MULTISTABILITY AND CHAOS BY PARAMETRIC EXCITATION OF MAGNETOSTATIC MODES

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Abstract. – Spinwave instabilities in YIG spheres have been studied by FMR within the coincidence regime of the first-order Suhl instability. Very complex sequences of auto-oscillations, period doublings, and quasi-periodicity were observed, ending up in chaos. A multimode model considering the specific properties of magnetostatic modes is discussed.

Introduction

The recent progress of nonlinear dynamics has stimulated the reexamination of high power FMR experiments under the special aspect of universality in chaotic behaviour. Different bifurcation routes showing period doublings [1, 2], irregular periods [3] and quasi-periodicity [4] have been observed, and simplified nonlinear models, based on Suhl's spinwave instabilities [5] have been offered for their interpretation. In this paper we present our recent observations of bifurcation routes and chaos on yttrium iron garnet (YIG) spheres driven by a transverse microwave field. We confine to the coincidence regime of the first-order Suhl instability ($1.7 \leq \omega/2\pi \leq 3.4$ GHz), where extremely small thresholds in the order of only 100 μW can be obtained, if both the resonance condition for the spinwaves $(\omega = 2\omega_{\mathbf{k}})$ and for the uniform mode $(\omega = \omega_0)$ can be satisfied simultaneously.

Experimental results

For the transverse excitation and detection of the uniform mode we used two micro-coils with perpendicular orientation instead of a microwave cavity to suppress disturbing reactions of the radiation field on the sample. The transmitted and rectified signal is proportional to the square amplitude $|a_0|^2$ of the uniform mode, i.e. to the transmitted power P_{tr}. By means of a digital oscilloscope and an integrating voltmeter we could directly record the time variation of $P_{\text{tr}}(t)$ and measure its average value $\bar{P}_{\text{tr}}$.

Increasing the microwave excitation P_{in} as control parameter, we found auto-oscillations within a frequency range from some 10 kHz up to MHz and various sequences of bifurcations ending up in chaos. Three different regimes could be distinguished: (i) above the saturation field of domains, 600 Oe $\leq H \leq$ 730 Oe (regime A), P_{tr} up to the Suhl threshold increases linearly with P_{in} but then remains constant for a range of nearly 10 dB. No oscillation is observed until finally a sudden decrease of $\bar{P}_{\text{tr}}$ occurs and the signal becomes chaotic; (ii) for 730 $\leq H \leq$ 950 Oe (regime B) $\bar{P}_{\text{tr}}$ shows a variety of sudden jumps starting directly above the threshold. These jumps are accompanied by a very complex sequence of oscillations including quasiperiodic behaviour, period doublings and intermittency (cf. Fig. 1). By increasing and decreasing the microwave power hysteresis effects and well reproducible multistabilities were observed; (iii) for 950 $\leq H \leq$ 1200 Oe (regime C) we observed a similar behaviour as in regime A. Above 1200 Oe the first-order coincidence regime is exceeded, and only the second-order Suhl instability can be excited at a much higher threshold.

To get some idea of the minimum number of relevant degrees of freedom involved in the time evolution we have analysed our data by means of the Grassberger-Procaccia method [6]. To this end we have evaluated a large number of time series of $P_{\text{tr}}(t)$ (up to 16 000 points each) taken from the whole investigated parameter range. For regular signals we found correla-

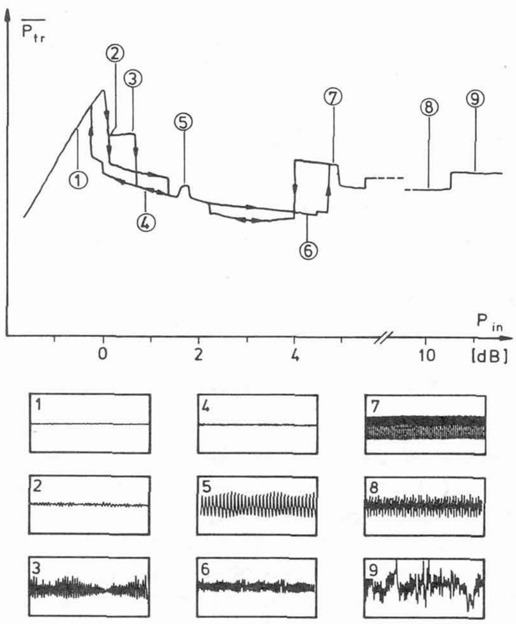

Fig. 1. – Transmitted FMR signal of a YIG sphere with respect to the exciting microwave power (regime B: $\omega/2\pi$ = 2.372 GHz, H = 838 Oe). Inserts show corresponding time dependencies of $P_{\text{tr}}(t)$.

tion dimensions D_2 close to 1 or 2, in some cases even close to 3. Irregular signals, in general, showed higher fractal dimensions ranging from 3 to 6.5. Changes of D_2 were strongly correlated to the sudden jumps of $\bar{P}_{\rm tr}$. Therefore, one might be tempted to interpret such a jump as a change of the involved degrees of freedom.

Discussion

According to [5], regime A is characterized by the parametric excitation of spinwaves with rather large wavevectors (10^4–$10^5 \mathrm{cm}^{-1}$). In regime B, however, extremely longwave modes should be excited. Then, for a realistic description, one has to consider discrete magnetostatic modes [7] rather than a spinwave continuum. For higher magnetic fields the resonant magnetostatic modes are increasingly separated in frequency, and, on average, the resonance condition $\omega = 2\omega_k$ better holds for inhomogeneous modes with larger k. Therefore, we conclude that in regime C the excitation of spinwaves ($k \geq 10^4\ \mathrm{cm}^{-1}$) should be more efficient.

Our experimental findings can be interpreted in terms of the following multimode model: starting with the eigenmodes $\mathbf{m}_j(\mathbf{r})\ \exp(-i\omega_j t)$ of the linearized system, nonlinear coupling terms give rise to a collective motion of some of these modes. In addition to the uniform mode ($j = 0$) which is excited by a microwave field $h\cdot\cos\omega t$, we consider both spinwaves ($j = \mathbf{k}$) and magnetostatic modes ($j = \{n, m, r\}$). By analogy with [5] but confining to three-magnon processes only, we have derived the following equation of motion:

$$\dot{a}_0 = -\left[\eta_0 + i\left(\omega_0 - \omega\right)\right] a_0 - \sum_{j,\ell=1}^{N} \rho^*_{j\ell} a_j a_\ell - i\gamma h/2 \quad \text{(1a)}$$

$$\dot{a}_j = -\left[\eta_j + i\left(\omega_j - \omega/2\right)\right] a_j + \sum_{\ell=1}^{N} \rho_{j\ell} a_0 a^*_\ell \quad \text{(1b)}$$

$a_0(t)$ and $a_j(t)$ denote the slowly variing amplitudes of the uniform and of the inhomogeneous eigenmodes, $\mathbf{m}_j(\mathbf{r})$ being normalized to unity. η_0 and η_j denote the respective damping constants and γ the gyromagnetic ratio. The dipolar coupling coefficients are given by

$$\rho_{j\ell} = -i\gamma/4V \int \mathrm{d}^3 r \mathbf{m}^*_j(\mathbf{r})\, \mathbf{m}_0(\mathbf{r})\, H^z_{\rm dip}\left\{\mathbf{m}_\ell(\mathbf{r})\right\}. \quad (2)$$

In contrast to Suhl's theory [5] and related work [4] equation (1) includes the mutual coupling of inhomogeneous modes described by the non-diagonal terms $\rho_{j\ell}$, $j \neq \ell$. If we only consider the parametric excitation of spinwaves such coupling terms $\rho_{\mathbf{kk}'}$, $\mathbf{k} \neq \mathbf{k}'$ will not occur owing to the orthogonality of Fourier components with different wavevectors. (Note that $\mathbf{m}_k(\mathbf{r}) \sim \exp(i\mathbf{kr})$ and $H^z_{\rm dip}\left\{\mathbf{m}_{\mathbf{k}'}(\mathbf{r})\right\} \sim \exp\left(i\mathbf{k}'\mathbf{r}\right)$). In case of eigenmodes with very small k this argument does not hold any longer and their mutual coupling can be rather strong. Solving equation (1) by numerical methods and considering two or more inhomogeneous modes without a direct coupling (regimes A or C), above the threshold only a fixed point occurs, whereas, introducing the mutual coupling between these modes (regime B), a_0 starts oscillating directly at the threshold.

In addition, we assume an amplitude dependent renormalization of the eigenfrequencies ω_j

$$\omega_j \rightarrow \omega_j + \sum_{\ell=0}^{N} T_{j\ell} \left|a_\ell\right|^2, \quad (3)$$

which could either result from the reduction of the longitudinal magnetization with increasing excitation [5] or from sample heating affecting both anisotropy and demagnetizing fields. Including such frequency renormalization in our equation of motion (1) the numerical solutions show the same kind of sudden jumps of $|a_0|^2$ as observed in experiment. Similar effects, however, could be obtained if instead of equation (3) the indirect excitation of the magnetostatic 430-mode (which is nearly degenerate with the uniform mode) is taken into account [8]. Our simulations show that these discontinuities of $\bar{P}_{\rm tr}$ coincide with a sudden decrease or increase of certain inhomogeneous modes. So, we conclude that the direct coupling of magnetostatic modes combined with one of these mechanism results in "switching on and off" certain modes with respect to the control parameters and the previous state of the system. This – in accordance with the results of our Grassberger-Procaccia analyses – could explain the complex multistable behaviour observed in regime B, but also the non-appearance of such effects in the regimes A and C, where the excited modes should be of spinwave type.

Acknowledgments

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