with reverse polarity, we obtained a current varying as V^2 (Fig. 1). We are most likely dealing here with positive ions created at the inner cylinder (stainless steel). This injection is of moderate but fairly constant relative level C ($C = Q_0 d^2/\varepsilon V$, Q_0 being the charge density at the injector).

Fig. 1. — Log-log plot, *versus* applied voltage, of the current I and rotation θ relative to the inner cylinder (radius $R_1 = 6$ cm, height $H = 15$ cm). I_0 represents the space charge limited current including the turbulent convective contribution.

4. **Results and discussion.** — The results plotted in figure 2 show the relative increase of θ which is proportional to the mean torque on the inner cylinder and therefore proportional to the viscous friction on the walls. (θ_0 is the value in absence of electric field). To a fair approximation, $\Delta\theta/\theta_0$ is independent of the angular velocity and has a linear variation in V^4 ($V \leq 150$ V) :

$$\Delta\theta/\theta_0 = \text{constant} (V^4 - V_0^4) \quad (8)$$

V_0 , defined by extrapolation, is the voltage for which the phenomenon seemingly appears. The dependence on V^4 which was predicted for space-charge-limited injection also holds for moderate injections, provided that the non-dimensional injection parameter C is constant ($\overline{QE}$ then varies like V^2).

The voltage $V_0 \simeq 80$ V is certainly very close to the instability voltage threshold. Such a critical value corresponds to an injection level C lying between 1 and 1.5 [7]. This C value is in good agreement with that estimated from the comparison of the measured current with the space-charge-limited current [9] taking into account the convection contribution [8]. At the present time however, we are not quite sure that V_0 is the instability threshold. First the law of variation of w *versus* V could be different for low amplitudes of w ($\leq KE$) because of a different coupling

Fig. 2. — Relative increase of θ as a function of V^4 for three different angular velocities.

between w and q (in that case $q < \overline{Q}$). We most presumably should expect a slightly higher threshold due to the discontinuous character of the phenomena occurring at critical voltage as observed for space charge limited unipolar injection [3]. In order to clarify this point, more accurate torque measurements are required.

Beyond a voltage of about 150 V, $\Delta\theta/\theta_0$ departs from the variation as expressed by (8). This might be due to the transition to a turbulent flow. In the case of space-charge-limited unipolar injection without primary flow ($U_0 = 0$), the conditions for which viscous and inertial effects are of the same order are characterized by $MR \simeq 30$ [8]. Here $V_1 \simeq 150$ V corresponds to $MR \simeq 50$, a plausible transition value for a moderate injection ($C = 1$ to 1.5).

5. **Conclusion.** — In the particular case considered here, the electric cylindrical viscosimeter permitted the observation of an increase of the viscous friction proportional to $V^4 - V_0^4$ in the hydrodynamic regime where the viscous effects are dominant. The noticeable enhancement of the torque on the inner cylinder (about 3 times the pure Couette flow torque at 700 V) illustrates once more the vigour of the convection induced by bulk Coulomb force. Moreover the apparatus employed provides an interesting way of determining, by objective means, the instability thresholds in electro-hydrodynamic cases where the motion induces undetectable increase of the total electric current.

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