QUANTITATIVE SPECIFICATION OF MICROSTRUCTURAL ANISOTROPY

C. Hartley

To cite this version:

C. Hartley. QUANTITATIVE SPECIFICATION OF MICROSTRUCTURAL ANISOTROPY. Journal de Physique Colloques, 1990, 51 (C1), pp.C1-173-C1-177. <10.1051/jphyscol:1990126>. <jpa-00230284>

HAL Id: jpa-00230284
https://hal.archives-ouvertes.fr/jpa-00230284
Submitted on 1 Jan 1990

HAL is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers.

L’archive ouverte pluridisciplinaire HAL, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d’enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.
QUANTITATIVE SPECIFICATION OF MICROSTRUCTURAL ANISOTROPY

C.S. HARTLEY

Department of Materials Engineering, University of Alabama at 
Birmingham, UAB Station, Birmingham, AL 35294, U.S.A.

Abstract—The microstructural anisotropy of a multiphase material can be 
expressed quantitatively by the spatial distribution function of the 
normals to the internal interfaces. Two-dimensional sections of such 
structures can be analyzed to obtain the spatial variation of the mean 
linear intercept of interfaces with a test line as a function of the 
orientation of the test line. For a continuous distribution of surface 
normals the three-dimensional representation of the mean linear intercept 
can be represented in terms of a second rank tensor, the Microstructural 
Anisotropy Tensor (MAT), which in turn defines principal directions of 
the microstructure. The components of this tensor are appropriate 
averages of the distribution of surface normals over all orientations of 
surfaces.

1 - INTRODUCTION

The anisotropic nature of oriented microstructures was first characterized 
quantitatively by Saltykov using a figure called the "rose of the number of 
intersections" /1/. This is simply a polar plot of the number of 
intersections with the traces of oriented features in an observation plane per 
unit length of a test line, \( P_i(t) \), as a function of the orientation of a unit 
vector parallel to the test line, \( \mathbf{t} \). For internal interfaces this quantity is 
equal to the total internal boundary surface area per unit volume projected on 
a plane normal to the test line, \( S_v(t) \). Other scalar measures express the 
degree of orientation as a ratio of the length of features oriented parallel 
to a particular axis to the total length of such features in the plane of 
observation /2/.

Harrigan and Mann /3/ pointed out that the variation of mean linear intercept 
with orientation of test line in three dimensions can be characterized by an 
ellipsoid whose principal axes define the principal directions of 
the microstructure. The ellipsoid has the general equation

\[
\mathbf{L}^2(t) = M_{ij} t_i t_j = P_L^2(t),
\]

where summation from 1 to 3 over repeated latin suffixes is implied and the 
coefficients \( M_{ij} \) form a second rank tensor called the Microstructural 
Anisotropy Tensor (MAT). The effects of processes which alter the nature and 
extent of the anisotropy of the microstructure can then be expressed in terms 
of their effects on the eigenvectors of the MAT.
The present work discusses the relationship of the MAT to the distribution of normals to internal boundaries in space. The description of the microstructure in terms of the eigenvectors of the MAT and the interpretation of the eigenvectors are discussed.

2 - THEORETICAL DEVELOPMENT

The interface boundary area per unit volume, $S_V$, is related to the expected number of intersections per unit length of a randomly applied test array, $P_L$, by the equation /1,2/

$$S_V = 2P_L.$$  \hspace{1cm} (2)

The average value of $P_L^2$ given by equation 1 is obtained by integrating over all possible orientations of the test line and normalizing,

$$<P_L^2> = \frac{1}{4\pi} \int A M_{ij} t_i t_j dA,$$  \hspace{1cm} (3)

which yields

$$<P_L^2> = (M_{ij}/4\pi)(\delta_{ij}(4\pi/3) = M_{ii}/3.$$  \hspace{1cm} (4)

The appropriate average value, $P_L$, for calculating $S_V$ for an anisotropic microstructure is

$$P_L = \sqrt{M_{ii}/3} = S_V/2,$$  \hspace{1cm} (5)

which is an invariant of the MAT. Thus the total surface area of an anisotropic microstructure can be determined by measurements of $P_L$ on three mutually perpendicular but otherwise arbitrary planes of observation.

The three-dimensional character of the internal interfaces in a material can be described by a distribution function for internal boundary surface elements as a function of the orientation of the normals to the surface elements. Define the internal surface area per unit volume due to elements of area having outward normals parallel to the unit vector, $n$, as

$$S_V(n) = [|t.n|L(n,t)]^{-1} = P_L(n,t)/|t.n|$$  \hspace{1cm} (6)

where $L(n,t)$ is the mean linear intercept of a test line parallel to $t$ with internal surfaces having normals parallel to $n$, $P_L(t,n)$ is the number of intercepts per unit length of the test line with surfaces whose normal is $n$ and $|t.n|$ is the absolute value of the cosine of the angle between $t$ and $n$. In general $P_L(t)$ includes intersections with surfaces of all possible orientations. Thus the relationship connecting $P_L(t)$ with the distribution of surface elements in space can be written

$$P_L(t) = \frac{1}{8\pi} \int_A [S_V(n)]|t.n| dA$$  \hspace{1cm} (7)

where $A$ is the area of the unit sphere. The normalization factor, $1/8\pi$, arises because $S_V(n) = S_V(-n)$ and the integral of $dA$ over the unit sphere is $4\pi$. The rose of the number of intersections is a plot of the variation $P_L(t)$ with orientation of test line in a plane of measurement.

An alternative representation of the spatial variation of $L(t) = F^{-1}(t)$ with orientation is a plot of $L(t)$ vs. orientation rather than the relationship given by equation 7. A continuous distribution of surface elements results in an ellipsoid, centered at the origin with radii having a magnitude equal to
The general equation of such an ellipsoid is

\[ L^{-2}(t) = P_L^2(t) = M_{ij} t_i t_j, \]  

where the coefficients, \( M_{ij} \), are real and form a symmetric, second rank tensor. From equations 7 and 8 we find that

\[ P_L^2(t) = \frac{1}{16\pi} \int_A S_V^2(n) [n \cdot t]^2 \, dA \]  

or

\[ P_L^2(t) = \frac{1}{16\pi} \int_A S_V^2(n) n_i n_j t_i t_j \, dA. \]  

By equations 8 and 10 the coefficients \( M_{ij} \) are

\[ M_{ij} = \frac{1}{16\pi} \int_A S_V^2(n) n_i n_j \, dA \]  

relating them to the distribution of orientations of surface elements.

The anisotropic character of the distribution of interface boundary normals can also be described in terms of the isotropic and deviatoric components of the MAT. The isotropic component is related to the total boundary area per unit volume as shown by equation 5. Deviatoric components of the MAT quantitatively describe the anisotropic nature of the microstructure.

Principal directions of the MAT define principal directions of the microstructure normal to which the projected boundary areas are extrema. Principal values of the MAT represent the squares of the extreme values of the mean linear intercepts along principal directions of the microstructure and can be used in Saltykov's decomposition /1/ to determine the components of linear, planar and isometric elements in the microstructure. The total surface area per unit volume determined from Saltykov's treatment can be expressed as

\[ S_V = N_{\parallel} (2-\pi/2) + N_{\perp} (\pi/2-1) + N_{\perp}, \]  

where the suffixes refer to measurements of \( P_v \) parallel to the principal orientation axis, \( \parallel \), and along two mutually perpendicular axes normal to the principal orientation axis, \( \perp \) and \( \perp \). While equation 12 is not formally the same as equation 5 using the rms value for \( P_v \), the following section illustrates that the two approaches yield numerically similar values for \( S_V \).

3 - EXPERIMENTAL OBSERVATIONS

Recent studies of the variation of total grain boundary surface area per unit volume with strain in extrusion and rolling illustrate the comparison of equations 5 and 12 for the determination of \( S_V /4,5,6/ \). Extrusion experiments were conducted on solid copper rods at room temperature using extrusion ratios from 2 to 8. Table 1 compares the values of \( S_V \) obtained on extruded solid copper using the two analyses /4/.

Agreement of the two techniques is within experimental error and the two analyses exhibit the same behavior with increasing deformation.

Studies of the evolution of the grain boundary structure with deformation by rolling have been conducted on copper-clad mild steel and unclad copper /5/ and unclad mild steel /5,6/. In the earlier study measurements were made only in the rolling, long and short transverse directions, while in the later work, measurements were made at various angles to the principal directions of
deformation in planes normal to these directions and least squares fits were employed to obtain the values of the MAT. For mild steel specimens the P\(_c\) counts included only ferrite-ferrite grain boundaries. Diagonalization of the MAT so obtained verified that the principal directions of the deformed microstructure developed parallel to the principal directions of deformation. Results of these studies are summarized in Table 2, which compares the values of \(S_V\) obtained from equations 5 and 12.

### Table 1 Total surface area per unit volume from MAT and Saltykov analysis*

<table>
<thead>
<tr>
<th>Extrusion Ratio</th>
<th>(S_V) from MAT</th>
<th>(S_V) from Saltykov</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>127</td>
<td>130</td>
</tr>
<tr>
<td>2</td>
<td>140</td>
<td>139</td>
</tr>
<tr>
<td>4</td>
<td>216</td>
<td>209</td>
</tr>
<tr>
<td>6</td>
<td>261</td>
<td>253</td>
</tr>
</tbody>
</table>

*\(S_V\) is expressed in mm\(^{-1}\).*

### Table 2 Variation of \(S_V\) with rolling reduction for copper and mild steel*

<table>
<thead>
<tr>
<th>Spec.</th>
<th>% Red.</th>
<th>(S_V) from MAT</th>
<th>(S_V) from Saltykov</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cu</td>
<td>0</td>
<td>93</td>
<td>94</td>
</tr>
<tr>
<td>ref/5/</td>
<td>17.8</td>
<td>113</td>
<td>120</td>
</tr>
<tr>
<td></td>
<td>22.9</td>
<td>123</td>
<td>131</td>
</tr>
<tr>
<td></td>
<td>37.5</td>
<td>153</td>
<td>160</td>
</tr>
<tr>
<td>Steel</td>
<td>0</td>
<td>72</td>
<td>73</td>
</tr>
<tr>
<td></td>
<td>14.7</td>
<td>81</td>
<td>85</td>
</tr>
<tr>
<td>ref/5/</td>
<td>18.8</td>
<td>81</td>
<td>85</td>
</tr>
<tr>
<td></td>
<td>30.8</td>
<td>87</td>
<td>93</td>
</tr>
<tr>
<td></td>
<td>36.6</td>
<td>103</td>
<td>108</td>
</tr>
<tr>
<td>Steel</td>
<td>0</td>
<td>55</td>
<td>56</td>
</tr>
<tr>
<td></td>
<td>15</td>
<td>57</td>
<td>60</td>
</tr>
<tr>
<td>ref/6/</td>
<td>30</td>
<td>55</td>
<td>58</td>
</tr>
<tr>
<td></td>
<td>45</td>
<td>63</td>
<td>67</td>
</tr>
</tbody>
</table>

*\(S_V\) is expressed in mm\(^{-1}\).*

The values obtained from both analyses are comparable and they vary with strain in a parallel manner.

### 4 - DISCUSSION

Comparisons of \(S_V\) calculated by two techniques on anisotropic grain boundary microstructures produced by cold extrusion and cold rolling of single phase materials suggest that an appropriate average intercept count for the calculation of this quantity is the rms value of \(P_c\). The rms intercept count can be interpreted as the square root of the trace of the Microstructural Anisotropy Tensor of the material. This quantity is an invariant of the MAT, which means that the rms value of \(P_c\) can be obtained by measurements taken along any three mutually perpendicular directions in the structure.

The use of the MAT as a means of describing anisotropic microstructures offers a compact means of describing these complex configurations. The trace of this tensor provides a convenient measure of \(S_V\), and the deviatoric components can be used as measures of the degree of anisotropy of the structure.
ACKNOWLEDGEMENTS

The author wishes to acknowledge helpful discussions with B. R. Patterson and B. L. Adams.

REFERENCES


