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HELICAL TEXTURES IN $^3\text{He-A}$ IN THE PRESENCE OF SUPERFLOW AND MAGNETIC FIELD (*)

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Résumé. — On démontre théoriquement l'existence de textures en hélice dans $^3\text{He-A}$ en présence d'un courant superfluide et d'un champ magnétique. La texture en hélice produit un déplacement positif de la fréquence de résonance magnétique en plus de celui de Leggett.

Abstract. — The existence of helical texture in $^3\text{He-A}$ in the presence of both superflow and magnetic field is established theoretically. The helical texture gives rise to an additional positive shift to the Leggett shift both in the transverse and the longitudinal resonance frequency.

It has recently been shown by Kleinert, Lin-Liu, and Maki [1] (KLM) that, in the low temperature region where the uniform texture with $\hat{l} \parallel \mathbf{V}_s$ becomes unstable in the dipole-locked case [2] (i.e., $\hat{l} \not\parallel \hat{d}$) of $^3\text{He-A}$, there appear a class of helical textures with $\hat{l}$ winding around $\mathbf{V}_s$, the superflow, which are locally stable. In this paper we generalize the above analysis in the presence of a magnetic field $\mathbf{H}$. In this case $\hat{d}$ is no longer necessarily parallel to $\hat{l}$. For simplicity we limit ourselves to the case $\mathbf{V}_s \parallel \mathbf{H}$ and the Ginzburg-Landau regime. We find that, in strong contrast to the dipole-locked case [1], there is a large region in the v_s - H phase diagram where the helical texture is locally stable even in the Ginzburg-Landau regime.

Assuming that the texture depends only on z , the direction of the superflow, the free energy in reduced units is given by

$$g = -\frac{p^2}{2(1+s)} + \frac{1}{4}(3-2s)\beta_z^2 + \frac{1}{2}(1+s)\theta_z^2 +$$

$$+ \frac{\sqrt{1-s}}{(1+s)}p\gamma_z + s\left(\frac{1}{1+s} - \frac{1}{4}\right)\gamma_z^2 + \frac{1}{2}\sin^2\theta(1+s)\varphi_z^2 + [1 - (\cos\beta\cos\theta + \sin\beta\sin\theta\cos(\gamma-\varphi))^2] + h^2\cos^2\theta \quad (1)$$

where

$$s = \sin^2\beta, \quad p = (1+s)v_s/v_{s0}, \quad h = H/H_0$$

and

$$v_{s0} \left(\equiv \frac{h}{2m\xi_\perp} \simeq 0.1 \text{ cm/s} \right)$$

and

$$H_0 (\equiv \Omega_A/(\Delta\chi) \sim 20 \text{ Oe})$$

and p is the normalized super current, $\xi_\perp$ is the dipole coherence length and $\Delta\chi$ is the anisotropic part of the magnetic susceptibility. Here we parameterized

$$\begin{aligned} \hat{l} &= (\sin\beta\cos\gamma, \sin\beta\sin\gamma, \cos\beta) \\ \hat{d} &= (\sin\theta\cos\varphi, \sin\theta\sin\varphi, \cos\theta). \end{aligned} \quad (2)$$

(*) This paper has been presented at LT 15 Conference as a post deadline paper.

To study the stability of a uniform texture ($\hat{l} \parallel \hat{d} \parallel \mathbf{p}$) and helical textures (with small inclination β and θ),

it is more convenient to express g in terms of u, v, u' and v' as

$$\begin{aligned}
 g &= g_0 + g_1 + g_2 \\
 g_0 &= -\frac{1}{2}p^2 + p\gamma_z \\
 g_1 &= \frac{3}{4}(u_z^2 + v_z^2) - \frac{3}{2}p(uv_z - vu_z) + \frac{1}{2}(u_z'^2 + v_z'^2) + \\
 &\quad + \frac{p^2}{2}(u^2 + v^2) - h^2(u'^2 + v'^2) + (u - u')^2 + (v - v')^2 \\
 g_2 &= (u^2 + v^2) \left[\frac{1}{4}(u_z^2 + v_z^2) + \frac{11}{8}p(uv_z - vu_z) + \frac{1}{2}(u_z'^2 + v_z'^2) - \frac{1}{2}p^2(u^2 + v^2) \right] + \\
 &\quad + \frac{1}{2}(u' u_z' + v' v_z')^2 - \frac{5}{4}(uv_z - vu_z)^2 + [u(u - u') + v(v - v')] [u'(u - u') + v'(v - v')]
 \end{aligned} \tag{3}$$

where

$$u = \sin \beta \cos \gamma, \quad v = \sin \beta \sin \gamma, \quad u' = \sin \theta \cos \varphi, \quad v' = \sin \theta \sin \varphi \tag{4}$$

and we kept up to quartic terms in u, v , etc.

Then eq. (3) is simplified by assuming that :

$$\begin{aligned}
 u &= u_0 \cos(kz), \quad v = u_0 \sin(kz), \\
 u' &= u'_0 \cos(kz), \quad v' = u'_0 \sin(kz)
 \end{aligned}$$

$$\langle g \rangle = \langle g_0 \rangle + A(k) u_0^2 + \frac{1}{2} B(k) u_0^4 \tag{5}$$

$$\begin{aligned}
 A(k) &= \frac{p^2}{2} + \frac{3}{4}k^2 - \frac{3}{2}pk + \\
 &\quad + 1 - \left(1 - h^2 + \frac{1}{2}k^2\right)^{-1} \\
 B(k) &= 2\left(h^2 - \frac{1}{2}k^2\right)^2 \left(1 - h^2 + \frac{1}{2}k^2\right)^{-3} + \\
 &\quad + k^2 \left(1 - h^2 + \frac{1}{2}k^2\right)^{-2} - p^2 - 2k^2 + \frac{11}{4}pk
 \end{aligned}$$

and

$$u'_0 = \left(1 - h^2 + \frac{1}{2}k^2\right)^{-1} u_0 \tag{6}$$

where $\langle g \rangle$ is the spatial average of g . From the free energy (5), we conclude that :

1) When $A(k) > 0$ for all k , a uniform texture with $\hat{l} \parallel \hat{d} \parallel \mathbf{p}$ is locally stable.

2) When $A(k) \geq A(k_m) = 0$ and the equality is satisfied by a single $k = k_m$, and furthermore $B(k_m) > 0$, the uniform texture becomes unstable against appearance of the helical texture with pitch $k = k_m$. Therefore this condition gives the phase boundary.

3) When $A(k_m) < 0$, and $B(k_m) > 0$, the helical

texture with pitch $k = k_m$ and $u_0^2 = |A|/B$ is locally stable.

4) When $A(k) < 0$, and $B(k) < 0$, all of the above textures are unstable.

From the above criteria, we have constructed the phase diagram shown in figure 1. In the region I the uniform texture ($\hat{l} \parallel \hat{d} \parallel \mathbf{p}$) is locally stable. The region I is completely surrounded by the region II where the helical textures are locally stable. In the region enclosed by the broken curve another uniform texture ($\hat{l} \parallel \hat{d} \perp \mathbf{p}$) is locally stable.

In the present situation, where $\hat{d}$ and $\hat{l}$ are no longer parallel to each other, the existence of the

FIG. 1. — Phase diagrams, where p and h are the normalized super current and the magnetic field. I indicates the region where a uniform texture with $\hat{l} \parallel \hat{d} \parallel \mathbf{p}$ is stable, while III that for a texture with $\hat{l} \parallel \hat{d} \perp \mathbf{p}$. The shaded area (II) is the region where helical textures are locally stable. At the phase boundary k increases from $k = \frac{2}{3}p$ to $k = 0.863$ as p increases from 0 to $p_c (= 1.169)$.

helical texture can be detected by NMR. It is easily shown that for the helical textures the longitudinal and transverse resonance frequencies are given by

$$\omega_l^2 = \lambda_l \Omega_A^2, \quad \omega_t^2 = \omega_0^2 + \lambda_t \Omega_A^2,$$

$$\lambda_l = \sin \beta \cos (\beta - \theta) / \sin \theta$$

and

$$\lambda_t = \sin 2 \beta / \sin 2 \theta \quad (7)$$

where Ω_A is the Leggett frequency and ω_0 is the Larmor frequency. In the immediate vicinity of the phase boundary, where β and θ are still small ($\sin^2 \beta$ and $\sin^2 \theta$ increased linearly with the distance from the phase boundary) we have $\lambda_l = \lambda_t$. We have calculated λ_l along the phase boundary as a function of p (or v_s/v_{s0}) and shown in figure 2. The fact that $\lambda_l > 1$ may appear puzzling at first glance. It is, however, easy to see that the positive shift is due to the Coriolis force associated with the helical structure of the $\hat{d}$ vector. When the helical texture moves slowly with $\mathbf{v}_n$, it produces a periodic variation in $\hat{l}$ -field with a period $\tau (\equiv 1/v_n k)$. Since k is proportional to v_s for small p , the period τ is proportional to $(1 - T/T_c)$ for constant v_n . This may account for the periodic orbital motion observed by Paulson, Krusius and Wheatley [3]. We believe that the helical

FIG. 2. — $\lambda_l (= \lambda_t)$ is shown as a function of p along the phase boundary.

structures (as a locally stable structure) exist only in the limited region in the p - h diagram. When p or h becomes much larger than 1, the dipole energy is no longer sufficient for providing the stability of the helical textures. The present work is partially supported by the National Science Foundation under grant number DMR76-21032 ⁽¹⁾.

⁽¹⁾ After completing this work we have received a preprint by Takagi, who studied the stability of the uniform texture in the presence of both superflow and field. Although his analysis is limited to the dipole-locked case, his initial slope of the instability line for p and $h \rightarrow 0$ agrees with the present result.

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