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B(E2; $2^+ \rightarrow 0^+$) VALUES FOR HEAVY DEFORMED EVEN-EVEN NUCLEI

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Résumé. — Nous calculons les taux de transition $B(E2; 2^+ \rightarrow 0^+)$ pour la bande du fondamental de noyaux pair-pair très déformés par l'approche simplifiée de Kota [1]. Les valeurs BE2 expérimentales sont bien reproduites avec des charges effectives allant de 1,1 e à 0,6 e quand le nombre de neutrons de *parité naturelle* varie de 6 à 20.

Abstract. — $B(E2; 2^+ \rightarrow 0^+)$ values for the ground state band of heavy deformed even-even nuclei are calculated using the simplified approach of Kota [1]. It is found that the experimental BE2 values are well reproduced using effective charges ranging from 1.1 e to 0.6 e as the *natural parity* neutron number changes from 6 to 20.

To study the structure of the low lying states of even-even deformed nuclei having nucleons in different shells, a simplified approach (hereafter referred to as SAK) has been suggested by one of the authors [1]. This approach considers the natural parity neutrons (*) alone to be active and is prompted by the fact that the casimir invariant of the leading pseudo SU(3) representation corresponding to natural parity neutrons is much higher than that of the natural parity protons. The study of the $N = 66$ even- A nuclei [1] and that of the nuclei having eight natural parity neutrons in

$\tilde{\eta} = 4$ shell [4] supports this view point. The purpose of the present note is to calculate the $B(E2; 2^+ \rightarrow 0^+)$ values corresponding to the ground state band for all deformed even-even nuclei having natural parity neutrons in the $\tilde{\eta} = 4$ shell and see whether the experimental values can be reproduced with reasonable effective charges. We find that the predicted $B(E2; 2^+ \rightarrow 0^+)$ values are quite reasonable.

The classification of various nuclei Gd^{154} - Os^{190} according to natural parity neutron number is given in columns 1 and 3 of table I. The corresponding pseudo SU(3) representations are listed in column 2 of table I. In the present calculation, besides using SAK, a further simplification is made by assuming the 0^+ state in the ground state band to be

$$|(\tilde{\lambda}_1 \tilde{\mu}_1) \tilde{K} = 0 \quad \tilde{L} = 0 \quad \tilde{S} = 0 \quad J = 0 \rangle$$

and the 2^+ state to be

$$|(\tilde{\lambda}_1 \tilde{\mu}_1) \tilde{K} = 0 \quad \tilde{L} = 2 \quad \tilde{S} = 0 \quad J = 2 \rangle$$

(where $(\tilde{\lambda}_1 \tilde{\mu}_1)$ is the leading pseudo SU(3) representation corresponding to the natural parity neutrons). Taking only $T(\tilde{1}\tilde{1}) \tilde{K}_0 = 1 \quad \tilde{L}_0 = 2 \quad \tilde{S}_0 = 0 \quad J_0 = 2$ tensor part of $\tilde{Q}_{op}$ components (this part is shown to give 95 %-98 % of the static moments of odd- A nuclei in reference [5]) the BE2 values are calculated using the simplified expression

(*) In the case of heavy deformed even-even nuclei, the active neutrons will be filling the $1h_{9/2} \ 2f_{7/2} \ 2f_{5/2} \ 3p_{3/2} \ 3p_{1/2}$ and $1i_{13/2}$ orbits. In a deformed field, the Nilsson orbits that grow out of the spherical orbits $1h_{9/2} \ 2f_{7/2} \ 2f_{5/2} \ 3p_{3/2}$ and $3p_{1/2}$ can be assigned pseudo Nilsson quantum numbers with a major oscillator shell quantum number $\tilde{\eta} = 4$ (see references [2] and [3] for pseudo Nilsson classification). The Nilsson orbits that grow out of the $1i_{13/2}$ orbit will lie in the middle of the Nilsson orbits that grow out of the $\tilde{\eta} = 4$ shell orbits. When we fill the Nilsson orbits in order with a given number of neutrons, some of them will fill the Nilsson orbits that grow out of the $\tilde{\eta} = 4$ shell orbits, called *natural parity neutrons* and the rest will fill the Nilsson orbits that grow out of the $1i_{13/2}$ orbit, called *un-natural parity neutrons*. (Note that the parity corresponding to the $1i_{13/2}$ orbit is opposite to the parity corresponding to the orbits of $\tilde{\eta} = 4$ shell.) A more elaborate discussion of the concept of natural parity nucleons and un-natural parity nucleons can be found in reference [3].

$$\begin{aligned}
B(E2; 2^+ \rightarrow 0^+) = & \left[\left\{ e_{\text{eff}}^n (\hbar/m\omega_0) c / \sqrt{10} \right\} * (-1)^n * \right. \\
& * \sum_{(\lambda' \mu')} \sqrt{\frac{d(\lambda\mu)}{d(\lambda' \mu')}} \left\langle (\lambda\mu) S = 0 \parallel a_{(\eta 0) \frac{1}{2}}^+ \parallel (\lambda' \mu') S' = \frac{1}{2} \right\rangle^2 \\
& * \sum_{\rho} \langle (\lambda\mu) K = 0 L = 2 \quad (11) K_0 = 1 L_0 = 2 \parallel (\lambda\mu) K'' = 0 L'' = 0 \rangle_{\rho} \\
& \left. * U((\lambda\mu) (0\eta) (\lambda\mu) (\eta 0); (\lambda' \mu') (11)_{\rho}) \right]^2. \quad (1)
\end{aligned}$$

In the above expression $(\lambda\mu)$ is the leading pseudo SU(3) representation corresponding to the given number (N) of the natural parity neutrons. The summation is over all $(N - 1)$ particle pseudo SU(3) representations $(\lambda' \mu')$. The triple barred matrix elements are related through a $\sqrt{N}$ factor to the single particle SU(3) parentage coefficients (see Eq. (4) of ref. [6]). All the parentage coefficients required for the present calculation are available in reference [6]. The space symmetry partition labels are dropped from the triple barred matrix element, as they will be uniquely specified by the total spin quantum number (S). The SU(3) multiplicity labels are also dropped as they are redundant for the present calculation (see p. 139 of ref. [6]). The double barred matrix

element is the $SU(3) \supset R(3)$ Wigner coefficient and the U-coefficient is the SU(3) Racah coefficient [3]. ρ is the multiplicity label corresponding to the coupling $(\lambda\mu) \times (11) \rightarrow (\lambda\mu)$. $d(\lambda\mu)$ is the dimensionality of the SU(3) representation $(\lambda\mu)$, given by

$$d(\lambda\mu) = (\lambda + 1)(\mu + 1)(\lambda + \mu + 2)/2.$$

The factor c is the strength of the

$$T(\tilde{1}\tilde{1})\tilde{K}_0 = 1\tilde{L}_0 = 2\tilde{S}_0 = 0J_0 = 2$$

$\hbar/m\omega_0$ appearing in equation (1) is calculated in the usual way taking $R_0 = 1.4 \times 10^{-13}$ cm. e_{eff}^n is the neutron effective charge. In equation (1) the super-scripts ($\sim$) are dropped for brevity.

TABLE I
B(E2; $2^+ \rightarrow 0^+$) values for deformed even-even nuclei

Natural parity neutron number	Leading pseudo SU(3) representation	Nuclei	B(E2; $2^+ \rightarrow 0^+$)		Experimental	
			Calculated			
6	(18 0)	Gd ¹⁵⁴ Gd ¹⁵⁶ Dy ¹⁵⁶ Dy ¹⁵⁸ Er ¹⁶⁰ Yb ¹⁶²	0.799 (0.566), 0.813 (0.576), 0.840 (0.595),	0.813 (0.576), 0.826 (0.586), 0.854 (0.606).	0.75, 0.75, 0.83,	0.93, 0.96, 0.73.
8	(18 4)	Gd ¹⁵⁸ Gd ¹⁶⁰ Dy ¹⁶⁰ Dy ¹⁶² Er ¹⁶² Er ¹⁶⁴ Yb ¹⁶⁴ Yb ¹⁶⁶ HF ¹⁶⁶ HF ¹⁶⁸	0.976 (0.579), 0.992 (0.589), 1.008 (0.599), 1.025 (0.608), <u>1.042 (0.618),</u>	0.992 (0.589), 1.008 (0.599), 1.025 (0.608), 1.042 (0.618), <u>1.059 (0.628).</u>	0.93, 1.00, 1.10, 0.93,	0.96, 1.05, 1.10, 1.0.
10	(20 4)	Dy ¹⁶⁴ Er ¹⁶⁶ Yb ¹⁶⁸ HF ¹⁷⁰	1.081 (0.895), 1.118 (0.926),	1.100 (0.911), <u>1.136 (0.941).</u>	1.10, 1.10,	1.10,
12	(24 0)	Er ¹⁶⁸ Er ¹⁷⁰ Yb ¹⁷⁰ Yb ¹⁷² HF ¹⁷² HF ¹⁷⁴	1.044 (1.095), 1.061 (1.112), 1.077 (1.130),	1.061 (1.112), 1.077 (1.130), 1.094 (1.147).	1.10, 1.05, 0.92,	1.15, 1.25, 0.95.
14	(20 6)	Yb ¹⁷⁴ HF ¹⁷⁶ W ¹⁷⁸	1.190 (1.089), <u>1.222 (1.117).</u>	1.209 (1.105),	1.20,	1.20.
16	(18 8)	Yb ¹⁷⁶ HF ¹⁷⁸ HF ¹⁸⁰ W ¹⁸⁰ W ¹⁸² Os ¹⁸² Os ¹⁸⁴	0.797 (0.987), 0.821 (1.016), 0.834 (1.033), 0.846 (1.048).	0.809 (1.002), 0.821 (1.017), 0.834 (1.033),	0.97, 0.96, 0.84,	0.99, 0.75, 0.70,
18	(18 6)	Os ¹⁸⁶ W ¹⁸⁴ Pt ¹⁸⁸	0.612 (0.981), 0.621 (0.995).	0.604 (0.967)	0.6, 0.52,	0.72,
20	(20 0)	W ¹⁸⁶ O ¹⁸⁸ Os ¹⁹⁰	0.491 (0.887), <u>0.566 (0.913).</u>	0.499 (0.900),	0.51,	0.48.

B(E2; $2^+ \rightarrow 0^+$) values are expressed in units of $e^2 \times 10^{-48}$ cm⁴. All the experimental values are taken from reference [16]. There is one to one correspondence between the nuclei listed in column three and BE2 values listed in columns four and five. The values in the brackets (in column 4) are the BE2 values calculated with e_{OV}^n and the values outside the brackets are the BE2 values calculated with $e_{\text{eff}}^n(N)$. The values that are underlined are those for which there are no corresponding experimental values.

The $B(E2 ; 2^+ \rightarrow 0^+)$ values for all deformed nuclei are calculated using equation (1) and then a least square fit is made with the experimental values to obtain the over all best effective charge (e_{OVF}^n) which is found to be $\approx 0.838 e$. Besides this over all least square fitting, a regional fitting with respect to natural parity neutron number is also made (these effective charges are denoted by $e_{\text{eff}}^n(N)$). It can be seen from columns four and five of table I that the agreement with experiment is reasonable when we use e_{OVF}^n and in general good when we use $e_{\text{eff}}^n(N)$.

The success of the present calculation can be ascribed to the reasonable effective charges we obtained by the least square fit procedure. It can be seen from figure 1 that the effective charge varies from 1.1 to 0.62 as the natural parity neutron number changes from 6 to 20. When compared with the effec-

FIG. 1. — Variation of neutron effective charge (e_{eff}^n) with natural parity neutron number (N). e_{eff}^n are in units of e .

tive charges used for nuclei in various regions of the periodic table in different models, the effective charges obtained in a highly truncated calculation presented

here, are very reasonable. For instance, in the O^{16} core shell model calculations of oxygen isotopes by Halbert *et al.* [7] $e_{\text{eff}}^n \approx 0.5 e$ is used. In the shell model calculations of Ni isotopes by Cohen *et al.* [8] $e_{\text{eff}}^n \approx 1.5 e$ – $2.0 e$ is used. In the modified T.D.A. model of Pal *et al.* it is found that an effective charge of $1.35 e$ is to be used [9] to get good fit to experimental values in the case of Ni and Sn isotopes. In the shell model of heavy even-even nuclei by M. Baranger [10] where the method of linearized equation of motion is used, $e_{\text{eff}}^n \approx e$ is used to account for the experimental values. In the unified model when the core is replaced by harmonic oscillator which is coupled to external nucleons by a quadrupole force e_{eff}^n turns out to be $e_{\text{eff}}^n \approx 1.5 eZ/A$ as shown by Baranger and Kumar [11]. In the pairing + quadrupole model, Kisslinger [12] and Sorensen show that $e_{\text{eff}}^n \approx 1.0 e$. In the generalized pairing model, Hecht *et al.* [13] used $e_{\text{eff}}^n = 0.7 e$. In the $A = 70$ core VAP calculations by Warke and Gunye [14] $e_{\text{eff}}^n \approx 0.5 e$ is used. In literature one can find a variety of other model calculations but

$$e_{\text{eff}}^n \approx 0.5 e\text{--}1.0 e$$

is typical of almost all calculations.

One can see from figure 1 that the effective charge decreases on the average as the natural parity neutron number increases. This is understandable as the size of the core (protons + 82 neutrons + neutrons in $i_{13/2}$ orbit is the core in the present calculation) changes as A changes and we will be going towards the end of the $\eta = 5$ shell as A increases. In Cartesian representation $N = 6, 12, 20, 30$ correspond to closed orbits [15] in the $\tilde{\eta} = 4$ shell. Figure 1 shows that the e_{eff}^n has a minimum at closed orbits and increases with the addition of two neutrons and then decreases.

Structure calculations within the framework of SAK are under progress and will be reported in a future publication.

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References

- [1] KOTA, V. K. B., *J. Phys. Soc. Jpn* **42** (1977) 715.
- [2] HECHT, K. T., *Symmetry properties of nuclei*, Proc. XVth Solvay Conf. on Physics (Gordon and Breach), 1974, p. 301.
- [3] RATNA RAJU, R. D., DRAAYER, J. P. and HECHT, K. T., *Nucl. Phys. A* **202** (1973) 433.
- [4] KOTA, V. K. B., Dissertation, Andhra University, India (1976) Unpublished;
- KOTA, V. K. B. and RATNA RAJU, R. D. (to be published).
- [5] RATNA RAJU, R. D., SATYANARAYANA REDDI, M. and KOTA, V. K. B., *J. Phys. G* **13** No 8 (1977) 1063.
- [6] KOTA, V. K. B., *Pramana* **9** No 2 (1977) 129.
- [7] HALBERT, E. C., *et al.*, *Adv. Nucl. Phys.* (ed. M. Baranger and E. Vogt, Plenum Press, New York) **4** (1971) 315.
- [8] COHEN, S. *et al.*, *Phys. Rev.* **160** (1967) 903.
- [9] RAJ, R., GAMBHIR, Y. K. and PAL, M. K., *Phys. Rev.* **163** (1967) 1004.
- [10] BARANGER, M., *Phys. Rev.* **120** (1960) 957.
- [11] BARANGER, M. and KUMAR, K., *Nucl. Phys. A* **110** (1968) 490.
- [12] KISSLINGER, L. and SORENSON, S. A., *K. Dan. Vidensk. Selsk. Mat. Fys. Medd* **32** (1960) 9.
- [13] HECHT, K. T., MCGRORY, J. B. and DRAAYER, J. P., *Nucl. Phys. A* **197** (1972) 369.
- [14] WARKE, C. S. and GUNYE, M. R., *Phys. Rev. C* **12** (1975) 1647.
- [15] HECHT, K. T. and BRAUNSCHWEIG, D., *Nucl. Phys. A* **244** (1975) 365.
- [16] ANDREITSCHIEFF, W., SCHILLING, K. D. and MANFRASS, P., *At. Data Nucl. Data Tables* **16** (1975) 515.