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QUANTUM EFFECTS ON SOLITON PROPERTIES IN FERROMAGNETIC SPIN CHAINS

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Abstract. - Quantum effects on soliton-like excitations are investigated for the easy-plane ferromagnetic chain and for the anisotropic $xy$-chain. Our results for the soliton energy to order $1/S^2$ and for the soliton contribution to the specific heat indicate that the semiclassical approach reasonably describes quantum effects on solitons properties (in particular the strong reduction of the specific heat) for realistic spin chains.

1. Introduction

Solitons in quasi one-dimensional magnets predominantly have been investigated in the classical approximation [1]. Quantum effects on soliton-like excitations are of interest since spin values in magnetic chain materials are low ($S = 1$ in $\text{CsNiF}_3$) and since experiments, in particular on soliton contributions to the magnetic specific heat, show that a classical description cannot be completely adequate [2, 3]. In the following we use the perturbational approach to quantum effects (coupling constant $g \sim 1/S$) [4, 5] for two magnetic chain systems and discuss the validity of this semiclassical approach: in section 2 we formulate and evaluate a theory covering quantum effects in the easy-plane ferromagnetic chain including terms of $O(1/S^2)$, in section 3 we give a semiclassical theory of the discrete anisotropic $xy$-model in order to estimate the usefulness of the semiclassical approach for real quantum systems by comparison to the exact solution of this model for $S = 1/2$ [6].

2. Quantum solitons in the 1d easy-plane ferromagnet

Our subject in this section is the easy-plane ferromagnetic chain in an external field given by the Hamiltonian

$$H = -J \sum_n S_n S_{n+1} + A \sum_n (S_n^x)^2 - \mu B \sum_n S_n^z. \quad (2.1)$$

We investigate quantum effects in this system using the planar representation of spin operators ($S_n^x = \sqrt{S(S+1)}$)

$$S_n^+ = \epsilon^{i\phi_n} \sqrt{S^2 - S_n^z} (S_n^z + 1). \quad (2.2)$$

$$[\phi_m, S_n^z] = i \delta_{mn}. \quad (2.3)$$

We formulate the quantum properties of the model in terms of a weak coupling expansion in $1/S$ with the "classical lump" of Coleman [7] (corresponding to the Sine-Gordon (SG) soliton) as starting point. To obtain quantum corrections, fluctuations about the soliton have to be considered. Putting

$$\phi_n = \phi_{n0} + \varphi_n, \quad S_n^z = \hat{S} \beta_n \quad (2.4)$$

normal modes for the fluctuation amplitudes are obtained from a linear eigenvalue problem as described before [8]. The change of the soliton energy owing to quantum effects is obtained as a power series in $1/S$ by using (2.4) in $H$ and calculating the difference in energy between the one-soliton state and the vacuum. Using the continuum approximation we have to evaluate integrals involving $\langle \varphi^2(x) \rangle$ etc. for first order terms and $\langle \varphi^4(x) \rangle$ for second order terms. In order to go beyond the known [5] first order term, we calculate the soliton induced changes in $\langle \varphi^2(x) \rangle$, $\langle \varphi^4(x) \rangle$ etc., which then allow to directly perform the integration for higher orders. Working to leading order in $J/2A$ we find that $\langle \varphi^2(x) \rangle = \langle \beta^2(x) \rangle = 1/2S$ is unaffected by the the soliton whereas for the derivative terms one has

$$\langle \left( \frac{d^2 \varphi}{dx^2} \right)^2 \rangle = \langle \left( \frac{d \varphi}{dx} \right)^2 \rangle_0 + \frac{1}{2S} (ma)^2 \text{sech}^2 mz$$

$$\langle \left( \frac{d \beta}{dx} \right)^2 \rangle = \langle \left( \frac{d \beta}{dx} \right)^2 \rangle_0 + \frac{1}{2S} (ma)^2 \text{sech}^2 mz. \quad (2.5)$$

In the leading order considered here the main contributions come from wave vectors near the zone boundary – the lattice cutoff, which renders the present continuum theory finite, is essential. For the final result we have applied a discreteness correction to the fourth order derivative terms, replacing $q^2$ by $2(1 - \cos q)$. Our result for the soliton energy up to $O(1/S^2)$ then reads

$$E_{\text{sol}} = JS^2 8ma \left( 1 - \frac{1}{2S} + \frac{9}{16S^2} + \ldots \right). \quad (2.6)$$

The second order term thus is smaller than the first order term even for $S = 1$, which supports the expectation that this semiclassical expansion converges for spin values of interest with real materials. Quantitatively corrections from quantum effects to the zero...
temperature soliton energy are quite small in accordance with experimental observation.

Using the present method, results are also obtained for finite temperatures, for correlation functions and for moving solitons. These will be presented in detail in a separate publication [9].

3. Quantum solitons in the anisotropic xy model

In this section we investigate the influence of quantum effects on the soliton contribution to the specific heat for the anisotropic xy model defined by the Hamiltonian

\[ H = -J \sum_n \left\{ (1 + \gamma) S_n^x S_{n+1}^x + (1 - \gamma) S_n^y S_{n+1}^y \right\} \]

(3.1)

for varying spin length \( S \). We start from the static soliton solution for the discrete classical chain [10], which in polar coordinates reads

\[ \theta_n = 0, \quad \sin \varphi_n = \text{sech} \, m \, (n + \alpha) \]

(3.2)

with \( \cosh m = 1 + \gamma / (1 - \gamma) \) and arbitrary phase \( \alpha \). The corresponding energy is \( E_{\text{sol}} = 4 \sqrt{\gamma} JS^2 \). For \( \gamma \ll 1 \) the system (3.1) can be approximated by its continuum limit in planar approximation; it is then equivalent to a SG system with angular variable \( 2\varphi \). Attempting to go beyond the SG limit one finds that correction terms from out-of-plane terms are of the same order as those from the discrete lattice (note the difference to the easy-plane ferromagnet, where the strength of out-of-plane fluctuations is governed by the independent parameter \( J/2A \)). In the following we report the results of a semiclassical analysis for the soliton contribution to the magnetic specific heat starting from the discrete solution, whereas further details and results on the stability of the discrete soliton and its generalization to finite velocities will be published separately [11].

To calculate the specific heat we have to find the phase shift of a small oscillation (magnon) in the presence of a soliton [5]. It is obtained from the linearized equations of motion, which can easily be solved numerically for the discrete system (although Born approximation already gives quite good accuracy). Actually there exists a second bound state solution [11] in addition to the one related to translational invariance, which, however, is not of quantitative importance for the following.

From the phase shifts semiclassical corrections to the soliton energy and to the soliton contribution to the specific heat are obtained [5]. In figure 1 we show the resulting specific heat for several values of \( S \). To judge the validity of the semiclassical approximation we display for comparison the analogous quantity for the \( S = 1/2xy \) model making use of the exact solution [6] (after subtraction of the free Boson contribution of noninteracting magnons from the full specific heat). Figure 1 demonstrates that our semiclassical approximation reproduces surprisingly well the tendency of quantum effects to suppress the specific heat as observed experimentally. From our calculations for two different magnetic chain systems we conclude that the semiclassical approach is able to reproduce essential features of the quantum aspects of solitons in magnetic chains.

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References