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MULTI-FRACTALS OF STRANGE ATTRACTORS IN PARALLEL-PUMPED SPIN-WAVE INSTABILITIES

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Abstract. - Strange attractors beyond the critical point are numerically studied on the basis of a theoretical model for parallel-pumped spin-wave instabilities. In the transition region leading to the growth of a high dimensional attractor, a singularity spectrum is found to show interesting bifurcations.

Spin-wave nonlinear dynamics beyond the Suhl threshold is found to show rich structures of chaos and turbulence comparable to those in fluid dynamics [1]. The theoretical model which includes the effect of a cavity mode has proved to explain a variety of experimental results very well [2].

Concerning an essential feature of strange attractors, low-dimensional maps may be effective in analysing them below and at the critical point, i.e., onset of chaos. Above the critical point, however, more detailed informations about underlying equations of motion will be required so as to capture several dramatic changes in strange attractors.

In this paper, we shall analyse strange attractors at and beyond the critical point in a parallel-pumped spin-wave instability. In particular, the change in a multi-fractal structure of (numerically calculated) Poincare sections will be examined by increasing a driving power.

The equation of motion for spin-wave modes \( C_k \) is

\[
\dot{C}_k = -\gamma_k C_k - i\Delta \Omega_k C_k - iQF g_k C_k^* - i\{ 2 \sum_{k'} T_{kk'} |C_{k'}|^2 C_k \\
+ \sum_{k'} (S_{kk'} + E g_k^* g_k) C_{k'}^* C_k \},
\]

where \( \gamma_k, \Delta \Omega_k, g_k \) and \( F \) are damping constants for \( C_k \), frequency shifts, coupling between the cavity mode and \( C_k \), and a driving field, respectively. \( T_{kk'} \) and \( S_{kk'} \) denote the coupling among spin-waves. Equation (1) is formally identical to that employed in the case of the first-order perpendicular pumping [2]. In the present case, however, a cavity mode directly couples with spin-wave pairs. We have thereby new expressions for the quality factor \( Q \) and for \( E : Q = \omega / |\Gamma|, E = -i/2|\Gamma| \) with \( \Gamma \) being the damping constant for the cavity mode. We now confine to the case of two modes system where \( g_{k1} \neq 0 \) and \( g_{k2} = 0 \).

We find that period-doubling bifurcations accumulate at the critical point \( F = F_c = 1.92286 \). The strange attractor and corresponding Poincare section at \( F = F_c \) are given in figures 1a and 1b. When \( F \) is increased beyond \( F_c \), we see a gradual growth of a higher-dimensional attractor (see Figs. 1c and 1d).

We proceed to calculate the singularity spectra \( f(\alpha) \) [4] by using the Poincare section at and just above the critical point. Figures 2-4 corresponds to \( F = F_c, 1.9229 \) and 1.94, respectively. The curve \( f(\alpha) \) at \( F = F_c \) is found to be almost identical to the universal one due to a logistic model [4]. At \( F = 1.9229 \), however, \( t \)-dependence of \( \langle p_t^{\alpha-1} \rangle \) has a crossover region \( R_c \) at \( 2^{-12} < t < 2^{-9} \). (See the inset in Fig. 3.)

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Fig. 1. - (a) Strange attractor at \( F = F_c = 1.92286 \) (plot real \( C_1 \) vs. imaginary \( C_1 \)). (b) Poincare section at \( F = F_c \) (Inset is a partial magnification). (c) Strange attractor at \( F = 1.94 \). (d) Strange attractor at \( F = 2.20 \).
notations $\ell$, $p$, and $\alpha$ are the same as used in reference [4]. The scaling exponents $\tau_q$ below and above $R_c$ differ from each other. Consequently, a bifurcation of $f(\alpha)$ occurs in figure 3. The left-hand side curve is related to a larger-scale behavior, retaining a feature of the universal curve at the critical point. The right-hand curve is related to a small-scale one, describing a new fractal structure which cannot be described by a logistic model. At $F = 1.94$, we find no indication of a crossover region.

The curve in figure 4 has a larger fractal dimension $D_0 = 1.02$ and narrower width $D_{\infty} = 0.6$, $D_{-\infty} = 1.25$ than the right-hand side curve in figure 3. This implies that an increase in the fractal dimension of a strange attractor leads to more homogeneous distribution of measures. Further increase of $F$ has confirmed this observation.

In conclusion, the strange attractor above the onset of chaos in the parallel-pumped spin-wave instability cannot simply be described by a unique low-dimensional map. A new theoretical model indicates a route towards a higher-dimensional attractor via the transition region where $f(\alpha)$ shows remarkable bifurcations.


[3] Parameter values used in the present text are:
$g_{k1} = 0.2i \times 10^6$, $g_{k2} = 0$, $\Gamma = 1.0 \times 10^8$
$\gamma_{k1} = 0.1\times 10^8$, $\gamma_{k2} = 0.2 \times 10^8$, $T_{11} = -0.5 \times 10^{-8}$
$T_{22} = 0.25 \times 10^{-8}$, $T_{12} = T_{21} = 2.0 \times 10^{-8}$,
$S_{11} = -0.5 \times 10^{-8}$, $S_{22} = 0.25 \times 10^{-8}$, $S_{12} = S_{21} = 4.5 \times 10^{-8}$, $\Delta \Omega_1 = \Delta \Omega_2 = 0$ (s$^{-1}$).