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SENIORITY FOUR ADMIXTURES IN THE LOW-LYING O^+ STATES OF EVEN-MASS TIN AND LEAD NUCLEI

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Résumé. — Nous utilisons de nouvelles mesures statistiques de violation de symétrie pour évaluer les composantes de séniorité totale quatre dans les états O^+ de basse énergie des noyaux de masse paire d'étain et de plomb. Notre approche est basée sur les énergies des centroïdes et les largeurs partielles des distributions spectrales à séniorité totale et parité fixées. Nous trouvons des états de séniorité quatre étonnamment bas. Cependant, l'état fondamental est toujours un état très pur de séniorité nulle.

Abstract. — New statistical measures of symmetry breaking are used to evaluate the total seniority four admixtures in the low-lying O^+ states of even-mass tin and lead nuclei. This approach is based on the centroid energies and partial widths of fixed total seniority and parity spectral distributions. Some seniority four states are found to be surprisingly low. However the ground state is always a very pure seniority zero state.

For many years the concept of seniority has been extensively used to describe the spectra of single-closed shell nuclei, considered as identical nucleon systems [1]. The total seniority s , i.e., the sum of the seniorities relative to each subshell, is generally assumed to be a fairly good quantum number for the low-lying states of these nuclei. Accordingly only the lowest seniority parts of the shell model basis are retained. Various methods of approximation, such as the BCS theory [1] and the broken pair approximation [2, 3], are based on this truncation of the shell model basis.

The evidence concerning the validity of total seniority mainly comes from the available exact shell model calculations [1, 4-7]. However for numerical purposes the latter are always restricted to small single-particle bases and small numbers n of valence nucleons. Such limits do not exist for the spectral distribution method [8]. As fixed total seniority and parity distributions are now available [9, 10], we can use them to examine the validity of total seniority in those nuclei where the shell model basis is too large for an exact or even a lowest seniority diagonalization to be carried out. In this paper we study the seniority four admixtures in the low-lying seniority zero states (with angular momentum zero

and parity $+$) of the even-mass tin and lead isotopes. As a first attempt we neglect the admixtures of states with $s > 4$.

Let $S(n, \pi, s)$ denote the subspace of the n particle space characterized by parity π and total seniority s . Except for $s = 0$, $S(n, \pi, s)$ can be decomposed further into various subspaces $S(n, \pi, s, J)$ specified by a given value J of the angular momentum :

$$S(n, \pi, s) = \sum_J \oplus S(n, \pi, s, J), \quad s \neq 0, \quad (1a)$$

$$S(n, +, 0) = S(n, +, 0, 0). \quad (1b)$$

The states belonging to $S(n, \pi, s, J)$ are denoted by $|n\pi s \alpha J M\rangle$, where α specifies all the quantum numbers necessary to complete the classification.

We want to describe the low-lying O^+ states within the space $S(n, +, 0, 0) \oplus S(n, +, 4, 0)$. Neglecting for the moment the total seniority mixing, a diagonalization of the hamiltonian $\mathcal{H}$ within each of both subspaces $S(n, +, s, 0)$, $s = 0, 4$, can be simulated by applying Ratcliff's procedure [8] to the corresponding fixed s , parity $+$ and $J = 0$ spectral distribution. Assuming this distribution to be gaussian ⁽¹⁾,

⁽¹⁾ As mentioned in references [9] and [10], this distribution might be rather far from a gaussian. We think however that the error we introduce by assuming it gaussian is smaller than that coming from other approximations, especially from eq. (6).

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the energy $E_i(s)$ of the i th eigenstate of $\mathcal{H}$ within $S(n, +, s, 0)$ is given by the solution of the following equation :

$$F[n, +, s, 0, E_i(s)] \equiv [2 \pi \sigma^2(n, +, s \rightarrow s, 0)]^{-1/2} \times \int_{-\infty}^{E_i(s)} \exp \left\{ -\frac{1}{2} [E' - E(n, +, s, 0)]^2 / \sigma^2(n, +, s \rightarrow s, 0) \right\} dE' = i - \frac{1}{2}. \quad (2)$$

Here $E(n, +, s, 0)$ and $\sigma(n, +, s \rightarrow s, 0)$ are the centroid energy and the internal width of the distribution respectively. They are defined by

$$E(n, +, s, 0) = [d(n, +, s, 0)]^{-1} \times \sum_{\alpha} \langle n + s\alpha 00 | \mathcal{H} | n + s\alpha 00 \rangle, \quad (3)$$

and

$$\sigma^2(n, +, s \rightarrow s', 0) = [d(n, +, s, 0)]^{-1} \times \sum_{\alpha\alpha'} |\langle n + s'\alpha' 00 | \mathcal{H} | n + s\alpha 00 \rangle|^2 - \delta_{ss'} [E(n, +, s, 0)]^2, \quad (4)$$

where $d(n, +, s, 0)$ is the dimension of $S(n, +, s, 0)$.

As the moments of fixed s , π , and J distributions are still unknown, we have to make some assumptions about them. In references [9] and [10], we describe a procedure for calculating the centroid energies $E(n, \pi, s)$ and partial widths $\sigma(n, \pi, s \rightarrow s')$ of fixed s and π distributions, which are defined in a way similar to eqs. (3) and (4). For instance

$$E(n, +, s) = [d(n, +, s)]^{-1} \times \sum_{\alpha JM} \langle n + s\alpha JM | \mathcal{H} | n + s\alpha JM \rangle, \quad (5)$$

where $d(n, +, s)$ is the dimension of $S(n, +, s)$. Owing to eq. (1b), we have $E(n, +, 0, 0) = E(n, +, 0)$, and $\sigma(n, +, 0 \rightarrow 0, 0) = \sigma(n, +, 0 \rightarrow 0)$. For the seniority 4 and parity + subspace, we assume that the $J = 0$ states are distributed in the same way as the $J \neq 0$ states, i.e.,

$$\begin{aligned} E(n, +, 4, 0) &\approx E(n, +, 4), \\ \sigma(n, +, 4 \rightarrow 4, 0) &\approx \sigma(n, +, 4 \rightarrow 4). \end{aligned} \quad (6)$$

This statistical approximation has been tested in ^{62}Ni with the Auerbach interaction [6] starting from the exact shell model matrices, and has been found to be reasonable [9]. It should be still better in Sn and Pb nuclei, where the dimensions of the subspaces involved are much larger than in ^{62}Ni .

The solution of eq. (2), taking eq. (6) into account, always leads to a seniority zero ground state as it is expected [$E_1(0) < E_1(4)$]. In figures 1 and 2 we

FIG. 1. — Excitation energy of the second seniority zero and the first seniority four states in the Sn nuclei. The full (dot and dash) line corresponds to $s = 0$, the dashed (dotted) line to $s = 4$, and the interaction is the SDI (GI).

FIG. 2. — Excitation energy of the second seniority zero and the first seniority four states in the Pb nuclei. The full (dot and dash) line corresponds to $s = 0$, the dashed (dotted) line to $s = 4$, and the interaction is the SDI (GI).

show the excitation energies of the second $s = 0$ state, $E_2(0) - E_1(0)$, and the first $s = 4$ state,

$$E_1(4) - E_1(0),$$

calculated with a surface delta interaction (SDI) and a gaussian interaction with Serber mixture (GI), in Sn and Pb nuclei respectively. The parameter values of the interactions are the same as in references [9] and [10]. The first excited O^+ state is a seniority zero state only in ^{104}Sn , ^{106}Sn , ^{108}Sn (with GI only), ^{128}Sn (with SDI only), ^{200}Pb , ^{202}Pb , and ^{204}Pb . In the other nuclei, containing more particles or holes in the open shell, some seniority 4 states are found surprisingly low. The behaviour of seniority 0 and 4 states is qualitatively the same for both interactions. However some local differences may be non negligible. For instance the seniority 4 states are seen to be much lower in the heaviest lead isotopes with the GI than with the SDI.

When we turn on the interaction between $s = 0$ and $s = 4$ states, we have to take the mixing width $\sigma(n, +, 0 \rightarrow 4, 0)$ into account. Its value is known because it is equal to $\sigma(n, +, 0 \rightarrow 4)$ owing to eq. (1b) and the rotational invariance of $\mathcal{H}$. It is always found rather small in comparison with the internal width $\sigma(n, +, 0 \rightarrow 0, 0)$:

$\sigma(n, +, 0 \rightarrow 4, 0)/\sigma(n, +, 0 \rightarrow 0, 0) \leq 0.17$ and 0.24 in Sn and Pb respectively. However in many cases it is of the same order as or larger than the distance between the centroids of $S(n, +, 0, 0)$ and $S(n, +, 4, 0)$. The parameter

$$I^2 \equiv \sigma^2(n, +, 0 \rightarrow 4, 0) \times [E(n, +, 0, 0) - E(n, +, 4, 0)]^{-2} \quad (7)$$

takes values up to 27.81 in ^{124}Sn and 0.85 in ^{196}Pb with the SDI, and 23.71 in ^{124}Sn and 1.37 in ^{198}Pb with the GI. Therefore in the first order perturba-

tion approximation, the average intensity $(1 + I^2)^{-1}$ of the $s = 0$ components in the perturbed states arising from $S(n, +, 0, 0)$ is in general very small as can be seen in tables I and II. When they are considered as a whole, the $s = 0$ and $s = 4$ subspaces are thus completely mixed.

However as the internal widths $\sigma(n, +, 0 \rightarrow 0, 0)$ are very large, some low-lying $s + 0$ states might remain rather pure when the interaction between the $s = 0$ and $s = 4$ states is turned on. To check this possibility, we need a much finer measure of symmetry breaking than I^2 . In references [9] and [11] we showed that the set of parameters J_i^2 , defined in the present case by

$$J_i^2 \equiv [d(n, +, 0, 0)]^{-1} \sigma^2(n, +, 0 \rightarrow 4, 0) \times \sum_{j=1}^{d(n, +, 4, 0)} [E_i(0) - E_j(4)]^{-2}, \quad (8)$$

$$i = 1, 2, \dots, d(n, +, 0, 0),$$

TABLE I

*Intensity of the total seniority zero components in the low-lying O^+ states of the Sn nuclei.
The energy given between parenthesis is $E_1(4) - E_1(0)$*

A	SDI			GI		
	$E_i(0) - E_1(0)$	$(1 + J_i^2)^{-1}$	$(1 + I^2)^{-1}$	$E_i(0) - E_1(0)$	$(1 + J_i^2)^{-1}$	$(1 + I^2)^{-1}$
104	0	0.996	0.876	0	0.995	0.915
	2.701	0.983		2.725	0.986	
	(3.800)			(4.201)		
106	0	0.995	0.817	0	0.994	0.861
	2.959	0.961		3.028	0.975	
	(3.378)			(3.650)		
108	0	0.994	0.784	0	0.994	0.825
	(2.770)			3.317	0.005	
				(3.322)		
110	0	0.994	0.725	0	0.994	0.764
	(2.275)			(2.792)		
112	0	0.995	0.645	0	0.995	0.676
	(1.979)			(2.342)		
114	0	0.995	0.540	0	0.995	0.555
	(1.933)			(2.161)		
116	0	0.995	0.396	0	0.995	0.399
	(1.788)			(1.977)		
118	0	0.995	0.236	0	0.995	0.230
	(1.515)			(1.663)		
120	0	0.994	0.113	0	0.994	0.109
	(1.251)			(1.284)		
122	0	0.993	0.044	0	0.993	0.048
	(1.380)			(1.302)		
124	0	0.993	0.035	0	0.993	0.040
	(1.884)			(1.597)		
126	0	0.994	0.221	0	0.993	0.203
	(2.669)			(2.120)		
128	0	0.995	0.680	0	0.993	0.631
	2.769	0.958		(2.529)		
	(3.311)					

TABLE II

*Intensity of the total seniority zero components in the low-lying O^+ states of the Pb nuclei.
The energy given between parenthesis is $E_1(4) - E_1(0)$*

A	$E_i(0) - E_1(0)$	SDI $(1 + J_i^2)^{-1}$	$(1 + I^2)^{-1}$	$E_i(0) - E_1(0)$	GI $(1 + J_i^2)^{-1}$	$(1 + I^2)^{-1}$
204	0	0.995	0.958	0	0.991	0.845
	1.443	0.992		1.399	0.973	
	2.269	0.986		(2.188)		
	2.903	0.973				
	3.446	0.859				
	(3.640)					
202	0	0.993	0.871	0	0.988	0.655
	1.599	0.986		1.571	0.956	
	2.467	0.968		(1.945)		
	(2.905)					
200	0	0.992	0.748	0	0.989	0.492
	1.779	0.983		1.776	0.745	
	(2.478)			(1.857)		
198	0	0.991	0.625	0	0.989	0.423
	(1.816)			(1.482)		
196	0	0.989	0.540	0	0.989	0.443
	(1.208)			(1.071)		

where $E_i(0)$ and $E_j(4)$ are solutions of eq. (2), can be used for that purpose. Whenever $E_i(0) - E_1(4)$ is negative and not close to zero, $(1 + J_i^2)^{-1}$ should be a good approximation for the intensity of the $s = 0$ components in the region of the i th perturbed state arising from $S(n, +, 0, 0)$ [11].

The values obtained for this quantity in Sn and Pb nuclei are displayed in tables I and II respectively. The more important result is the extreme smallness of the seniority 4 admixtures (< 0.012) in the ground state of all the nuclei studied with both interactions. Let us stress once again that this property, which is a necessary (but not sufficient) condition for the validity of the BCS or the broken pair approximation, has never been tested before in heavy nuclei such as tin or lead, except in very restricted cases.

As above-mentioned, for excited states, our procedure does not allow us to find anything about the energy range above the first $s = 4$ state. Therefore we only get results for nuclei containing a very small number of particles or holes in the open shell, in which there are some $s = 0$ excited states below the first $s = 4$ state. In this case the $s = 4$ admixtures are larger than in the ground state but remain moderate.

We have made some additional calculations where

we have replaced J_i^2 by the more accurate measure K_i^2 defined in reference [11]. As a consequence of the large dimension of $S(n, +, 4, 0)$, the results remain essentially the same except for those states which are almost degenerate with the first excited $s = 4$ state; the differences are not significant anyway because this quasi-degeneracy is very sensitive to assumption (6). From this study we conclude that the measure J_i^2 is quite reliable within the statistical approximations we have made. To improve the results for the $s = 4$ admixtures significantly, it would be necessary to work on these approximations themselves. In particular it would be highly desirable to get fixed s , π and J distributions in order to be able to drop assumption (6).

The calculations reported in this paper show the usefulness of the measure J_i^2 of symmetry breaking, introduced in references [9] and [11], in a physically relevant case for the first time. It would be interesting too to apply this new measure to other physically relevant cases.

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