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SIZE DEPENDENCE OF THE PERCOLATION THRESHOLD OF SQUARE AND TRIANGULAR NETWORK (*)

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Résumé. — On étudie au moyen d'une simulation par ordinateur l'influence de la taille du réseau sur l'apparition du seuil de percolation pour des réseaux plans limités carrés et triangulaires.

Cette étude permet d'apprécier la taille minimale compatible avec une précision fixée pour la détermination du seuil.

L'hypothèse d'universalité fournit un moyen d'exprimer cet effet de taille en fonction d'une longueur de corrélation unique.

Abstract. — We study the size dependence of the percolation threshold of square and triangular networks by a computation simulation method.

This work gives an estimate of the accuracy on the threshold determination, which can be reached as a function of the size of the network.

This size effect can be explained using a single correlation length in accordance with the universality hypothesis.

Recently a number of studies based on computational (Monte-Carlo) [1] methods, or analog models [2, 3] have provided data on the percolation threshold p_c and onset of conduction above p_c in various 2 or 3 dimensional networks. We have obtained a description of the conduction problem for 3D percolation in a disordered mixture of conducting and insulating, but otherwise similar, spheroids [4]. In these studies, because of the finite number N of elements, transitions around p_c are smeared.

It is the purpose of this letter to discuss in more detail the size effect involved in site percolation. The study is based on a computer simulation on 2D square and triangular lattices. The size of the regular network is characterized by the number n ($20 \leq n \leq 1\,000$) such that the total number of elements is $N = n^2$. Each site has a probability p to be conducting obtained in a random way from a pseudo-random number generating subprogram. The program gives the number of clusters of conducting sites, their extent along two orthogonal directions and the number of sites in each cluster. If a cluster contains elements in both the first and last row the

corresponding configuration is said to be conducting [5].

For given size N and doping p , m configurations are drawn, l of which are conducting. The ratio l/m gives an estimate of the probability of conduction Π . For small networks : $n = 20$ we have produced $m = 100$ configurations. Because of the increase of computational time, m was limited to 25 for $n = 100$ and to 5 for $n = 1\,000$. (For this experiment the solution of a network requires 30 minutes on a PDP 11 computer). For the same reason we have limited n to 300 for triangular networks.

Figure 1 gives $\Pi(p)$ in square networks for different values of n . Let us consider one curve, say $n = 20$. For p smaller than 0.45 we find no conducting configuration whereas, for p larger than 0.70, all configurations are conducting. We characterize the limit of the transition regime by the values $p_1 = 0.52$ and $p_2 = 0.65$ such that $\Pi = 0.1$ and $\Pi = 0.9$. The transition region becomes narrower as n increases. The same features can be observed on all the curves plotted for square and triangular lattices.

In figure 2, we have plotted n versus p_1 and p_2 on a semilog scale. The curve indicates a divergence for infinite networks around the critical value p_c . Note the rather symmetrical shape of the curves around p_c . From the largest network data it is possible

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FIG. 1. — Gives the probability of conduction across a square network of n elements versus the probability of site conduction.

FIG. 2. — p_1 and p_2 are defined on figure 1 for $n = 20$. The inset gives the complete curve from $n = 1$ to 1000 for a square network. The values for small N have been computed numerically (for $n = 1$, $p_1 = 0.1$ and $p_2 = 0.9$; for $n = 2$, $p_1 = 0.23$, $p_2 = 0.83$).

to obtain an estimate of the percolation threshold from the inequality $p_1 < p_c < p_2$ with an absolute accuracy better than 10^{-2} . These results are in good agreement with the exact value for the triangular lattice ($p_c = \frac{1}{2}$) and with independent determinations of the percolation threshold for site problem in a square lattice [1, 6], $p_c = 0.59 \pm 0.01$.

In the description of the static critical behaviour around a second order phase transition, one expects size effects to scale in terms of a *single* correlation

length, here $\xi(p)$ [6], which should diverge at the critical threshold. We can understand our results qualitatively in terms of a cluster description of a phase transition. If one assumes that this is the case in the percolation problem, the $n(p)$ variation of figure 2 displays the expected divergence of the correlation length around p_c . More qualitatively one sees that the value $p_1(n) < p_c$ (pretransitional behaviour) corresponds to the growth of conducting clusters up to the size n . The value $p_2(n) > p_c$ (limit of the critical regime in a finite geometry) can be analyzed in a similar fashion if one considers the matching lattice. For $p > p_2$, the non conducting clusters are too small to prevent the formation of conducting channels through the limited geometry. We therefore consider the possibility of a critical dependence.

$$n(p_i) \propto \xi(p_i) = A/p_i - p_c / -\nu \quad i = 1, 2.$$

In figure 3 we have tested this dependence using a log-log plot. For triangular lattice, where $p_c = 1/2$ is known exactly (a fortunate but rare case in the determination of critical exponents !), the results above and below p_c are described by a straight line with the same slope $\nu = 1.06 \pm 0.1$.

FIG. 3. — Same as figure 2 in log-log units indicating the power divergence of a characteristic length for square networks ($\square$) and triangular networks ($\triangle$).

For square lattices, p_c has been adjusted to give the best linear fit; the value $p_c = 0.595$ obtained is consistent with the values known for p_c [1, 6]. The slope is $\nu = 1.1 \pm 0.1$. This insensitivity of the value of ν to the type of network points towards a certain

universality of the meaning of the scaling length in this problem.

Recently the percolation problem has been analyzed in the framework of the renormalization group approach [7] using the Potts model [8]. The value obtained here is not inconsistent with the estimates $\nu = 1.3 \pm 0.3$ given in [6] and also obtained with restricted geometry, and with the better results obtained by Padé's approximants method [9].

It is also possible to obtain from this simulation an estimate of the fraction of elements in the critical cluster. This result together with conduction data obtained on spheroid networks as a function of size will be presented in a forthcoming paper.

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