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Microscopic optical potentials of the nucleon-nucleus elastic scattering at medium energies

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Résumé. — Le premier terme du développement de Watson du potentiel optique est comparé à un modèle de diffusion multiple développé récemment par les auteurs. Des calculs numériques sont présentés et discutés pour la diffusion élastique p-⁴He et p-⁴⁰Ca dans le domaine des énergies intermédiaires.

Abstract. — The first order Watson optical potential is compared with the multiple scattering potential, recently developed by the present authors. The relations between the two potentials are discussed and numerically illustrated for p-⁴He and p-⁴⁰Ca elastic scatterings at intermediate energies.

The low energy (< 100 MeV) nucleon-nucleus elastic scattering data are usually being interpreted in terms of the *phenomenological* optical model potential [1]. At medium and high energies the application of the optical potential is less common, mainly due to the large number of partial waves involved in the analysis of the data. Moreover, in recent years, the optical model approach has been partly outshined by the success of the Glauber model [3] based upon the eikonal approximation. However, detailed analyses [4] revealed some discrepancies between the Glauber model and experiment, particularly at diffraction minima and large angles. The disagreement resembles the differences between the eikonal approximation and the exact treatment of the wave equation in potential scattering [2]. The use of the optical model which properly accounts for the effects of non-eikonal propagation may therefore be promising.

The resumption of the optical model approach at medium energies has also some theoretical foundations. At a sufficiently large incident energy one can apply closure to the intermediate target eigenstates neglecting their excitation energies. The complex coupled-channel scattering problem is thereby essentially simplified. This facilitates a construction of the *microscopic* optical potential which can be derived theoretically, and not only adjusted phenomenologically to the data.

The purpose of this work is a comparative analysis of various approaches to the theoretical optical potential. The emphasis will be put on the relation between the first order Watson potential [5] and the

multiple scattering potential developed recently in ref. [6].

The optical potential of elastic scattering can be expressed in terms of the nuclear ground state $|0\rangle$ and of the projectile-nucleon transition operators t_j , as follows :

$$V = T_{00}(1 + GT_{00})^{-1} = T_{00} - T_{00}GT_{00} + T_{00}GT_{00}GT_{00} - \dots \quad (1)$$

where G is free Green's function and T_{00} is elastic scattering matrix element as given by Watson's multiple scattering series [5] :

$$T_{00} = \left\langle 0 \left| \sum_{j=1}^A t_j + \sum_{j=1}^A \sum_{k \neq j}^A t_j G t_k + \sum_{j=1}^A \sum_{k \neq j}^A \sum_{l \neq k}^A t_j G t_k G t_l + \dots \right| 0 \right\rangle. \quad (2)$$

The two basic expressions for the microscopic optical potential can be derived in the following way. If the series in eqs. (1) and (2) are *truncated* after the first term then :

$$V_w = T_{00}^{(1)} = \left\langle 0 \left| \sum_{j=1}^A t_j \right| 0 \right\rangle. \quad (3)$$

In the momentum space (q -being the momentum transfer) eq. (3) reads :

$$V_w(q) = \left\langle 0 \left| \sum_{j=1}^A t_j e^{iq \cdot r_j} \right| 0 \right\rangle = -\frac{2\pi}{E_r} A f(q) F(q) \quad (3')$$

where $F(q)$ is the nuclear form factor and $f(q)$ — the projectile — nucleon scattering amplitude (assumed, for simplicity, as identical for neutrons and protons). E_r is a relativistic equivalent of the reduced mass; notice that the wave equation contains $2 E_r V$ hence it is independent of the choice of E_r .

This potential is referred to as the first order Watson potential [5]. Closely related to it is the Kerman-Mc Manus-Thaler (KMT) form of the optical potential [7]. Their prescription is to replace A in eq. (3') by $A - 1$, to solve the wave equation, and to multiply the resulting amplitude by $A/A - 1$. Obviously, the Watson potential gives more than a single scattering amplitude. Multiple scattering effects arise from higher order terms in the Born expansion of the scattering matrix. However, it can easily be seen that the Watson potential results in a scattering matrix which neglects the quasi-elastic shadowing [10], i.e. the virtual excitations of the target in intermediate states. Therefore, we may refer to the first-order potential as to the *static* optical potential.

The multiple scattering potential is obtained by working at the *eikonal limit* of quantum mechanics. The essential point is that the linearization of the Green's function, without affecting the interaction, allows solving of the inverse scattering problem [3]. Thus if at the eikonal limit ($G \rightarrow G_{\text{eik}}$) one has on energy-shell (p -being the incident c.m. momentum) :

$$T_{00} \rightarrow T_{\text{eik}}(q) = -2 \pi i p / E_r \int_0^\infty db b J_0(qb) \Gamma_{\text{eik}}(b), \quad (4)$$

then the eikonal form of eq. (1)

$$V = T_{\text{eik}}(1 + G_{\text{eik}} T_{\text{eik}})^{-1} \quad (1')$$

can easily be reduced, under assumption of a spherically symmetric interaction, to the following formula :

$$V(r) = - \frac{i p}{\pi E_r} \int_{-\infty}^{+\infty} dz \frac{d}{dr^2} \ln [1 - \Gamma_{\text{eik}}(\sqrt{r^2 + z^2})]. \quad (5)$$

Eq. (5) has been used in ref. [6] with the profile function $\Gamma_{\text{eik}}(b)$ as prescribed by the Glauber model :

$$\Gamma_{\text{eik}}(b) = 1 - \left\langle 0 \left| \prod_{j=1}^A [1 - \gamma_j(\mathbf{b} - \mathbf{s}_j)] \right| 0 \right\rangle \quad (6)$$

where the profiles γ_j of the target constituents ($\mathbf{s}_j$ -being their position in the plane of impact parameters) are related, as in eq. (4) to the elementary elastic scattering amplitudes f_j . This procedure is justified when realizing that the Glauber model can be treated as a transposition [8] of the rigorous Watson theory of multiple collision, under the usual closure assumption, to the eikonal mechanics. One should in principle use for the elementary profile γ the Fourier-Bessel trans-

form of the eikonal amplitude f_{eik} . However, the error introduced when the physical amplitudes f 's are inserted into the eikonal form of the Watson theory can be estimated by a perturbation expansion and appears to be small [9].

In contrast to the Watson potential, the multiple scattering potential, as given by eqs. (5) and (6), is *dynamic* since it admits the possibility of virtual excitations though only in the closure approximation of the target. Neglecting the quasi-elastic shadowing (NS) in the Glauber profile (6) would yield [10] :

$$\Gamma_{\text{NS}}(b) = 1 - \left[1 - \frac{1}{A} \Gamma^{(1)}(b) \right]^A, \quad (7)$$

$$\Gamma^{(1)}(b) \equiv \left\langle 0 \left| \sum_{j=1}^A \gamma_j(\mathbf{b} - \mathbf{s}_j) \right| 0 \right\rangle$$

producing the potential :

$$V_{\text{NS}}(r) = \frac{i p}{\pi E_r} \int_{-\infty}^{+\infty} dz \frac{\Gamma'^{(1)}}{1 - \frac{1}{A} \Gamma^{(1)}}, \quad (8)$$

$$\Gamma'^{(1)} \equiv \frac{d}{dr^2} \Gamma^{(1)}(\sqrt{r^2 + z^2}).$$

The Watson potential can be obtained by retaining in the expression (5) or (8) only the term linear in γ . This is equivalent to formally approximating :

$$\ln(1 - \Gamma_{\text{eik}}) \cong - \Gamma^{(1)}$$

in eq. (5) and

$$A \ln(1 - \Gamma^{(1)}/A) \cong - \Gamma^{(1)} \quad \text{in eq. (8)}.$$

The second assumption is obviously much weaker hence the Watson potential should be closer to the no-shadowing form (8) of the multiple scattering potential.

The effects of quasi-elastic shadowing, hence also the differences between the Watson and the multiple scattering potentials, depend strongly on the correlation structure of the target nucleus. For completely independent particles (IP) when :

$$|\langle \mathbf{r}_1 \dots \mathbf{r}_A | 0 \rangle|^2 = \prod_{j=1}^A \rho(r_j) \quad (9)$$

$\rho(r)$ being single particle density, there is of course no possibility of virtual target excitations. In fact the Glauber profile (6) and the multiple scattering potential (5) are given in this case by formulae :

$$\Gamma_{\text{IP}} = 1 - [1 - S]^A \quad (10)$$

$$V_{\text{IP}} = \frac{i p}{\pi E_r} \int_{-\infty}^{+\infty} dz \frac{A S'}{1 - S} \quad (11)$$

with

$$S(b) = \int d^3r \rho(r) \gamma(\mathbf{b} - \mathbf{s})$$

which are formally identical to the expressions obtained when neglecting the quasi-elastic shadowing. In order to expose the dynamical differences between the two potentials, one clearly should work with correlated wave functions.

In figures 1 and 2 we have shown elastic scattering of medium energy protons from the ^4He and ^{40}Ca nuclei. We are comparing there the microscopic optical potentials as given by eqs. (3), (5) and (8), referred to as the Watson, multiple scattering and no-shadowing potentials, respectively. The differential cross-sections resulting from these potentials are also presented.

Fig. 1. — The optical model potentials of the $p\text{-}^4\text{He}$ elastic scattering and the corresponding differential cross-sections at 348 MeV (right and top scales) and 1 050 MeV (left and bottom scales). Dotted curves refer to the Watson potential (eq. (3)), full curves — to the multiple scattering potential (eq. (5)). The dashed curves correspond to the no-shadowing approximation (eq. (8)) of the latter. The nuclear parameters were [11] $R = 1.265$ fm, $\lambda = 0.652$. The parameters of the nucleon-nucleon interaction were [6]: $\sigma = 28$ mb, $\alpha = -0.45$, $a = 1.5$ GeV^{-2} at 348 MeV, and $\sigma = 44$ mb, $\alpha = -0.30$, $a = 5.0$ GeV^{-2} at 1 050 MeV.

Fig. 2. — The optical model potentials of the $p\text{-}^{40}\text{Ca}$ elastic scattering and the corresponding differential cross-sections at 1 044 MeV in the α -particle model of the ^{40}Ca target. Dotted curves refer to the Watson potential (eq. (3)), full ones to the multiple scattering potential (eq. (5)), and the dashed ones — to the no-shadowing approximation (eq. (8)) of the latter. The nuclear parameters were obtained from fitting to the ^{40}Ca elastic form factor: $R = 2.583$ fm, $\lambda = 0.728$. The parameters of the $p\text{-}\alpha$ interaction were [6]: $\sigma = 150$ mb, $\alpha = -0.26$, $a = 27$ GeV^{-2} .

We have used for the nuclear ground state density the model of correlated pairs [11]:

$$|\langle \mathbf{r}_1 \dots \mathbf{r}_A | 0 \rangle|^2 = \prod_{j=1}^A \rho(r_j) \left\{ 1 + \sum_{l=1}^{[A/2]} (2^l l!)^{-1} \times \right. \\ \left. \times \sum_{j_1 \neq k_1 \neq \dots \neq j_l \neq k_l}^A [\Delta(j_1, k_1) \dots \Delta(j_l, k_l)] \right\} \quad (12)$$

$$\Delta(j, k) \equiv G^2(r_{jk}) - 1.$$

The successive terms of (12) correspond to the expansion in numbers of correlated pairs: independent

particles, one correlated pair etc... The two input quantities of the model have been assumed as being :

$$\begin{aligned} \rho(r) &= \pi^{-3/2} R^{-3} \exp(-r^2/R^2) \\ G^2 &= \frac{1}{M} [g^2 + (M-1)g], \\ g(r_{jk}) &= 1 - \exp\left(-\frac{\lambda r_{jk}^2}{R^2}\right) \end{aligned} \quad (13)$$

where the coefficient M is determined by the normalization. The parameters R and λ can be established from the analysis of the elastic charge form factor. Assumptions (13) make possible to eliminate unambiguously the spurious centre-of-mass coordinate [12] from the nuclear density (12). It should be emphasized that for a small A the correlations induced by the condition of the translational invariance are extremely important for the evaluation of the optical potentials [6].

The elementary profiles have been taken as :

$$\gamma(b) = \frac{\sigma(1-i\alpha)}{4\pi a} \exp\left(-\frac{b^2}{2a}\right) \quad (14)$$

which corresponds to a Gaussian q -dependence of the elastic scattering amplitude $f(q)$. The parameters σ (the total cross-section), α (the Re/Im ratio of the forward amplitude) and a (the slope) are, in general energy dependent.

The correlated model (12) has been applied to the ^4He and ^{40}Ca nuclei in a different manner. The ^4He nucleus was treated as a system of four nucleons, while the ^{40}Ca nucleus has been assumed to be composed of ten α -particles. That is why for ^4He the parameters σ , α , and a refer to the proton-nucleon interaction, while those for ^{40}Ca to the p - α interaction.

Our results show a distinct difference between the multiple scattering and the Watson potentials. The dynamic potential is stronger and less diffuse at the nuclear surface. Therefore it gives rise to greater values of the cross-section, the difference being pronounced at large q . The *no-shadowing* potential produces results which generally lie in-between the predictions of the two former potentials. With the increasing momentum transfer they become, however, quite close to the Watson result.

In the case of the p - ^4He scattering the multiple scattering potential shows a characteristic waving at small distances. This is a joint result of the quasi-elastic shadowing and of the strong c.m. correlations. In fact, these ripples of the potential no longer appear in the no-shadowing approximation. They are also absent for the ^{40}Ca nucleus in spite of a considerable shadowing effect.

It should be stressed that for the p - ^4He scattering the results of the multiple scattering potential are in a very good agreement with the experimental data [6]. On the other hand, neglecting the quasi-elastic sha-

dowing which is inherent in the Watson and KMT approaches leads to a considerable underestimation of the cross-section at large momentum transfers.

In light nuclei the differences between the multiple scattering and no-shadowing potentials arise mainly from the c.m. constraint. The role of the pair correlations in enhancing multiple scattering is less important. Thus if they are switched off or if their action is simulated by a change in the single-particle density $\rho(r)$ the large shadowing effect still persists. This is illustrated in figure 3 where we compared the three microscopic potentials for the nuclear density of eq. (9) with the condition of the translational invariance imposed upon. The single-particle density is assumed to be in a double Gaussian form :

$$\begin{aligned} \rho(r) &= \pi^{-3/2} (R_1^3 + \delta R_2^3)^{-1} \times \\ &\times \left[\exp\left(-\frac{r^2}{R_1^2}\right) + \delta \exp\left(-\frac{r^2}{R_2^2}\right) \right] \end{aligned} \quad (15)$$

which allows for reproducing a diffraction structure

Fig. 3. — The optical potentials of the p - ^4He elastic scattering and the corresponding differential cross-sections at 1 050 MeV. Dotted curves refer to the Watson potential, full ones to the multiple scattering potential and the dashed ones — to the no-shadowing approximation of the latter. The parameters of the density (15) were [10] : $R_1 = 1.25$ fm, $R_2 = 0.77$ fm, $\delta = -1.0$. The elementary parameters : $\sigma = 44$ mb, $\alpha = -0.30$, $a = 5.0$ GeV^{-2} .

of the elastic charge form factor of ^4He . In this case the elimination of the c.m. spuriousity is, in contrast to a single Gaussian, no longer unique. In our calculations the c.m. correlations have been introduced into eq. (9) by means of the Gartenhaus-Schwartz transformation [12].

Figure 2 exemplifies that the differences between the static and dynamic character of the interaction may show up also for heavier targets. However, in describing heavy nuclei one usually neglects both the c.m. and short-range correlations. In that situation the intermediate excitations of the target are excluded.

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