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PYRAMID SPECTRAL ANALYSIS OF NARROW-BAND SIGNAL

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Abstract - In this paper, a novel scheme using pyramid is proposed which computes the narrow-band spectrum with good resolution while requiring only modest computation and storage.

1. Introduction

The calculation of the spectrum of a narrow-band signal which is embedded in a broad-band sequence usually requires substantial computation and storage if executed by performing an FFT or DFT's directly on the broad-band sequence.

In previous works, The zoom transform and a modified zoom were proposed to compute the narrow-band spectrum with less storage than the FFT approach. Another proposed scheme [1] first modulates the signal with complex exponentials, then decimates, and finally analysis the decimated signal. Liu and Mintzer [1] proposed a good scheme first decimates the signal, then uses multirate bandpass filtering. The signal band of interest can be shifted to low frequencies, the sampling rate can be reduced to twice the band-width of the desired signal, and a low-order FFT analysis can be performed, giving the desired high-resolution, narrow-band spectral data. It combines the advantages of both a small computation rate and a small storage requirement.

In this paper, we proposed a novel scheme for pyramid spectral analysis of a narrow-band signal. In a pyramid spectral analysis, an original signal is first bandpass filtered to isolate the desired band of interest, and the resulting signal is successively pyramid low-pass filtered and decimated to form a compact representation of the signal. To obtain the desired high-resolution, narrow-band spectrum, a low-band FFT analysis can be performed on pyramid data.

2. Gaussian Pyramid

Burt and Adelson [2] have proposed a Gaussian filter to form the pyramid. The Gaussian pyramid is a sequence of signal in which each is a low-pass filtered copy of its predecessor. Let sequence x_0 be the original signal; this becomes the bottom, or zero level of the pyramid. Each node of the next pyramid level, sequence x_1 , is obtained as a weighted average of x_0 . Nodes within a 5-window. Each node of x_2 is then obtained from x_1 by applying the same pattern of weights. Note that the sample distance in each level is double that in the previous level. As a result, each signal in the sequence is represented by a sequence which is half as large as its predecessor. If we imagine these sequence stacked one above another, the result is the tapernig pyramid data structure.

The process which generates each sequence from its predecessor will be called a REDUCE operation since both resolution and sample density are decreased. Let x_0 be the original signal and let x_M be the top level of the pyramid. Then for $0 \leq l \leq M$, we say

$$x_l = \text{REDUCE}[x_{l+1}] \quad (1)$$

which means

$$x_l(i) = \sum_{m=-2}^2 w(m) x_{l-1}(2i+m) \quad (2)$$

The weighting function w is called the generating kernel. This is chosen subject to three constraints. It is

- (i) Normalized: $\sum w(m) = 1$
- (ii) Symetric: $w(m) = w(-m)$
- (iii) Equal contribution: $w(0) + w(-2) + w(2) = w(-1) + w(1)$

Combining constraints we have

$$\begin{aligned} w(0) &= \text{free variable} \\ w(-1) &= w(1) = 0.25 \\ w(-2) &= w(2) = 0.25 - 0.5w(0); \end{aligned}$$

Iterated pyramid generation is equivalent to convolving the signal x_0 with a set of "equivalent weighting function" h_l :

$$x_l = h_l \otimes x_0 \quad (3a)$$

$$\text{or} \quad x_l(i) = \sum_{m=-M_l}^{M_l} h_l(m) x_0(i \cdot 2^l + m) \quad (3b)$$

The size M of the equivalent weighting function doubles from one level to the next, as does the distance between samples. The effect of convolving a signal with one of the equivalent weighting function h_l is to blur, or low-pass filter, the signal. The pyramid algorithm reduces the filter band limit by an octave from level to level, and reduces the sample interval by the same factor. This is a very fast algorithm, requiring fewer computational steps to compute a set of filtered signals than are required by FFT to compute a single filtered signal.

3. Narrow-Band, High-Resolution Spectral Analysis Using Pyramid method

One of the most fundamental signal processing operations is spectral analysis: that is evaluation of the Z-transform of a sequence over a band of frequencies on the unit circle. Formally, if we define an N -point sequence, $x_0(n)$, $n=0, \dots, N-1$, with Z-transform

$$x_0(z) = \sum_{n=0}^{N-1} x_0(n) z^{-n} \quad (4)$$

then the spectrum of $x_0(n)$ at frequency ω is

$$x_0(e^{j\omega}) = x_0(z) \Big|_{z=e^{j\omega}} = \sum_{n=0}^{N-1} x_0(n) * e^{-j\omega n} \quad (5)$$

Although a variety of techniques have been proposed for evaluating $x_0(e^{j\omega})$ of Eq. (5), perhaps the most commonly used one is the class of FFT algorithms. These algorithms evaluate, in an efficient manner, the Z-transforms of a finite sequence at a set of equal spaced frequencies

$$\omega_k = k * \frac{2\pi}{N}, \quad k=0, 1, \dots, N-1 \quad (6)$$

giving

$$x_0(e^{j\omega_k}) = x_0(k) = \sum_{n=0}^{N-1} x_0(n) * e^{-j(2\pi/N)nk} \quad (7)$$

From the set of FFT coefficients $x_0(k)$, $k=0, 1, \dots, N-1$, the original sequence can be calculated via an inverse FFT relation of the form

$$x_0(n) = \frac{1}{N} \sum_{k=0}^{N-1} x_0(k) * e^{j(2\pi/N)nk} \quad (8)$$

The set of FFT and inverse FFT relations, of Eqs.(7) and (8), provide a powerful framework for wide-band digital spectral analysis of signals, and have been used in this manner in a wide variety of applications.

One major disadvantage of the FFT algorithm is that it automatically provides spectral information over a uniformly spaced set of frequencies covering the entire band of the signal of interest. For some applications only the spectral and for such cases it is highly inefficient to make all the calculations of Eq.(7) and then to use a small subset of them. By using the narrow-band filtering and pyramid reduce, the signal band of interest can be shifted to low frequencies, and a low-order FFT analysis can be performed, giving the desired high-resolution, narrow-band spectral data.

Fig.1 illustrates the principle used to derive a pyramid structure for high-resolution, narrow-band spectral analysis. In Fig.1.(a) we show the pyramid structure for bandpass filtering for an integer-band signal. The decimation rate of the multilevel pyramid reduce operations is M_l , defined as

$$M_l = \frac{\pi}{\omega_{su} - \omega_{sl}} \quad (9)$$

where ω_{sl} and ω_{su} are the lower and upper frequencies of interest in the bandpass signal. The signal is first bandpass filtered to isolate the desired band of interest, and the resulting is decimated by a factor of M_l . Since we assumed integer-bands, we have

$$\omega_{sl} = \frac{l\pi}{M_l} \quad (10a)$$

$$\omega_{su} = \frac{(l+1)\pi}{M_l} \quad (10b)$$

for some value of l , $1 \leq l \leq M-1$. The resulting signal, $x_l(m)$, has a lowpass spectrum as shown in Fig.1.(b). For the lowpass signal (assuming negligible aliasing), the frequency ω' corresponds to the frequency ω in $x_0(e^{j\omega})$ such that

$$\omega' = \frac{(\omega - \omega_{sl})\pi}{\omega_{su} - \omega_{sl}} \quad (11)$$

To obtain the desired high-resolution, narrow-band spectrum, the system of Fig.1.(c) is used, in which an N' -point FFT is performed on $x_l(m)$, yielding spectral values for frequencies

$$\omega' = \frac{2\pi k}{N'}, \quad k=0, 1, \dots, N-1 \quad (12)$$

(a)

(b)

Fig.1 Multilevel pyramid structure for performing high-resolution narrow-band spectral analysis.

and these values are translated back to the ω range using Eq.(11). The value of N' is, approximately,

$$N' = \frac{N}{M_1} \quad (13)$$

since the pyramid decimated signal has only N/M_1 samples. A correction network is shown which can correct for the imperfect passband response of the passband filter.

The improvement in efficiency derives from the use of a pyramid reduction operation and a considerably smaller value of N' than N , whenever M_1 is large, thereby reducing the FFT computation, which is proportional to $N \log N$. However, there is extra computation due to the bandpass filter. For most cases of interest bands where $M_1 \gg 10$, there is a real reduction in computation using the pyramid structure of Fig.1.(c) over the direct FFT computation.

4. Discussion

Pyramids, in general, are data structures that provide successively condensed representations of the information in the input signal. Many basic signal operations may be performed efficiently within pyramid structures. Pyramid reduction operation can generate low-pass filtered signals at a fraction of the cost of FFT. Local signal properties can be estimated with equal efficiency within Gaussian-like windows of many sizes. Here the pyramid will be viewed primarily as a computational tool for spectral analysis.

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