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CRITICAL INDICES TO $O(1/n^2)$ FOR A THREE DIMENSIONAL SYSTEM WITH SHORT RANGE FORCES

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Résumé. — Les exposants critiques λ et μ associés respectivement au propagateur d'énergie et au vertex à deux points avec une insertion en φ^2 sont calculés à l'ordre $O(1/n^2)$ pour un système à $d = 3$ dimensions et avec des interactions à courte portée. Ces résultats combinés avec ceux d'Abe pour l'exposant η permettent de calculer tous les exposants critiques à l'ordre $1/n^2$ et de vérifier les lois d'échelle.

Abstract. — $O(1/n^2)$ corrections to the critical exponents λ and μ associated with the energy propagator and the two point vertex with one φ^2 insertion, respectively, are calculated for a $d = 3$ dimensional system with short range forces. Combined with Abe's previous result for η they yield the other indices and confirm a scaling law to the same order.

We report results of the calculation of the $O(1/n^2)$ corrections to the critical exponents for a $d = 3$ dimensional, n component system with $O(n)$ symmetric short range interaction. This work is motivated by the following facts : a wealth of information as to the critical indices, scaling functions, etc. to $O(1/n)$ has been available for years now [1]. Results going beyond the first order are scarce, however. In particular, the only index known to the second order is η [2]. Clearly, calculating any other independent exponent to $1/n^2$ enables one to obtain the rest through scaling laws. Combining field theoretic methods with powerful resummation techniques, spectacular progress has been made recently in calculating critical exponents for real systems [3, 4, 5]. It is self-evident that it would be extremely important to reach the physical region (say $d = 3$ and $n = 1, 2, 3$) also by the complementary technique of the $1/n$ expansion. At the present time, both the essential ingredients of the success of the expansion directly in terms of the coupling constant [3, 4, 5] are lacking in the $1/n$ case : we have no clear idea as to the asymptotic behaviour of the series and no long $1/n$ series are available. Nevertheless, we do hope that it will be possible to make some progress

in this field and we regard our $1/n^2$ results below as a step in that direction.

All calculations have been performed at the critical point. Two critical exponents have been calculated : λ -the exponent of the energy propagator and μ -the exponent of the two point vertex with one φ^2 insertion (all notations are the same as in [6]). Combined with Abe's result for η [2], either of the two can be used to calculate $1/n^2$ corrections to the indices α , β , γ , ν by making use of the scaling laws. The coefficients of n^0 , n^{-1} and n^{-2} are given in table I for the indices mentioned. Results for η and δ (determined by η itself through hyperscaling) have been included for completeness. (It may be worth mentioning that we recalculated η to $1/n^2$ as an exercise and recovered Abe's result).

The coefficients listed in table I are not small. If one extrapolated the $O(1/n^2)$ results to any physical value of n such as $n = 3$ for example, one would soon discover that the series are rather badly behaved. Though η and δ get somewhat closer to their Heisenberg model values [7] at the second order, the other indices deviate considerably ; in fact, the agreement is definitely worse than at the previous order. This is not

TABLE I

Critical exponents to $O(1/n^2)$ for a $d = 3$ dimensional system with short range forces. The entries give the coefficients of the indicated powers of n . The first corrections were calculated by several authors, see reference [1], and are quoted here for comparison. Of the $1/n^2$ corrections η is due to Abe [2], λ and μ are the main results in this paper, the rest was obtained through scaling laws.

Order Index	n^0	n^{-1}	n^{-2}
λ	1	$-64/3 \pi^2$	$2\,752/27 \pi^4 - 64/\pi^2$
μ	1	$-40/3 \pi^2$	$1\,888/27 \pi^4 - 32/\pi^2$
α	-1	$32/\pi^2$	$-13\,344/27 \pi^4 + 96/\pi^2$
β	1/2	$-4/\pi^2$	$176/3 \pi^4 - 16/\pi^2$
γ	2	$-24/\pi^2$	$3\,392/9 \pi^4 - 64/\pi^2$
δ	5	$-16/\pi^2$	$1\,408/9 \pi^4$
ν	1	$-32/3 \pi^2$	$4\,448/27 \pi^4 - 32/\pi^2$
η	0	$8/3 \pi^2$	$-512/27 \pi^4$

unlike the situation found in the ε expansion beyond the first two orders [8]. (Technically, i.e. for the number and complexity of graphs, the calculation at $O(1/n^2)$ may roughly be compared with an $O(\varepsilon^4)$ calculation.) The extrapolation of the $1/n$ series down to small values of n is, of course, quite illusory. What one really expects is that at higher orders wild oscillations set in and one will have to use resummation techniques similar to those applied in [3-5] to get meaningful results.

There is another point that vaguely reminds one of the situation in the ε expansion : one single graph (Fig. 1) gives an anomalously large contribution to λ (and the corresponding vertex to μ), indeed so large that this sole contribution dominates over the sum of the rest and is responsible for the terms proportional to π^{-2} in the third column of table I.

FIG. 1. — Diagram giving an anomalously large contribution to the critical index λ .

SOME COMMENTS ON THE CALCULATION. — The reader is recommended to consult reference [1] and also [6] for background material and basic techniques relating to the $1/n$ expansion. There is only one technical point we have to mention : in a perturbative calculation of critical exponents beyond the first order, one has to ensure that the condition for exponentiation

is fulfilled and that the delicate cancellation among cutoff dependent contributions coming from various graphs takes place. That means one has to handle the problem of regularization more carefully than at the first order. Having considered a number of possible cutoff procedures we came to the conclusion that the one that would introduce the least complication into an already complicated scheme consisted of cutting off the bubble string (the effective interaction in the $1/n$ expansion) for large momenta. This procedure can be given by a rather natural interpretation : introducing a nonlocal (momentum dependent) potential $u(k)$ instead of the single coupling constant u_0 into the interaction Hamiltonian will clearly cut off the theory for any reasonable choice of the function $u(k)$. It is also easy to show that to $1/n^2$ critical indices are universal with respect to $u(k)$ —not an unexpected result. In particular, we are free to choose a step function like $u(k)$, which then generates the regularization mentioned above.

Details of the calculation are completely impossible to give here. We restrict ourselves to a few comments only : the number of graphs contributing to λ and μ is already fairly large at this order, their structure is also quite complex : one has to face diagrams going up to the 6 loop level. The fact that we are calculating at $d = 3$ is crucial : some vertices such as the triangle and the quadrangle graph can be calculated explicitly for $d = 3$. This is the fact that makes the calculation (by hand) of the set of graphs containing them as building blocks feasible at all. As mentioned in [3], the same fact played an important role also in calculating the long series in the coupling constant. As a matter of fact, due to the simplifications taking place at $d = 3$, quite a few graphs that would yield infrared logarithms otherwise simply stay finite, while some others are effectively reduced to integrals with a lower number of loop momenta. (We note that similar reductions take place already at $O(1/n)$ for $d = 3$.) Having exploited all these simplifications we were still left with a considerable number of diagrams, some of them very complicated. These were treated as follows : the integrals were broken down to several pieces according to the usual ordering of the momentum variables and the energy denominators were expanded in Gegenbauer polynomials. The relevant terms could then be collected and the remaining integrations could in most of the cases be done relatively easily. In a number of cases, however, the following trick had to be applied : the graph was Mellin-transformed with respect to the cutoff, the long wavelength limit was then obtained by calculating the residue of the pole at the origin of the Mellin variable.

In order to minimize possible errors (and to facilitate locating those that unavoidably occurred) we calculated not only the single logarithms, but all the terms that were more singular than the naive dimension of the graph in question. This way, we were able to check carefully, the criteria of exponentiation and

the cancellation of cutoff dependence from the final result. The latter imposes more than a single condition on the calculation : the graphs can be grouped into a few classes such that the cutoff dependent terms cancel within each class separately. These internal consistency conditions, though very important, are not enough to control all the graphs contributing to a given critical index. One of the main motivations for calculating *two* independent indices besides the existing result for η was just to seek a condition that was able to embrace the totality of graphs we evaluated. We think that the complexity of the calculation

does justify this precaution, despite the considerable amount of labour that went into this extra check. Viewed from another angle, the mutual consistency of the results for three independently calculated exponents confirms the scaling law connecting them, namely $2\mu = \lambda + 4 - d - 2\eta$, to $O(1/n^2)$.

A detailed account of this work will be published elsewhere.

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