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To cite this version:

HAL Id: jpa-00229469
https://hal.archives-ouvertes.fr/jpa-00229469
Submitted on 1 Jan 1989

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COLLECTIVE BEHAVIOURS OF GRANULAR MASSES UNDER VERTICAL VIBRATIONS

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Abstract - We report two types of collective behaviour of a horizontal layer of rigid particles under vertical vibrations. Beyond a threshold of the acceleration a convective flow of particles is observed and the horizontal free surface becomes unstable. A heap with a constant slope is generated; the slope decreases at large accelerations, and localized waves propagate in an erratic fashion on the free surface, which is on the average roughly horizontal. However, we found intervals in excitation frequency and amplitude, where a standing surface wave is generated at half the excitation frequency, and forms a perfectly ordered surface pattern.

1 - INTRODUCTION

The dynamical properties of media composed of rigid particles are of great interest. Granular masses behave like deformable solids in the static case, but they can also flow like liquids. A well-known example is the avalanche flow down a dune that occurs when the slope exceeds the limiting static friction angle. Although granular masses are used in many industrial situations, only a few attempts have been made to investigate their dynamical properties, and a complete theory is presently beyond our scope. Hard spheres in a high state of agitation have been successfully used to simulate fluids in pedagogical experiments, but several phenomena become important at high density and low agitation, and give rise to unusual hydrodynamic properties. Important questions thus concern the effect of friction between particles and inelasticity of collisions. We present here a simple experiment on granular flows. It consists of a vertically vibrating horizontal layer, composed of rigid particles. We report two types of collective behaviour of the particles. One, at high acceleration, consists of a standing wave, analogous to the Faraday instability of a vertically vibrating horizontal fluid layer. The other, at lower external excitation, is a convective motion of the particles; the horizontal free surface becomes unstable, and a heap with a constant slope is generated. Although there exists a flow of particles, the behaviour of the granular mass in that case, differs completely from the convection of a simple fluid.

2 - EXPERIMENTAL SET-UP

The experimental set-up consists of a vertically vibrating plexiglass vessel of dimensions 78 * 17 * 35 mm³, containing a layer of height H composed of rigid particles; different types...
of salt, glass spheres, fillite, have been used, and thus the observations reported here have been made with particles of different sizes (0.05 to 0.5 mm), and different shapes (spherical or irregular). The size dispersion was not controlled, except in one experiment with glass spheres. We have observed the same types of flow regime in all cases, and thus the collective behaviours we report are generic for powders under vertical vibrations. A Bruel & Kjaer 4809 vibration exciter produces a clean vertical acceleration waveform (horizontal acceleration less than 1%), and is driven by a frequency synthesizer (frequency precision better than $10^{-6}$). The acceleration, which is the relevant external constraint, is measured with an accelerometer. The experiments are conducted by increasing the acceleration at a fixed value of the frequency. Most of the observations have been made in the frequency range $10 - 100$ Hz, but in one experiment on the convective regime threshold measurement, frequencies up to $400$ Hz have been considered. In the wavy regime, observations and films have been made using a stroboscope.

### 3 - THE CONVECTIVE FLOW REGIME

At low accelerations the layer of particles is (loosely) rigid and the horizontal free surface is stable. Above a critical acceleration, the particles begin to move and the horizontal free surface becomes unstable. At the instability threshold the disturbances appear at the lateral boundaries, and then the particles migrate toward the center of the cell and form a mound (see figure 1). When the acceleration is abruptly increased above the instability threshold, several small mounds can be generated, but the stable configuration at the instability threshold usually consists of a heap close to one lateral boundary. There is an avalanche flow along the surface of the heap, compensated by an internal circulation of the particles from the bottom toward the top of the heap. This phenomenon is known for a long time and similar observations were made by Faraday in 1831 /1/, but the origin of this convective motion is still unexplained. The convective flow have been observed with different types of periodic excitation of a granular mass /2, 3, 4, 5, 6/ but there is a lack of quantitative studies.

Fig. 1 - photograph of the convective heap.

The first measurement one should do concerns the threshold of the convective regime. The external control parameters are the vibration amplitude, $A$, and frequency, $V$. In a large enough cell, the shape (circular or rectangular) and the horizontal dimensions of the cell have little effect on the convective regime. Another geometric parameter, which connects a macroscopic size with a microscopic one, is $N = H/d$, where $H$ is the height of the layer and $d$ the characteristic size of the particles. For a given powder, on the frequency range $10 < V < 100$ Hz, and for $35 < N < 140$, we have found that the convection threshold corresponds to a critical value of the acceleration, $4\pi^2 V^2 A$, or of the dimensionless parameter $\Gamma = 4\pi^2 V^2 A/g$, where $g$ is the acceleration of gravity. The convective regime is not observed when the...
layer is too thin; the minimum height is much larger with glass spheres than with salt grains. The threshold $T_c$ increases at high frequency; this effect depends on the height of the layer, but has not been systematically studied. The critical value $T_c$ increases when the grains are very small; it is for instance nearly three times larger with glass spheres (average size 0.07 mm) than with salt grains (average size 0.3 mm), but this is probably a cohesion effect. With large enough grains (salt or glass spheres), $T_c$ is slightly larger than one, indicating that the effective gravity needs to be reversed to reach the convective regime. Close to the instability onset, the characteristic time for the heap formation diverges; transient regimes lasting more than one hour have been observed. This makes difficult precise threshold measurements.

Above the convection onset, we have measured the slope $\theta$ of the stationary heap as a function of the acceleration. The results are displayed on Figure 2 for a layer of salt grains (average size 0.3 mm, height 12 mm) vibrating at 50 Hz; the slope decreases as the acceleration is increased. At about 5 $T_c$, the layer is nearly horizontal, but of course with a larger height than at rest because of particles agitation. When the acceleration is increased further, time-dependent regimes appear (see below). With glass spheres, we first observe an increase of the slope above $T_c$; when the acceleration is increased, $\theta$ reaches a plateau and then decreases at high accelerations like in the case of salt grains. Time-dependent regimes occur before the slope vanishes, and the layer is horizontal only on an average at large accelerations.

![Figure 2](image)

**Fig. 2** - The slope $\theta$ of the convecting heap of salt grains as a function of the acceleration amplitude of the cell ($V = 50$ Hz, $N = 40$).

4 - DISCUSSION

Faraday's explanation for the convective motions in the powder pile was as follows: when the base of the pile loses contact with the plate, "it forms a partial vacuum, into which the air, round the heap, enters with more readiness than the heap itself; and as it enters, carries in the powder at the bottom edge with it." This effect modifies the slope of the heap as shown by experiments performed in vacuum, but the convective motions persist.

Hydrodynamic theory of granular flows have been developed in the past years and evolution equations for the density, velocity and "temperature" have been derived using kinetic theory /7, 8/. The inelasticity of collisions was taken into account but the description was limited to moderate solid volume fraction. As pointed out recently by de Gennes /9/, the powder behaves like a fluid only on the time interval where the effective gravity is reversed, but is solid during the remaining interval (except at the free surface where avalanches occur). Thus the particles are periodically in close rubbing contact and the standard approximations of kinetic theory are no more valid.
In the description of the instability an important step is to understand how the time-periodic external forcing generates a stationary convective flow. A well known mechanism is the steady streaming generated by wave attenuation in acoustics or in oscillatory boundary layers /10, 11/; a time-dependent motion gives rise to a steady flow through quadratic non-linearities. It was recently shown by Savage /6/ that this mechanism is responsible for the generation of convective motions in a granular material fluidized by a wave created by an oscillating membrane.

In our case the instability mechanism is as follows: for accelerations larger than \( g \), at a certain instant of the vibration cycle the particles are lifted, leaving a small gap between the bottom of the layer and the vessel base; the gap becomes thicker, reaches a maximum and decreases again until the layer collides with the vessel base; the layer and the vessel remain in contact until the next cycle when the process repeats. During the free flight time interval of the layer, its lower boundary undergoes an instability and thus a wave develops and modulates the layer lower boundary at the excitation frequency. The convective flow is the steady streaming associated with this wave; indeed the grains ascending motion is located at the maxima of vibration of the lower boundary. We will report elsewhere the quantitative study of this lower interface instability.

The slope of the convecting heap is a significant measurement since it is connected to the intensity of the convective flow, but also to the fluidisation of the granular material. Its decrease at large accelerations can be understood as follows: first, there is an optimum value of the free flight time interval (resonance condition) to achieve a maximum energy transfer to the convective motion; second, the fluidisation increases at large accelerations, and thus, even for a given convective velocity, the slope decreases.

5 - THE STANDING WAVE REGIME

When the acceleration is increased further, time-dependent instabilities of the free surface develop. They consist of localized waves that propagate in an erratic fashion. However, there exist intervals in the excitation amplitude and frequency, where a standing surface wave is parametrically amplified at half the excitation frequency, and forms a perfectly ordered pattern (see figure 3).

![Photograph of the standing surface wave](image)

The successive surface deformations and the corresponding motions of the grains are sketched on figure 4, and show that there is a resonance condition for the development of the waves; the two relevant time scales are the external forcing period and the characteristic avalanche time. This resonance mechanism selects the wavelength; indeed, at higher frequency, the mounds have smaller wavelength and amplitude, and thus the time to recover a nearly horizontal layer when the grains collide with the vessel base is smaller. This wavy regime at half the excitation frequency is analogous to the Faraday instability of ordinary fluids, but steeper surface gradients easily develop because of the absence of surface tension.
Fig. 4 - Sketch of the successive surface deformations and corresponding motions of the grains and of the cell (indicated by arrows).

REFERENCES