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HIGH-FREQUENCY MONOSTATIC ECHOES FROM FINITE-LENGTH CYLINDERS

D. BRILL(1) and G.C. GAUNAURD (*, (1))

U.S. Naval Academy, Physics Department, Annapolis, Maryland 21402, U.S.A.

*Naval Surface Warfare Center, Research Department, (R43) White Oak, Silver Spring, Maryland 20903-5000, U.S.A.

ABSTRACT - We study the scattering of plane monochromatic acoustic waves of frequency $f$ by rigid, finite-length cylinders of various $L/D$-ratios, submerged in a fluid, at arbitrary incidence angles $\theta$. The cylinder end-caps are either flat or hemispherical. The (monostatic) backscattering cross-sections are determined by the physical optics (P.O.) or Kirchhoff method, and in some instances by saddle-point techniques. The far-field scattered pressures as functions of $\theta$ are used to obtain target strength (TS) estimates in decibels. We derive the pertinent formulas and display the resulting TS as functions of $\theta$ and $f$ in various relevant situations. We compare results for various terminations and by various methods. The results are valid down to relatively low frequencies.

1. INTRODUCTION

One of the oldest and simplest methods to determine high-frequency target-strength estimates, is the method of physical optics (or Kirchhoff) method [1,2]. Although the method is based on physically wrong assumptions (viz., the shadow region is perfectly dark and no energy ever leaks into it, i.e., there can be no diffraction!) and does not emerge as the first term of any asymptotic theory of diffraction [3], it gives surprisingly good estimates of the diffracted fields even in the low-frequency region - where it was not designed to work - with relative ease. This state of affairs must be very annoying to pure mathematicians. The method has been extended by the works associated with the names of Fock, Keller and Ufimsev, but we will not deal here with these modifications. We use the approach in its old simplicity to predict the scattering of plane waves by impenetrable cylinders of finite-length (the method can not handle penetrable bodies) in water at any angle of incidence, for various types of cylinder-terminations. We display many calculations.

2. THEORY.

The physical optics method (it is called like that even in acoustics - since the term "physical acoustics" is normally used for something else) predicts the form-function of a body of arbitrary shape by the expression:

$$f(\theta = \pi) = \frac{1}{\lambda} \int_{S'} \hat{k} \cdot \hat{r} e^{2\pi i k \cdot r} dS'$$

(1)

where $\lambda$ is the wavelength of the incident radiation, $\hat{k}$ is the wavevector, and $S'$ is the

(1) Supported by NSWC, the ONR and NOSC.
insonified portion of the scatterer's surface. The unit outward normal to \( S' \) is \( \hat{n} \) and \( dS' \) is the surface element. For a sphere of radius \( a \) insonified from the North pole: 
\[
\hat{k} \cdot \hat{n} = -\cos\theta, \quad \hat{n} = a = (\sin\theta \cos\phi, \sin\theta \sin\phi, \cos\theta)
\]
and 
\[
dS' = a \sin\theta d\phi d\theta.
\]
In this case, Eq. (1) yields 
\[
f(\pi) = \frac{1}{\lambda} \int_{\pi/2}^{\pi/2} \int_{-\pi/2}^{\pi/2} (-\cos\theta) \exp \left[ -2ik(r\cos\theta) \right] \sin \theta d\phi d\theta,
\]
which can be integrated by parts and simplified to give, eventually,
\[
f(\pi) = -\frac{a}{2} e^{-2ikr} \left[ e^{2ix} - e^{-ix} \right]
\]
where \( x = ka \). The backscattering or sonar cross-section \( \sigma \) is defined by
\[
\sigma = 4\pi \left| f(\pi) \right|^2.
\]
in terms of the form-function, in 3-D. The Target-Strength (TS) is defined as \( \sigma/4\pi \) expressed in decibels. Substituting into (4) yields,
\[
\frac{\sigma}{\pi a^2} = 1 - \frac{\sin 2\pi}{x} + \left( \frac{\sin x}{x} \right)^2 = \left[ 1 - \frac{i}{2x} \left( 1 - e^{-2ix} \right) \right]^2,
\]
which is the (normalized) sonar cross-section for a sphere by this method.

For a circular cylinder of radius \( a \) and finite length \( 2\ell \) under arbitrary incidence along \( \hat{k} = k (\sin\theta, 0, \cos\theta) \), the contribution to Eq. (1) from the curved side of the cylinder is:
\[
f(\pi) = \frac{1}{2\pi} \int_{-\pi/2}^{\pi/2} \int_{-\pi/2}^{\pi/2} (-\sin\phi) \cos \phi \exp \left[ -2ix\sin\theta\cos\phi \right] e^{-2ik\ell\cos\theta} d\phi dz
\]
where we have used \( \hat{k} \cdot \hat{n} = -\sin\theta \cos\phi \) and \( \hat{r}' = a \sin\phi \hat{\phi} + a \cos\phi \hat{z} \). The result is:
\[
\text{(side)}(\pi) = \pm i x \ell \sin \theta J_0(2k\ell \cos \theta) \left\{ \frac{2}{\pi} H_1(2x\sin \theta) - i J_1(2x\sin \theta) \right\}.
\]
The contribution from the top surface (assumed flat) is, analogously,
\[
f(\pi) = \frac{1}{2\pi} \int_{-\pi/2}^{\pi/2} \int_{-\pi/2}^{\pi/2} (-\cos \phi) \exp \left[ -2ik(r'\sin\theta \cos\phi + \ell \cos \theta) \right] r' dz' d\phi
\]
where \( r' = rsin\theta \) and \( k = k (\sin\theta \hat{\phi} + \cos \theta \hat{z}) \). Using the integral definitions of \( J_0(z) \) and \( J_1(z) \) we find the flat-top contribution:
\[
\text{(top)}(\pi) = \pm i \left\{ \frac{a}{2} \cos \theta \exp \left[ 2ik\ell \cos \theta \right] J_1(2x\sin \theta)/x\sin \theta \right\}.
\]
We note that this result contains that of a circular flat disk (i.e., \( \ell = 0 \)) which yields the sonar cross-section
\[
\sigma = \pi a^2 J_1(2x\sin \theta)/\tan^2 \theta.
\]
At normal incidence (i.e., \( \theta = 0 \)), Eq. (10) yields: \( \sigma = 4\pi \left( A/\pi \right)^2 \), where \( A = \pi a^2 \) is the area of the circular disk. At normal incidence we obtain the same result as in Eq. (9) when the termination is hemispherical. All bodies with the same shadow-boundary produce the same shadow-forming wave. Away from normal incidence, the contribution from a hemisphere, if evaluated by the saddle-point method, yields only the specular contribution which is:
\[
f(\pi) = 1 - 1/2x \quad \text{(from Eq. (5))}.
\]
Clearly, for long cylinders, the influence of various end-caps is felt only for incidences close to the axial direction i.e., within angles \( \theta = \tan^{-1}(D/L) \), which are small for large \( L/D \) values.

We note that a finite-length cylinder of length \( \ell \) with a hemispherical end-cap insonified from below at the arbitrary incidence angle \( \theta \) leads to an integral of the type in Eq. (1) with
\[
\hat{k} \cdot \hat{n} = (\sin \theta \sin \phi \cos \theta - \cos \theta \cos \phi)
\]
\[
\hat{k} \cdot \hat{r}' = kr + ka \left( \sin \theta \sin \phi \cos \theta + \cos \theta \cos \phi \right) - k\ell \cos \theta.
\]
FIGURE 1. Angular plot of the target strength \( \text{TS}=10 \log |\bar{f}(\theta)|^2 \) of a finite-length cylinder with flat ends. Here, \( L=120'' \), and \( L/D=6 \). Also, \( f=20 \text{ kHz} \). TOP: Combined top side and curved side contributions. BOTTOM: Curved side contribution alone.

FIGURE 2: Same as Fig. 1 but now \( f=3 \text{ kHz} \). TOP: Curved side and flat-top contributions combined. CENTER: Side contribution alone. BOTTOM: Isolated contribution from a hemispherical end-cap.

FIGURE 3: The effect of increasing the frequency. Same as in Fig. 1, adding the curved side and the flat-top contributions. Here \( L=120'' \& L/D=6 \). TOP: \( f=3 \text{ kHz} \). CENTER: 20 kHz. BOTTOM: 50 kHz.

FIGURE 4: Analogous to Figs. 1 and 2. Here \( L=6'' \). Side and flat-top contributions added. TOP: \( L/D=2 \& f=50 \text{ kHz} \). BOTTOM: \( L/D=2 \), and \( f=20 \text{ kHz} \).
Such an integral admits a saddle-point evaluation of the second-order [4], that will be discussed at the Conference, and elsewhere [5].

3. NUMERICAL RESULTS.

The above results were programmed for numerical evaluation. For a (long) cylinder of \( L/D = 6 \), the target strength is \( TS = 10 \log |f(\omega)^2| \), with \( f(\omega) \) as given by Eq. (7) (top), or by Eqs. (7) plus (9) (bottom), and it is shown in Fig. 1 at \( f = 20 \) kHz. Here \( \lambda = 3^\circ \) which is small compared to both \( L \) and \( D \). Near normal incidence \( (\theta = 90^\circ) \) the plots show no differences. For small incidence angles the differences are substantial if one accounts for the end-cap contribution or not. The actual shape of the end-cap makes relatively little difference. In Fig. 1 the end-cap is flat. At lower frequencies, viz., \( f = 3 \) kHz, for the same size cylinder, Fig. 2 shows the pertinent plots. The top graph accounts for the side and flat-top contributions. The side contribution is displayed alone in the center, and the contribution from a hemispherical cap is shown isolated in the bottom graph. At 3 kHz the wave-length is \( \approx 20^\circ \), which is about the size of the cylinder's diameter. Here, the end-cap contribution is substantial at nearly all incidences away from the normal. The effect of increasing the frequency for a cylinder with flat heads and \( L/D = 6 \) is shown in Fig. 3. Here, the frequency increases from 3 kHz (top) to 20 (center) to 50 (bottom), respectively. As the frequency increases the scattering pattern exhibits progressively more lobes. The peak values at normal incidence also increase from 35, to 43.7 to 47.7 dB. The peak values at axial incidence decrease from 70, to 50, to 44 dB, respectively. For shorter cylinders, viz., \( L/D = 2 \), the peak values of TS for axial incidence decrease with increasing frequency from 46 dB down to 41 dB. For normal incidences the peak values increase from 9.3 dB to 13.2 dB. These numerical values are representative of bodies of these dimensions at these aspect angles and frequencies [6]. Although the exact levels are slightly off and the lobe structure of the patterns is somewhat displaced, the simple predictions of the physical optics method give estimates of the TS with sufficient accuracy to serve as inputs to any detection or classification scheme. We remark that more sophisticated predictions can and have been generated for the various bodies discussed here, particularly those based on Keller's Geometrical Theory of Diffraction [7,8].

4. CONCLUSIONS.

We have generated predictions for the monostatic cross-sections of some simple-shaped bodies that are based on the physical optics method. The results have been displayed in various relevant instances. In spite of its limitations, the method yields satisfactory estimates of the T.S. values.

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