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MAGNETIC FIELD EFFECTS ON THE HELICAL INSTABILITY OF THE $\hat{l} // V_s$ TEXTURE IN $^3\text{He-A}$ (*)

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Résumé. — Nous étudions l'effet du champ magnétique H , dans le domaine où l'énergie dipolaire est prépondérante, sur la stabilité de la texture avec $\hat{l} // V_s$ dans $^3\text{He-A}$. Pour $H \perp V_s$ (resp. $H // V_s$) cette texture peut être stabilisée (resp. déstabilisée).

Abstract. — We have studied the effect of a magnetic field H , in the dipole-locked regime, on the stability of the $\hat{l} // V_s$ texture in $^3\text{He-A}$. An $H \perp V_s$ ($H // V_s$) tends to stabilize (destabilize) this texture.

Recently there has been much interest in the question of the $\hat{l}$ -texture in $^3\text{He-A}$, in the presence of a superflow. Bhattacharyya *et al.*, [1] and Cross and Liu [2] established that, in the weak-coupling dipole-locked Ginzburg-Landau (GL) regime, the texture with $\hat{l}$ parallel to the superfluid velocity V_s barely satisfies the stability criterion, but that at some lower temperature it is unstable with respect to circularly polarized $\hat{l}$ -fluctuations at some finite wave number k_z along the flow. In addition, they showed that in the spin-orbit decoupled GL regime (which holds for channels narrower than the dipole length [3]), the texture with $\hat{l} // V_s$ is unstable. Fetter [4] and Kleinert *et al.*, [5] then considered the nature of the new stable texture, finding that the uniform texture continuously distorts into a helical structure, which is stable with respect to further small $\hat{l}$ -fluctuations.

It is the purpose of this paper to point out several effects of a magnetic field H , in the dipole-locked regime, on the stability of the uniform $\hat{l} // V_s$ texture in $^3\text{He-A}$. First, by applying a sufficiently large H along V_s , in the case when the uniform texture with $\hat{l} // V_s$ is stable, it is possible to induce this texture to destabilize and form a helical texture; furthermore, one may *tune* the apex angle of the helical texture by suitably adjusting $|H|$. Secondly, by applying a sufficiently large H perpendicular to V_s (in a regime

where $\hat{l} // V_s$ is unstable), it is possible to stabilize the $\hat{l} // V_s$ texture and to control the onset temperature of the instability. We have studied the case of an $H \perp V_s$ which rotates about V_s with angular frequency much greater than the inverse of the orbital ($\hat{l}$) relaxation time, and the case of a static $H \perp V_s$.

We first consider the case with $H // V_s \parallel \hat{z}$. To the free energy employed in reference [1] we add the anisotropic part of the magnetic free energy,

$$F_H = \frac{1}{2} \chi_a H^2 \int d^3r (\hat{l} \cdot \hat{z})^2, \quad (1)$$

where $\chi_a = \chi_{\perp} - \chi_{\parallel} > 0$, and due to the assumption that the system is in the dipole-locked regime, we have taken $\hat{l}$ parallel to the spin vector $\hat{d}$. Repeating the analysis of references [1] and [2], the new stability criterion for the texture $\hat{l} // V_s$ is found to be :

$$\frac{(C_0 + \frac{1}{2} \rho_s^{\parallel})^2}{\rho_0 K_b} + \frac{\chi_a}{\rho_0} \left(\frac{H}{w} \right)^2 < 1, \quad (2)$$

where $w = |V_s|$. The wavevector k_z at which an $\hat{l}$ -fluctuation first becomes unstable occurs with the field-independent magnitude

$$|k_z^0| = [(C_0 + \frac{1}{2} \rho_s^{\parallel}) / K_b] w. \quad (3)$$

A critical field $H_{c\parallel}$ may be defined by setting the left hand side of eq. (2) equal to unity. If

$$(C_0 + \frac{1}{2} \rho_s^{\parallel})^2 > \rho_0 K_b,$$

so that $H_{c\parallel}^2 < 0$, the $\hat{l} // V_s$ texture is unstable for

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all H . If $(C_0 + \frac{1}{2}\rho_s^{\parallel})^2 < \rho_0 K_b$, so that $H_{c\parallel}^2 > 0$, the $\hat{l} \parallel \mathbf{V}_s$ texture is unstable only for $H > H_{c\parallel}$. This latter case includes the weak-coupling GL regime.

In both cases the texture deforms to a helical one with polar angle θ_0 given, for small θ_0 (and H near $H_{c\parallel}$, k_z near k_z^0), by :

$$\theta_0^2 = \frac{K_b \left(C_0 + \frac{1}{2}\rho_s^{\parallel} \right) \left\{ \chi_a (H^2 - H_{c\parallel}^2) / \left[\left(C_0 + \frac{1}{2}\rho_s^{\parallel} \right)^2 w^2 / K_b \right] - \left(\frac{k_z - k_z^0}{k_z^0} \right)^2 \right\}}{\left(2 K_t - \frac{1}{2}\rho_s^{\parallel} \right) \left(C_0 + \frac{1}{2}\rho_s^{\parallel} \right) - \frac{3}{2}\rho_s^{\parallel} K_b - K_b (\chi_a H^2 / w^2)} \quad (4)$$

Note that $\theta_0 = 0$ corresponds to $\hat{l} \parallel \mathbf{V}_s$, and that (4) applies even when $H_{c\parallel}^2 < 0$. We have not performed a stability analysis of this texture, but we note that the analysis for $H = 0$ guarantees stability for small values of $H_{c\parallel}$ [4, 5]. Thus, by suitably adjusting H it should be possible to tune θ_0 (cf. Eq. (2)). In the GL regime, at $P \approx 20$ atm, we find that $H_{c\parallel} \approx 60 w$, for w in cm/s and H in gauss. Such a value of $H_{c\parallel}$ is easily realizable.

Next we consider the effect of an $\mathbf{H} \perp \mathbf{V}_s$. We have considered the two cases which we report below.

In the case where $\mathbf{H} \perp \mathbf{V}_s$ rotates in the x - y plane rapidly compared to the inverse of the orbital relaxation time, we may employ the time-average of eq. (1) to obtain

$$\begin{aligned} \bar{F}_H^t &= \frac{1}{4} \chi_a H^2 \int d^3r [(\hat{l} \cdot \hat{x})^2 + (\hat{l} \cdot \hat{y})^2] \\ &= \frac{1}{4} \chi_a H^2 \int d^3r [1 - (\hat{l} \cdot \hat{z})^2]. \end{aligned} \quad (5)$$

For our purposes the isotropic term may again be neglected. It is then clear that this problem may be solved by employing eqs. (2-4) with the replacement $\chi_a \rightarrow -\frac{1}{2}\chi_a$. Thus eq. (2), with $\chi_a \rightarrow -\frac{1}{2}\chi_a$, may be used to define an $H_{c\perp}^2$. If $(C_0 + \frac{1}{2}\rho_s^{\parallel})^2 < \rho_0 K_b$, so that $H_{c\perp}^2 < 0$, the $\hat{l} \parallel \mathbf{V}_s$ texture is stable for all H . If $(C_0 + \frac{1}{2}\rho_s^{\parallel})^2 > \rho_0 K_b$, so that $H_{c\perp}^2 > 0$, the $\hat{l} \parallel \mathbf{V}_s$ texture is stable for $H > H_{c\perp}$; for $H < H_{c\perp}$, where the helical structure is appropriate, the effect of H is to decrease θ_0 .

Lastly, we consider the case where $\mathbf{H} \parallel \hat{x}$ and $\mathbf{V}_s \parallel \hat{z}$, which again tends to stabilize the $\hat{l} \parallel \mathbf{V}_s$ texture. To discuss this less trivial case we define the quantity

$$\varepsilon = \frac{\chi_a H^2}{4(C_0 + \frac{1}{2}\rho_s^{\parallel})^2 w^2 / K_b}. \quad (6)$$

By setting $\varepsilon = 1$ we obtain a value of H which we denote H_p . For $H > H_p$ the least stable $\hat{l}$ -fluctuations with $\mathbf{k} = k_z \hat{z}$ are restricted to the y -direction, and are stable. For $H < H_p$ the least stable $\hat{l}$ -fluctuations

with $\mathbf{k} = k_z \hat{z}$ lie in the x - y plane, and have the Fourier component

$$\delta \hat{l}_k = e^{i\varphi} (\alpha \hat{x} + i\beta \hat{y}), \quad (7)$$

where φ is arbitrary, and the optimum value of k_z has

$$|k_z^{(0)}| = [(C_0 + \frac{1}{2}\rho_s^{\parallel}) / K_b] [1 - \varepsilon^2]^{1/2}. \quad (8)$$

In this case

$$\alpha^2 / \beta^2 = (1 - \varepsilon) / (1 + \varepsilon). \quad (9)$$

Note that the signs of α , β , and $k_z^{(0)}$ must satisfy $\alpha\beta k_z^{(0)} > 0$. The energy of the optimum fluctuation is found to be positive (and hence the $\hat{l} \parallel \mathbf{V}_s$ texture is stable) if

$$\begin{aligned} \frac{(C_0 + \frac{1}{2}\rho_s^{\parallel})^2}{\rho_0 K_b} - \frac{\chi_a}{2\rho_0} \left(\frac{H}{w} \right)^2 + \\ + \frac{K_b \chi_a^2}{16\rho_0 (C_0 + \frac{1}{2}\rho_s^{\parallel})^2} \left(\frac{H}{w} \right)^4 < 1. \end{aligned} \quad (10)$$

Replacing the inequality in (10) by an equality gives a quadratic equation defining a single physical value of H^2 which we denote $(H_c^*)^2$, given by

$$(H_c^*)^2 = H_p^2 [1 - (\rho_0 K_b)^{1/2} / (C_0 + \frac{1}{2}\rho_s^{\parallel})]. \quad (11)$$

(The second solution of the quadratic equation gives $\varepsilon > 1$, and thus unphysical values of $|k_z^{(0)}|$ and α .) If $(C_0 + \frac{1}{2}\rho_s^{\parallel})^2 < \rho_0 K_b$, so that $(H_c^*)^2 < 0$, the $\hat{l} \parallel \mathbf{V}_s$ texture is stable for $H = 0$, and finite H only tends to stabilize it further. If $(C_0 + \frac{1}{2}\rho_s^{\parallel})^2 > \rho_0 K_b$, so that $(H_c^*)^2 > 0$, the $\hat{l} \parallel \mathbf{V}_s$ texture is stable for $H > H_c^*$, and unstable for $H < H_c^*$. We have not yet examined the nature of the new stable texture when the $\hat{l} \parallel \mathbf{V}_s$ texture is unstable. However, from our knowledge of the optimum fluctuation (cf. eq. (9)) we anticipate that a distorted helical texture will be formed. In the limit of $H/H_c^* \approx 1$, the distortion should become greatest; nevertheless the texture is not expected to be planar because $\varepsilon < 1$ even for $H = H_c^*$.

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