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ID-LOCALIZATION OF SURFACE ACOUSTIC WAVES BY QUASIPERIODICALLY CORRUGATED SURFACES

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Abstract: Specific properties of the propagation of surface acoustic waves on quasiperiodically corrugated solids are reviewed. This problem corresponds to the critical regime of the Anderson localization transition, characterized by critical proper modes which are neither extended nor localized and which exhibit remarkable scaling features. The spectrum is also predicted to have a Cantor-like structure. The experimental system is made of a thousand grooves engraved according to a Fibonacci sequence. For the first time, the self-similar spatial structure of the critical proper modes is observed through an optical diffraction experiment. Signatures of the fractal spectrum are also reported. These results are explained in terms of the asymptotic approximation of the quasicrystal by periodic systems of increasing periods.

1. INTRODUCTION

Rayleigh surface acoustic waves (SAW) may become localized in presence of a rough surface. This localization phenomenon of SAW has recently been studied theoretically /1-3/ but no experimental evidence is available up to now. Here, we present the first experimental evidence of this effect for SAW.

Wave propagation in an inhomogeneous medium leads to the phenomenon of Anderson localization /4,5/ at any non-vanishing disorder for space dimension d=1 and d=2 and at sufficiently high disorder for d=3. The localization regime is a subtle non-perturbative effect involving coherent interferences between all the wavelets partially reflected by the quenched disordered set of scatterers, for which exist only partial theoretical scenarios /4,5/. Quasiperiodicity, neither true periodicity nor randomness, can be considered intermediate between these two extremes /6/. Its study is interesting, in particular, in a one dimensional (d=1) quasiperiodic systems, since a transition exists between an extended and a localized regime similarly to what occurs in a (d=3)-disordered system /7/. The existence of a transition between two regimes is always exciting because one can hope that understanding the crucial features which trigger the transition will allow to unravel the physics of the different regimes. The existence of such a critical Anderson localisation transition accounts for the self-similarity of the spectrum and of the proper modes which we now describe /8/.

2. DESCRIPTION OF THE EXPERIMENTS

A Rayleigh surface acoustic wave (SAW) propagates at the quasiperiodically corrugated surface of a piezoelectric lithium niobate (YZ-LiNbO3) substrate of total length L=15984µm, with N=10^3 identical grooves engraved, using the well known micro-lithographic techniques /9/. The average distance 'a' between the grooves is thus a=L/(N-1)=16µm. Each groove has a width w=5 µm, a depth h=0.3)xm and a well characterized inverse plateau profile. The lateral scale of each groove (the so-called opening) is E=2150µm. The groove centers are positioned on the sites of the Fibonacci sequence built recursively from successive concatenation of lower order patterns /10/

\[ S_{j+1}=S_{j}+S_{j} \]  \hspace{2cm} (1)

One has \( S_0=(a) \), \( S_1=(c) \), \( S_2=(cc) \), \( S_3=(ccsc) \) and so forth. \( s=(11.6\pm0.1)\mu m \) and \( c=(18.7\pm0.1)\mu m \) are the two elementary tiles of our one-dimensional quasicrystal. We have \( s/c=0.620\pm0.005 \) near the golden mean \( t=(\sqrt{5}-1)/2=0.6180 \) . The ±0.05µm precision of the position of the grooves corresponds to the limitation of the electronic etching technique.

Our typical surface acoustic wave set-up is composed of electro-mechanical transducers, laid out at the surface of the YZ-LiNbO3 crystal, which surround the array of etched grooves. They perform the launching and detection of the surface acoustic waves both in reflection and transmission. The Rayleigh wave has well-known characteristics /9/. It is a mixture of longitudinal and transverse acoustic modes. It propagates along the solid-air plane boundary and is evanescent away from the

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solid boundary with a typical excursion of the order of the wavelength ($\lambda=20\mu\text{m}$ at typical frequencies around 170MHz). Its phase velocity $c_R=3490\text{m/s}$ is slightly less than the transverse wave velocity $c_t$ and its dispersion relation is linear in the absence of surface corrugation. In the presence of a single groove, the SAW is partially reflected with a reflection amplitude coefficient given by $\mu=0.6(\pi/\lambda)\sin(2\pi w/\lambda)=10^{-2}$ for our frequency range. Furthermore, a fraction $p=20\%$ of the SAW energy is detrapped and converted into longitudinal and shear bulk acoustic waves $\beta,\gamma$. A full description of the experiments can be found in /8/.

3 - THE SPECTRUM

The description of the SAW propagation on the corrugated solid has been developed within a transfer matrix theory /3/. Using the mapping approach of Kohmoto et al. /10,11/ recalled and adapted to our system /3/, the spectrum has been numerically calculated /8/. In an infinite system, our results are comparable to previous ones obtained for a quasiperiodic Schrödinger equation with a step potential /10,11/ since the transfer matrix formalism is the same in both cases. In particular, we learn from this analogy that this system exactly corresponds to the so-called "critical" regime intermediate between the extended and the localized regimes. This means that the spectrum is a Cantor set of zero measure. Thus, gaps (or stop bands) are present at all frequency scales.

These asymptotic properties are not really observed in their totality in our experiments. This comes from the finite length of the system which contains (only) $N=10^5$ grooves. Also, due to the fact that each groove is shallow ($h=0.3\mu\text{m}$) compared to the typical value of the wavelength $\lambda=20\mu\text{m}$, most of the stop bands of the spectrum are too small and will not be observed.

**Fig.1** : Dependence of the SAW reflection modulus as a function of frequency. Each peak probes the existence of a stop band.

Fig.1 gives the dependence of the SAW reflection coefficient obtained experimentally as a function of frequency $f$ in the range 153-193 MHz. The large scale bell-like shape corresponds to the transfer function of the measuring transducers. The informations relevant to the study of the (d=1)-quasicrystal are the peaks which decorate this structure. We observe the existence of particular frequencies for which the reflection coefficient is significantly increased. Note that reflection peaks appear as alternate doublets due to interference with the direct transducer-transducer propagation. These peaks can be interpreted as the largest stop-bands of the system. We have checked that the pattern of peaks obtained under reflection was exactly recovered under transmission /8/.

We have computed numerically the "finite size" spectrum determined from the transfer matrix formalism /3,8/ and the mapping described in /10,11/ with the criterion that only those frequencies yielding a value of the trace of the transfer matrix for the whole system larger than 2 are selected as "effective" stop bands. We have taken explicit account of the finite size $N=10^5$ of the system and of the small value of the amplitude reflection coefficient $\mu$ in order to predict which features of the Cantor set should remain observable in our experiments. The agreement is found to be quite good. We note furthermore that the most prominent peaks on fig.1 in the neighborhood of the central frequency $f=175$ Mhz can be indexed by a single integer $n$ such that the variable $a/\lambda$ is of the form

$$a/\lambda=n/(2\pi n)$$

For instance, the central peak occurs for $a/\lambda=0.5010$ at $f=175$ Mhz and a wavelength $\lambda=19.7\mu\text{m}$. On fig.1, one can identify the different frequencies giving the smallest $n$ for the rational expression (2), for instance, of $a/\lambda = n/6$: $f=191.35\text{Mhz}$; $\lambda=18.12\mu\text{m}$, $n=7$; $a/\lambda=7/13$; $f=189.35\text{Mhz}$; $\lambda=18.35\mu\text{m}$, etc... These special frequencies, which correspond to the largest stop bands in the finite size system, are those for which the transmission is small enough or conversely the reflection is large enough in order to be detected.

Qualitatively, the values of these gap frequencies can be related to the following essential property of our quasicrystal:
it is the asymptotic limit of a series of periodic systems of larger and larger periods \( a_1 \), each corresponding to the successive "words" of the iterative concatenation process (1). It is well known that, for any periodic sub-system, there should be a gap at half of any reciprocal lattice vector \( 2\pi/b \), of the crystal, and all gaps and differences of the reciprocal lattice vectors of the periodic structures are reciprocal lattice vectors. As the system becomes higher and higher order periodic, the smallest reciprocal lattice vector of the system gets smaller and smaller. Thus, in the almost periodic limit, there should be a gap in the vicinity of every wave vector. This explains intuitively the highly fragmented Cantor-like band structure. We can go further and rationalize the existence of the observed series (2) as follows.

Consider first the smallest period \( a_0 \) corresponding to the complete pattern of letters "s" and "c". Since "s" (resp. "c") occurs with relative frequency \( t/(t+1) \) (resp. \( 1/(t+1) \)), \( a_0 \) is given by

\[
a_0 = s \frac{t}{t+1} + c \frac{1}{t+1}
\]

Thus \( a_0 = \alpha \), the average period of the system. We expect a stop band at the Bragg condition \( 2a_0 = \lambda \), i.e., at a value of the reduced parameter \( \alpha = 1/2 \). From the self-similar structure of the infinite quasicrystal which is invariant under the following transformation in the tiling

\[
c \rightarrow s \quad \text{and} \quad sc \rightarrow c
\]

a second period \( a_t = \alpha t \) comes out. The discussion is easily generalized to the larger periods \( a_i = \alpha^i a_0 \), obtained by replacing the word \( S_j \) by "c" and \( S_{j-1} \) by "s" in the infinite quasicrystal. Note the remarkable scaling invariance of the quasi-crystal with respect to this transformation. This leads to gaps at frequencies such that

\[
\alpha^{t^2} = \lambda/2
\]

Expression (5) gives only a part of the whole spectrum. Indeed, it must be generalized to take account of the interactions between the different periods in the system. Consider the two reciprocal lattice vectors \( 2n/a_0 \) and \( 2n/a_t \). Their sum is \( 2(n/a_0)(1+t)=2n/a_0t \) which is again a reciprocal lattice vector with \( b=a_0t \). The Bragg condition applied to it yields a gap for \( a_0/\lambda = 1/2 \). This is the main gap observed experimentally on fig.1. Repeating the argument for the two reciprocal lattice vectors \( 2n/a_0 \) and \( 2n/a_t \) yields a reciprocal vector equal to \( 2n/a_0^2 \), and so on... One thus generates gaps at all wavelengths for which eq.(5) holds but with \( j<0 \). These gaps are of course a few among the infinite set of the singular continuous spectrum. However, they are the largest since they correspond to successive periodic approximations of the quasicrystal with the smallest periods. These periods will give the largest reflection and smallest transmission since this will correspond to the largest number of periods per unit length in the effective periodic lattice.

We can now understand the existence of the full series (2) observed experimentally in the finite system. Until now, we only have considered combinations of the fundamental reciprocal wave vectors \( 2\pi/b \). Of course, if \( 2\pi/b \) is a reciprocal lattice vector, \( 2\pi/(2\pi+1)b \) is also a reciprocal lattice vector since it corresponds to a period \( 2\pi+1 \) which is a multiple of the fundamental period \( b \). For example, if we take \( b = a \) and consider the following combination \( 4\pi n/(2\pi+1)\alpha^{t^1} + 4\pi n/(2\pi+1)a = 2\pi (2\pi/(2\pi+1))/\alpha \). The Bragg condition for this reciprocal lattice vector exactly yields the series (1). Such combinations of reciprocal vectors allow to understand the peculiar role played by experimentally observed frequencies such that \( \alpha^{t^2}/\lambda = 1/2 \) rational.

4 - THE SPATIAL STRUCTURE OF THE PROPER MODES

Along the lines of the transfer matrix formalism briefly described above, Thouless and Niu /12/ have proposed that the absolute value of a wave corresponding to a proper mode should be a product of periodic functions, the period of each function being the approximate period of the almost periodic problem obtained by cutting off the continued fraction representation of the relevant incommensurate number at a particular stage.

![Fig. 2: Spatial structure (SAW amplitude obtained with an heterodyne technique) of five modes obtained with two SAW incident from the left and from the right onto the system (condition of "zero energy flux") as a function of position of the laser spot along the sample. These modes corresponds to five close frequencies in the neighborhood of the main stop band shown on fig.1 (at/\lambda = 1/2).](image)

Experimentally, our observations are reported on fig.2 which shows the spatial structure of the SAW at particular frequencies in the close neighborhood of the main gap (such that \( at/\lambda \) is in the close neighborhood of \( 1/2 \) ). The SAW envelope amplitude for different frequencies is plotted as a function of the distance \( x \) from one extremity of the lattice. These spatial proper mode structures are obtained from an optical diffraction experiment using the so-called Raman-Nath effect which has been described elsewhere /8/. The figures correspond to a "zero flux" condition, i.e. to the superposition of two counterpropagating SAW's of the same frequency, amplitude and phase launched from the two transducers on both sides of the system. We have checked on many examples /8/ for different frequencies that the characteristic mode structure is not changed by using the "zero flux" condition.
In agreement with the theoretical predictions, we have observed that the modes contain several sinusoidal components of widely separated spatial periods. Consider for example the mode depicted on fig.2 whose frequency is \( f = 176.66 \text{MHz} \). Then, we obtain \( \lambda = 19.784 \text{nm} \) taking a value for the SAW velocity \( c_p = 3495 \text{ms}^{-1} / \text{MHz} \). This yields a value \( \alpha = 8/17 \). This shows that the spatial mode at this frequency is obtained by superposition of two periods since \( \alpha = \frac{1}{500} \). The difference of the reciprocal wave vector \( 2\pi/\lambda \) with \( b = 2a \) and \( 500a \). This frequency is placed on the right of the stop band for \( \alpha \approx 1/2 \) which explains the minus sign. Thus, one should observe a period equal to \( 2a \) corresponding to the first period \( b = 2a \) and a period equal to \( 500a \) corresponding to the third period \( b = 500a \). Therefore, the largest scale structure exhibits 1000/500=2undulations, each one being decorated by 500/2=250 smaller undulations. These different periods 2 and 500 correspond to a total number of oscillations over the system length. We can analyse in a similar fashion the modes measured in the vicinity of \( \alpha = 8/17 \).

This with this theory, we can understand the rapid change of the largest spatial modulation with the very small variation of SAW frequency. On going from \( f = 176.66 \text{MHz} \) to 176.9MHz, the large scale structure changes from having two undulations to five. Without adjusting any parameter, we can describe the spatial structure of all these modes. For \( f = 176.73 \text{MHz} \), \( \lambda = 19.746 \mu \text{m} \), with \( \alpha = 0.5008 \) and \( \alpha = \frac{1}{500} \). This corresponds to three lobes since 1000/331=3. This is well in agreement with fig.2. For \( f = 176.90 \text{MHz} \), \( \lambda = 19.727 \mu \text{m} \), with \( \alpha = 0.5012 \) and \( \alpha = \frac{1}{500} \). This corresponds to five lobes since 1000/202=5. For \( f = 178.60 \text{MHz} \), \( \lambda = 19.605 \mu \text{m} \), with \( \alpha = 0.5044 \) and \( \alpha = \frac{1}{500} \). This corresponds to seventeen lobes since 1000/58=17. This is well in agreement with fig.2 where we count 16±1 lobes. Note that this series is characterized by a single large scale modulation in agreement with the observation, since the small period is to small to be detected.

The existence of the large scale structure is the most typical signature of the criticality of the proper modes. The figures therefore constitute the first direct experimental test of the theoretical models describing wave propagation in quasi-periodic media. We can analyse in a similar fashion the modes measured in the vicinity of \( \alpha \) given by eq.(2) as well as all other modes. In particular, by picking up the right values of \( \alpha \), we can observe almost any spatial structure as rich or complex as desired for the critical proper modes. This complexity is completely determined from the rational expansion of \( \alpha \) as a sum of reciprocal wavevectors. In other words, by a continuous variation of the surface acoustic wavelength \( \lambda \), we are able to sample the rich patterns coded by "irrational" numbers.

5 - CONCLUSION

In summary, we have given an experimental characterization of the critical proper modes and of the spectrum in a \((d=1)\)-quasiperiodic system. These critical properties can be simply interpreted in terms of successive approximation of the reduced variable \( \alpha \), as a sum of reciprocal lattice vectors of decreasing norms, where \( \alpha \) is the average lattice period, \( \lambda \) the surface acoustic wavelength and \( t \) the inverse golden mean. A much extended version of this work is presented in /8/ where the time impulse response is also discussed. In the future, we will report on similar experiments on fully random corrugated substrates.

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