Classification
 Physics Abstracts
 7.730

ON THE SUPERFLUID PHASES OF ^3He IN A MAGNETIC FIELD

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(Reçu le 24 avril 1975, accepté le 3 juin 1975)

Résumé. — On étudie l'effet d'un champ magnétique sur l'hélium-3 dans l'hypothèse de paires de Cooper dans l'état p. On prévoit en particulier une nouvelle phase où le paramètre d'ordre n'est pas uniforme dans l'espace mais sinusoïdal. Ces résultats sont étendus de manière à tenir compte de certains effets de couplage fort et l'on montre comment des mesures effectuées sous champ magnétique peuvent fournir des indications sur les paramètres phénoménologiques de Landau.

Abstract. — The behaviour of superfluid ^3He in the presence of a magnetic field is discussed assuming an $L = 1$ pairing. In particular we predict a new phase with a spatially non uniform (sinusoidal) order parameter. These results are extended in order to take into account strong-coupling effects and we show how measurements under a magnetic field may provide information on the phenomenological Landau coefficients.

A complete experimental study of the effect of a magnetic field on the superfluid phases of ^3He would be a good test of the p-wave assumption which is commonly accepted. This effect has been theoretically investigated by Ambegaokar and Mermin [1], within the framework of the weak-coupling model, and for small values of the field. Takagi [2] has studied the general case for the A_1 and A_2 phases. We shall examine the problem with less restrictive assumptions: we first consider the weak-coupling model for arbitrary fields; in the second part we show how the weak-coupling results have to be modified to take into account strong-coupling effects in all the phases, in the limit of small fields.

1. The weak-coupling model. — The general form of the condensation amplitudes in the different phases may be seen from symmetry considerations alone. The invariance group of the normal phase is generated by the operators $N_\uparrow$, $N_\downarrow$, L [11]. If we assume, following Ambegaokar and Mermin, that a transition to a state with up spin pairs will take place first, the order parameter in the A_1 phase is of the form

$$\langle a_{-\mathbf{k}\uparrow} a_{\mathbf{k}\uparrow} \rangle \propto Y_1^m(\hat{\mathbf{k}}).$$

In the weak-coupling theory $m = \pm 1$ are the stable solutions. If, for instance, $m = +1$, one easily sees that the invariance group of the A_1 phase is generated by $N_\downarrow$ and $-\frac{1}{2}N_\uparrow + \hbar^{-1}L_z$. Then the order parameter of the second order transition to the A_2 phase which breaks this invariance is

ter of the second order transition to the A_2 phase which breaks this invariance is

$$\langle a_{-\mathbf{k}\downarrow} a_{\mathbf{k}\downarrow} \rangle \propto Y_1^{\pm 1}(\hat{\mathbf{k}}).$$

In the strictly weak-coupling model the solutions $Y_1^{\pm 1}$ are degenerate; a small deviation from this model stabilizes either Y_1^1 (*ABM like state*) or Y_1^{-1} (*planar like state*). These phases are still invariant under the change $k_z \rightarrow -k_z$.

If a second order transition may occur to a phase with $S_z = 0$ pairs, then the corresponding order parameter is

$$\langle a_{-\mathbf{k}\uparrow} a_{\mathbf{k}\downarrow} \rangle \propto \hat{k}_z.$$

Ambegaokar and Mermin have shown that such a transition is expected for fields $> H_1$ (≈ 1 kG). For $H < H_1$ the transition becomes first order.

However, several years ago Fulde and Ferrell [3] suggested that a conductor might go into a superconducting state in which the Cooper pairs have a non zero total momentum, when an exchange field acts on the spins of the conduction electrons; in such a state the translation invariance is also broken. Sarma and Saint-James [4] have shown that the Fulde-Ferrell state could only exist in the case of *clean* superconductors, i. e. when the motion of the electrons is not affected by impurity scattering. The transition to the normal state is second order. Such a situation

is expected in ^3He , which can be viewed as a *clean* superfluid [9], with an order parameter :

$$\langle a_{-\mathbf{k}+\mathbf{q}/2\uparrow} a_{\mathbf{k}+\mathbf{q}/2\downarrow} \rangle \propto \hat{k}_z \delta(\mathbf{q} \pm \mathbf{Q}).$$

Therefore we shall examine the antiparallel pairing for high values of the field. Allowing for the degeneracy of the axial and planar solutions, we take the gap parameters in the A_2 phase as

$$\Delta_+(\mathbf{k}) = \Delta_+ \cdot (-\hat{k}_x + i\hat{k}_y),$$

$$\Delta_-(\mathbf{k}) = \Delta_- \cdot (\hat{k}_x + i\hat{k}_y) \quad (\text{planar})$$

which gives the highest critical temperature for the antiparallel pairing. The linearized equation for the gap Δ_0 is written as

$$\Delta_0(\mathbf{k}, \mathbf{q}) = T \sum_n \sum_{\mathbf{k}'} V_{\mathbf{k}\mathbf{k}'} \left[G_{\uparrow} \left(\mathbf{k}' + \frac{\mathbf{q}}{2}, \omega_n \right) G_{\downarrow} \left(-\mathbf{k}' + \frac{\mathbf{q}}{2}, -\omega_n \right) \Delta_0(\mathbf{k}', \mathbf{q}) + F_{\uparrow} \left(\mathbf{k}' + \frac{\mathbf{q}}{2}, \omega_n \right) F_{\downarrow} \left(-\mathbf{k}' + \frac{\mathbf{q}}{2}, -\omega_n \right) \Delta_0^*(\mathbf{k}', -\mathbf{q}) \right] \quad (1)$$

where G_{σ} and F_{σ} are the single particle propagators in the A_2 phase [8]

$$G_{\sigma}(\mathbf{k}, \omega_n) = - \frac{i\omega_n + \varepsilon_{\mathbf{k}\sigma}}{\omega_n^2 + \varepsilon_{\mathbf{k}\sigma}^2 + |\Delta_{\sigma}(\mathbf{k})|^2}, \quad F_{\sigma}(\mathbf{k}, \omega_n) = \frac{\Delta_{\sigma}(\mathbf{k})}{\omega_n^2 + \varepsilon_{\mathbf{k}\sigma}^2 + |\Delta_{\sigma}(\mathbf{k})|^2}$$

$\varepsilon_{\mathbf{k}\sigma} = \varepsilon_{\mathbf{k}} + \sigma\gamma H$ being the quasi-particle energies in the normal phase. Far from T_{c0} (i. e. for high fields) we make the approximation $\Delta_+ \simeq \Delta_- = \Delta$.

1.1 THE A_2 -B TRANSITION. — The solution of (1) with $\mathbf{q} = \mathbf{0}$ yields the A_2 -B line, given in reference [1] near T_{c0} within the Landau theory :

$$1 = vT \sum_n \sum_{\mathbf{k}} \hat{k}_z^2 \frac{\varepsilon_{\mathbf{k}}^2 - (i\omega_n + \gamma H)^2 + |\Delta(\mathbf{k})|^2}{(\omega_n^2 + E_{\mathbf{k}}^2)(\omega_n^2 + E_{\mathbf{k}}^2)}.$$

Performing the frequency sum, one gets :

$$\frac{1}{vN(0)} = \int_{-\omega_c}^{+\omega_c} \frac{d\varepsilon}{4\varepsilon} \int_0^{\pi} d\theta \sin \theta \cos^2 \theta \times \\ \times \left[\frac{\varepsilon + \gamma H}{[(\varepsilon + \gamma H)^2 + \Delta^2 \sin^2 \theta]^{1/2}} \text{th} \frac{1}{2T} [(\varepsilon + \gamma H)^2 + \Delta^2 \sin^2 \theta]^{1/2} + \frac{\varepsilon - \gamma H}{[(\varepsilon - \gamma H)^2 + \Delta^2 \sin^2 \theta]^{1/2}} \text{th} \frac{1}{2T} [(\varepsilon - \gamma H)^2 + \Delta^2 \sin^2 \theta]^{1/2} \right].$$

The solution $H(T)$ of this implicit equation has been numerically computed after solving the gap equation for Δ and is plotted in figure 1. It exhibits a smooth bump as in the case of superconductors [10] ; this suggests an instability of the second order transition, as will be shown in what follows.

1.2 THE SINUSOIDAL STATE. — The physical explanation for a Fulde-Ferrell state in ^3He is the following (at $T = 0$) : in the presence of an external magnetic field the Fermi surface splits into 2 spheres $S_{\uparrow}$ and $S_{\downarrow}$; in the A_2 phase the pairing takes place independently on each of these surfaces and mostly involves the particles lying in the xy plane since

$$|\Delta_{\pm}|^2 \propto \hat{k}_x^2 + \hat{k}_y^2.$$

Therefore the region along the z axis is available for antiparallel pairing. Figure 2 shows the usual BCS pairing ; but in order to associate a particle on $S_{\uparrow}$

FIG. 1. — Paramagnetic critical field for the $\uparrow\downarrow$ pairing versus temperature. The dashed part of the curve is unstable.

FIG. 2. — BCS pairing. For the sake of clarity we have only represented a part of the region involved in the $\uparrow\uparrow$ pairing (shaded region) and a part of the region involved in the $\uparrow\downarrow$ pairing (heavily shaded region).

with one on S_1 one may form a pair with a non-vanishing center of mass momentum along the z direction, i. e. parallel to the angular momentum (Fig. 3). For sufficiently high fields such a phase is expected to have a lower energy. Moreover in the second case the solid angle containing the particles to be paired is smaller than in the BCS case and the corresponding transition would not be so strongly affected by the $\Delta_{\pm}$ components.

FIG. 3. — Antiparallel pairing with non zero total momentum.

Therefore we look for solutions of (1) of the form

$$\Delta_0(\mathbf{k}, \mathbf{Q}) = \Delta_0^*(\mathbf{k}, -\mathbf{Q}) \propto \hat{k}_z.$$

The Fulde-Ferrell state is favorable when

$$K(\mathbf{q}, \Delta) > K(0, \Delta)$$

where

$$K(\mathbf{q}, \Delta) = vT \sum_n \sum_{\mathbf{k}} \hat{k}_z^2 \left[G_{\uparrow} \left(\mathbf{k} + \frac{\mathbf{q}}{2}, \omega_n \right) \times \right. \\ \times G_{\downarrow} \left(-\mathbf{k} + \frac{\mathbf{q}}{2}, -\omega_n \right) \\ \left. + F_{\uparrow} \left(\mathbf{k} + \frac{\mathbf{q}}{2}, \omega_n \right) F_{\downarrow} \left(-\mathbf{k} + \frac{\mathbf{q}}{2}, -\omega_n \right) \right].$$

We may write

$$K(\mathbf{q}, \Delta) = K(0, \Delta) + [K(\mathbf{q}, \Delta) - K(0, \Delta)].$$

The term between brackets does not depend strongly on Δ as previously seen [5] so that we expand it up to second order in Δ

$$K(\mathbf{q}, \Delta) - K(0, \Delta) \simeq K(\mathbf{q}, 0) - K(0, 0)$$

terms proportional to $|\Delta(\mathbf{q})|^2$.

The transition point is first reached for $|\Delta(\mathbf{q})|^2 = 0$, i. e. $\mathbf{q} \parallel \text{Oz}$.

The kernel calculated in the normal phase is

$$K(\mathbf{q}, 0) = vT \sum_n \sum_{\mathbf{k}} \hat{k}_z^2 \frac{1}{\varepsilon_{\mathbf{k} + \frac{\mathbf{q}}{2}\uparrow} - i\omega_n} \frac{1}{\varepsilon_{-\mathbf{k} + \frac{\mathbf{q}}{2}\downarrow} + i\omega_n}.$$

Taking $\varepsilon_{\mathbf{k} + \mathbf{q}/2} \simeq \varepsilon_{\mathbf{k}} + \mathbf{q} \cdot \mathbf{v}$ as is usually done [8]

$$K(\mathbf{q}, 0) = 2\pi v T N(0) \times \\ \times \text{Re} \sum_{\omega > 0} \int_0^\pi \frac{\cos^2 \theta \sin \theta d\theta}{\omega_n + i(qv \cos \theta + \gamma H)}.$$

The curvature $\partial^2 K / \partial q^2$ ($q = 0$) becomes positive for $\gamma H / kT > 1.911$ as shown in reference [4]. The Fulde-Ferrell state is stable if $T < T^* \simeq 0.45 T_{\infty}$ and if $\gamma H > \gamma H^* \simeq 0.85 T_{\infty}$, T^* and H^* being the coordinates of the intersection point of the straight line $\gamma H / kT = 1.911$ and of the curve $H(T)$ computed in 1.1. The transition from the F-F state to the uniform (BW) state is first order. The sinusoidal phase has been represented in figure 1 (however the A_2 -FF and FF-B lines have not been calculated exactly. In summary this state is characterized by a longitudinal wave

$$\Delta_0(\mathbf{k}, \mathbf{x}) = \Delta_0 \cos(\mathbf{Q} \cdot \mathbf{x}) \hat{\mathbf{k}} \cdot \mathbf{l} \quad (2)$$

where $\mathbf{l}$ is a unit vector along the direction of the angular momentum, thus perpendicular to the field $\mathbf{H}$, and $\mathbf{Q}$ is a function of H and corresponds to the maximum of $K(\mathbf{q}, 0)$: $\partial K / \partial q(\mathbf{Q}, 0) = 0$. From an experimental point of view, the two lines A_2 -FF and FF-B could be deduced from specific heat measurements. Evidences for the sinusoidal state would be given by neutron diffraction or NMR since a spin density wave is associated with the gap (2), or by acoustic attenuation measurements. In a thin film experiment, with a field parallel to the film, one may observe a resonance

for the values of the field yielding a wavelength commensurate with the thickness of the film ⁽¹⁾.

2. Extension of the weak-coupling model. — We restrict ourselves to the case of weak fields (i. e. close to T_c). The free energy reads :

$$\mathcal{F} = \frac{3}{2} \int d\Omega(\hat{\mathbf{k}}) \times [\alpha_+ |\Delta_+(\mathbf{k})|^2 + \alpha_- |\Delta_-(\mathbf{k})|^2 + \alpha_0 |\Delta_0(\mathbf{k})|^2] + \mathcal{F}^{(4)}.$$

We take for the fourth-order contribution $\mathcal{F}^{(4)}$ the zero field terms of reference [6] :

$$\begin{aligned} \mathcal{F}^{(4)} = & \beta_1 |\text{tr } A\tilde{A}|^2 + \beta_2 (\text{tr } A A^\dagger)^2 + \\ & + \beta_3 \text{tr} [A\tilde{A}(A\tilde{A})^*] + \beta_4 \text{tr} (A A^\dagger)^2 \\ & + \beta_5 \text{tr} [A A^\dagger (A A^\dagger)^*]. \end{aligned}$$

The matrix A is defined by

$$\begin{bmatrix} \Delta_+ & \Delta_0 \\ \Delta_0 & \Delta_- \end{bmatrix} = \sum_{\mu\nu} (\sigma_\mu A_{\mu\nu} \hat{k}_\nu) i\sigma_y.$$

We have to consider the 3 solutions (using continuity arguments with the weak-coupling case) :

axial (ABM) :

$$\begin{aligned} \Delta_+(\mathbf{k}) &= \Delta_+ \cdot (-\hat{k}_x + i\hat{k}_y) \\ \Delta_-(\mathbf{k}) &= \Delta_- \cdot (-\hat{k}_x + i\hat{k}_y) \quad \Delta_0(\mathbf{k}) = 0 \end{aligned}$$

planar (P) :

$$\begin{aligned} \Delta_+(\mathbf{k}) &= \Delta_+ \cdot (-\hat{k}_x + i\hat{k}_y) \\ \Delta_-(\mathbf{k}) &= \Delta_- \cdot (\hat{k}_x + i\hat{k}_y) \quad \Delta_0(\mathbf{k}) = 0 \end{aligned}$$

BW :

$$\begin{aligned} \Delta_+(\mathbf{k}) &= \Delta_+ \cdot (-\hat{k}_x + i\hat{k}_y) \\ \Delta_-(\mathbf{k}) &= \Delta_- \cdot (\hat{k}_x + i\hat{k}_y) \quad \Delta_0(\mathbf{k}) = \Delta_0 \hat{k}_z. \end{aligned}$$

The corresponding $\mathcal{F}^{(4)}$ energies are

$$\begin{aligned} \mathcal{F}_{\text{ABM}}^{(4)} &= (\beta_2 + \beta_4) (\Delta_+^4 + \Delta_-^4) + \\ &+ 2(\beta_2 + \beta_4 + 2\beta_5) \Delta_+^2 \Delta_-^2 \\ \mathcal{F}_{\text{P}}^{(4)} &= (\beta_2 + \beta_4) (\Delta_+^4 + \Delta_-^4) + \\ &+ 2(2\beta_1 + \beta_2 + \beta_3 + \beta_5) \Delta_+^2 \Delta_-^2 \\ \mathcal{F}_{\text{BW}}^{(4)} &= (\beta_2 + \beta_4) (\Delta_+^4 + \Delta_-^4) + \\ &+ 2(2\beta_1 + \beta_2 + \beta_3 + \beta_5) \Delta_+^2 \Delta_-^2 + \beta_0 \Delta_0^4 \\ &+ 2\beta_2 (\Delta_0^2 \Delta_+^2 + \Delta_0^2 \Delta_-^2) + 4\beta_1 \Delta_0^2 \Delta_+ \Delta_- \end{aligned} \quad (3)$$

$$\left(\text{we set } \beta_0 = \sum_{i=1}^5 \beta_i \right).$$

We have seen that the axial and planar solutions are degenerated in the weak-coupling limit. More generally this degeneracy occurs when

$$2\beta_1 + \beta_3 - \beta_4 - \beta_5 = 0. \quad (4)$$

The hyperplane (Π) defined by (4) divides the space of the parameters β_i into 2 regions. If

$$2\beta_1 + \beta_3 - \beta_4 - \beta_5 > 0$$

(region I) the axial state has the lower energy ; in this case a continuous change to the B type phase is impossible and the transition becomes first order (Fig. 4).

FIG. 4. — Energies of the P, BW and ABM states versus temperature.

If $2\beta_1 + \beta_3 - \beta_4 - \beta_5 < 0$ (region II) the planar state is stable and the transition is still second order. The behaviour of the phase diagram is thus different according to whether the physical manifold

$$\beta_i = \beta_i(T, P)$$

cuts the hyperplane (Π) or not. This is shown in figures 5 and 6.

FIG. 5. — a), b) Evolution of the phase diagram P, T at increasing applied field if the point $\{\beta_i(T, P)\}$ belongs to region I for all values of T and H . The thin lines stand for 2nd order transitions, the thick lines for 1st order transitions. The sinusoidal phase has not been represented.

The same arguments hold for the transition to the sinusoidal state : it is first order in region I, second order in region II. However the Fulde-Ferrell state is expected to vanish at high pressure, when spin fluctua-

⁽¹⁾ De Gennes P. G., private communication.

FIG. 6. — a), b), c) Evolution of the phase diagram P, T at increasing applied field if the manifold $\beta_i(T, P)$ crosses (II) .

tions are significant [12], actually when the three energies $\mathcal{F}_{ABM}$, $\mathcal{F}_{BW}$ and $\mathcal{F}_{FF}$ become equal.

Remark : since the representative point of the weak-coupling model is lying on (II) it corresponds to an instability threshold for the second order transition. The specific heat jump given in reference [1] is not valid and actually diverges for all values of the field because of the fluctuations of the order parameter in the set of the degenerated solutions (the degeneracy was arbitrarily removed in reference [1]).

From the previous considerations, one may derive some quantitative results for the first order transition (for weak fields).

From (3) one gets the energy of the ABM state

$$\mathcal{F}_{ABM} = \frac{\alpha^2}{2} \left(-\frac{t^2}{B_1} + \frac{\eta^2 h^2}{2\beta_5} \right)$$

and the energy of the BW state assuming a small Δ_0

$$\mathcal{F}_{BW} = \frac{\alpha^2}{2} \left(-\frac{t^2}{B_2} + \frac{\eta^2 h^2}{B_3} \right) - \frac{\tilde{\alpha}_0^2}{\beta_0},$$

where we have set $\alpha_{\pm} = \alpha(t \pm \eta h)$ following reference [1],

$$B_1 = 2(\beta_2 + \beta_4 + \beta_5),$$

$$B_2 = 2\beta_1 + 2\beta_2 + \beta_3 + \beta_4 + \beta_5,$$

$$B_3 = 2\beta_1 + \beta_3 - \beta_4 + \beta_5,$$

$$\tilde{\alpha}_0(t, h) = \alpha_0(t, h) - \frac{2\alpha\beta_2 t}{B_2} - 2\alpha\beta_1 \left(\frac{t^2}{B_2^2} - \frac{\eta^2 h^2}{B_3^2} \right)^{1/2}$$

$\alpha_0(t, h)$ is of the form $\alpha t - bh^2$.

The parameters α , η , B_1 and β_5 can be fitted with measurements on the A_1 and A_2 transitions [2], [7]. The other parameters b , β_1 , β_2 , β_3 , β_4 may be obtained from the shape of the A_2 -B line whose equation is given by

$$\mathcal{F}_{ABM} = \mathcal{F}_{BW}$$

and from the entropy jump

$$\left(-\frac{\partial \mathcal{F}_{BW}}{\partial T} + \frac{\partial \mathcal{F}_{ABM}}{\partial T} \right),$$

magnetic susceptibility, etc... The validity of these formulas is restricted to low pressure, i. e. when the deviation from the weak-coupling model is small (one may guess $P < P_{PCP} \simeq 20$ bar).

Acknowledgments. — I am indebted to L. de Seze who did the numerical computation, to G. Sarma who suggested the possibility for a Fulde-Ferrell state, to P. G. de Gennes, N. Boccara, R. Balian and especially J. M. Delrieu for illuminating discussions.

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