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LOCALIZATION EFFECTS ON NEUTRON

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Abstract. — Multiple scattering effects are studied on the cross section of elastic scattering of neutron. The cross section is found to be enhanced within a narrow angle centered around the backscattering direction due to interference effects. The spin-flip scattering causes an antienhancement of intensities.

Anderson localization of light as well as electron has recently been attracted much attention. Light scattering experiments have been extensively done to find that the scattered intensity is enhanced within a narrow angle around the backscattering direction [1, 2]. This enhancement is interpreted as the result of the constructive interference between the time-reversed path and the original path for backscattering geometry. Note that this interference is closely related to Anderson localization, because it leads to a reduction of the diffusion constant.

In this paper, we study the coherent backscattering of neutron, which may be different from that of light. First we note that neutron's mean free path is usually very long. Using an expression for the mean free path \( l \approx \frac{4\pi |b|^2}{V} \), where \( b \) and \( N \) denote the scattering length and the number of nuclei that scatter neutron in the sample with volume \( V \), we find that \( l \) is typically of order 0.5 cm, which is usually the same order of the sample size. This situation is quite different from the localization problem of light and electron where \( l \) is much shorter than sample size. Accordingly we can calculate sufficiently the multiple-scattering contribution to the cross section by using the second-order Born approximation.

Second we note that neutron interacts strongly with nuclear spin and can easily change its spin state. The interaction between neutron and matter may be expressed as

\[
U(r) = \frac{2\pi \hbar^2}{m} \sum_j b_j \delta(r - r_j),
\]

where \( (1/2) \sigma, m, \) and \( r \) denote the spin operator, the mass and the position vector of neutron, while \( \sigma \) and \( r_j \) denote the spin operator and the position vector of the j-th nucleus. Considering amorphous substances or alloys, we assume that \( A_j \) is a random variable with the average \( A_0 \) and the variance \( (\Delta A)^2 \). The effects of lattice vibration are neglected for neutrons with long wavelength in low temperatures. We also assume that nuclear spins are pointing in random direction due to thermal effects, because the energy concerned with nuclear spin is usually much smaller than thermal energy.

In the first-order Born approximation, the cross section of elastic incoherent scattering is expressed as

\[
\frac{d}{d\Omega} \sigma = N \left[(\Delta A)^2 + (1/4) \frac{B^2 I}{I + 1}\right].
\]

This forms a constant background in the cross section. The double scattering processes in the second-order Born approximation are shown in figure 1; in process a, for example, a neutron with spin \( \uparrow \) and momentum \( k_i \) is scattered by nucleus \( j \), changes its spin from \( \uparrow \) to \( \downarrow \), propagates from the site \( j \) to \( i \), is scattered by nucleus \( i \), changes its spin from \( \downarrow \) to \( \uparrow \), and comes out with momentum \( k_f \). The probability amplitude for each process may be expressed as

\[
\psi^{j1}_{a} = \frac{(1/4) B^2 \exp(-ik_i \cdot r_j)}{\sqrt{(I + m_j + 1)(I - m_i + 1)}},
\]

\[
\psi^{j1}_{b} = (A_j + Bm_j/2)(A_l + Bm_l/2),
\]

\[
\psi^{j1}_{c} = \frac{(1/2) B \sqrt{(I + m_j + 1)(I - m_i)} \exp(-ik_f \cdot r_l)}{\sqrt{(I + m_i + 1)(I - m_i)}},
\]

where

\[
\phi_{jl} = \exp(ik \cdot r_j) \frac{\exp(i\phi_j)}{\exp(-i\phi_j)},
\]

with \( k = |k_i| = |k_f| \). Since nuclear spin states are not changed in process \( b \), a constructive interference occurs between \( \psi^{j1}_{b} \) and \( \psi^{j1}_{c} \) near the backscattering direction.

![Fig. 1. - Diagrams for double scattering processes. Solid lines represent the propagation of neutron. The arrows denote the spin states of neutron, while \( j \) and \( l \) denote nuclei that scatter neutron. Diagrams connected by double arrows interfere with each other in the backscattering direction.](http://dx.doi.org/10.1051/jphyscol:19888939)
As for spin-flip processes (c, d), one may think that there is no interference effect because changing nuclear spin states corresponds to a measurement of the position of nuclei. This statement, however, is not correct; interference can occur between \( \psi_d^j \) and \( \psi_d^j \), because the same final state are realized after these processes. We note from (4) that this interference is destructive for large \( B \). Taking configurational average and thermal average over \( m_j \) and \( m_l \), we find the cross section in the second-order Born approximation,

\[
\frac{d\sigma_2}{d\Omega} = \sum_{j,l} \left| \psi_a^j \right|^2 + \left| \psi_b^j \right|^2 + \left| \psi_c^j + \psi_d^j \right|^2
\]

\[
= F \left[ (\Delta A)^2 + (1/4) B^2 I (I + 1) \right]^2 + G \left\{ \left[ (\Delta A)^2 + (1/12) B^2 I (I + 1) \right]^2 + (1/3) B^2 I (I + 1) \left[ (\Delta A)^2 - (1/12) B^2 I (I + 1) \right] \right\},
\]

\[
F = \sum_{j,l} \frac{1}{|r_j - r_l|^2}, \quad G = \sum_{j,l} \cos \left( \frac{(k_f + k_i) \cdot (r_j - r_l)}{|r_j - r_l|} \right). \tag{7}
\]

We evaluate \( F \) and \( G \) using the continuum approximation for cylindrical sample with radius \( R \) and height \( h \), assuming that the incident neutron beam is normal to the \( c \)-axis of the cylinder. Figure 2 shows the cross section as a function of scattered neutron’s direction which is assumed to be in the plane normal to the \( c \)-axis. We can clearly see the enhancement of intensity, whose angular width is of order \( \delta \theta < \lambda/R \) near the backscattering direction, where \( \lambda \) is the wavelength of neutron. For the case that \( B \) is large, we can see even an antienhancement of intensity due to destructive interference. Unlike the effects of the spin-orbit coupling in the weak localization of electron, this antienhancement may be reduced when multiple scattering higher than the double scattering is effective. Note that the shapes of the intensity curves remain nearly unchanged with increasing the height of cylinder or altering shape of sample from cylinder to sphere [3].

When configurational average is not justified, intensities may fluctuate with changing scattering angle for the case that \( B \) is small. To suppress such fluctuations, it is better to rotate samples. Since thermal average on nuclear spin states is usually justified, intensity fluctuations should not appear for the case that \( B \) is large.

For observing such coherent enhancement of intensities over appreciable angle, a small sample and a large wavelength of neutron are favorable, whereas a large sample is favorable to increase the double scattering intensity. For ultra-cold-neutrons with a wavelength \( \lambda \approx 100 \text{ Å} \), we expect that \( \delta \theta \approx 10^{-4} \text{ rad} \) and \( \sigma_2/\sigma_1 \approx 10^{-2} \), for \( R \approx 0.1 \text{ mm} \), \( h \approx 0.5 \text{ mm} \), \( V/N = (2 \text{ Å})^3 \), and \( (\Delta A)^2 + (1/4) B^2 I (I + 1) = (10^{-4} \text{ Å})^2 \).

Note that ratio \( \sigma_2/\sigma_1 \) is mainly determined by \( R \), and nearly independent of \( h \), for \( h > R \). The effects of magnetic scattering neglected in this paper may complicate the situation. Accordingly magnetic atoms should be excluded, and compounds of hydrogen may be better for experiments, since the nucleus has a large incoherent (spin-flip) cross section.

In summary, we have found that the elastic incoherent cross section is enhanced near the backscattering direction due to interference effects, and even antienhancement can appear, provided that the spin-flip scattering is dominant.