DYNAMICAL CRITICAL BEHAVIOUR OF HEISENBERG FERROMAGNETS FOR $T \geq T_c$

H. Iro

To cite this version:

H. Iro. DYNAMICAL CRITICAL BEHAVIOUR OF HEISENBERG FERROMAGNETS FOR $T \geq T_c$. Journal de Physique Colloques, 1988, 49 (C8), pp.C8-1563-C8-1564. <10.1051/jphyscol:19888716>. <jpa-00228954>
DYNAMICAL CRITICAL BEHAVIOUR OF HEISENBERG FERROMAGNETS FOR $T \geq T_c$

H. Iro

Institut für theoretische Physik, Universität Linz, A-4040 Linz, Austria

Abstract. From an $O(\varepsilon)$-calculation ($\varepsilon = 6 - d$) of the dynamical spin correlation function in a Heisenberg system for $T \geq T_c$, an expression for that function applicable in dimension $d = 3$ is deduced. It comprises the expected limiting behaviour in different sectors of the critical region.

It has been shown that the Ma-Mazenko model of a Heisenberg ferromagnet [1], evaluated within an $\varepsilon$-expansion, $\varepsilon = 6 - d$ [2, 3], accounts very well for certain features found by neutron scattering in Fe, Ni and EuO [4-6] at $T_c$. To investigate the applicability of the model also for $T > T_c$, one has to calculate the spin correlation function for arbitrary values of $q$, $\omega$, and the correlation length $\xi$. Since the calculation involves an expansion around $d = 6$ [7], the result must be processed further in order to apply in $d = 3$ [8].

The equation of motion for the order parameter $S$ in a Heisenberg system with effective Hamiltonian

$$\mathcal{H} = \frac{1}{2} \int d^d x \left[ r S^2 + (\partial_i S)^2 \right]$$

is given by [1]

$$\dot{S} = \lambda S \times \frac{\delta \mathcal{H}}{\delta S} + \lambda \nabla^2 S + \zeta \tag{1}$$

with the Gaussian distributed random noise $\zeta$. For the spin correlation function $C(q, \omega, \xi)$, expressed via the fluctuation dissipation theorem by the self energy $\Pi(q, i\omega)$ of the response function, which can be more conveniently calculated, one has [7] ($x = q\xi$, $w = \omega/\lambda q^2$)

$$C(q, \omega, \xi) = q^{-2} X(x) \frac{1}{\lambda q^2} \text{Re} \left[ \frac{2}{-i\omega + [X(x) \Pi(x, i\omega)]^{-1}} \right]. \tag{2}$$

Here $X(x) = (1 + 1/x^2)^{-1}$ is the scaling function for the static susceptibility (i.e. $\xi^{-2} = r$) and the dynamical critical exponent $z$ equals $d+2 - 4 - \varepsilon/2$ [1]. (In $X(x)$ and in $z$ we have neglected $\eta$; $\lambda$ has been renormalized.) The self energy $\Pi$ has been calculated to first order in $\varepsilon$ ($\varepsilon$ is related to the fixed point value of $f^*$ by $f^* = (96\pi^3 \varepsilon)^{1/2}$ [1])

$$\Pi(x, i\omega) = 1 - \varepsilon F(x, i\omega) \tag{3}$$

where $F(x, i\omega)$ is a rather involved expression, which can be found explicitly in reference [7]. We quote only the two limits of $F(x, i\omega)$ used in the following. At $T_c$ one finds for large values of $\omega$

$$F(\infty, i\omega) = -\frac{1}{8} \ln (-i\omega) + \frac{1}{8} \ln 2 + O(1/\sqrt{\omega}) \tag{4a},$$

and in the hydrodynamic region, i.e. $x \ll 1$, for $\omega = 0$ the limit is

$$F(x, 0) = \frac{1}{2} \ln x - \frac{3}{8} + O(x^2). \tag{4b}$$

This asymptotic behaviour of $F(x, i\omega)$ has to match the known asymptotic power law behaviour of $\Pi(x, i\omega)$ [8], namely

$$\Pi(\infty, i\omega) \sim w^{(4-z)/z} \quad \text{for} \quad w \to \infty \tag{5a},$$

and

$$\Pi(x, 0) \sim x^{z-4} \quad \text{for} \quad x \to 0. \tag{5b}$$

In the same spirit as in reference [3] this can be achieved by replacing $F(x, i\omega)$ by a simpler function showing the logarithmic behaviour of equations (4a, b)

$$F(x, i\omega) \simeq -\frac{1}{8} \left[ \left(1 + \frac{b}{x^2}\right)^2 - ai\omega \right]. \tag{6}$$

where

$$a = 0.46 \quad \text{and} \quad b = 3.16. \tag{7}$$

By exponentiating the right hand side of equation (3) with $F$ given by (6) and observing conditions (5a, b) one arrives at the final expression for $\Omega(x, i\omega)$

$$\Pi(x, i\omega) = \left[ \left(1 + \frac{b}{x^2}\right)^{2-\varepsilon/4} - ai\omega \right]^{\varepsilon/(3-\varepsilon)} \tag{8}$$

Insertion into equation (2) and setting $\varepsilon = 3$ gives the final form for the correlation function $C(q, \omega, \xi)$ of isotropic Heisenberg ferromagnets [9].

For a comparison with neutron scattering results one has to express $C(q, \omega, \xi)$ in terms of the measured quantities

$$C(q, \omega, \xi) = \chi(q, \xi) \frac{1}{\omega_c(q, \xi)} \Phi \left( q\xi, \omega/\omega_c \right). \tag{9}$$
Here \( \omega_c(q, \xi) \) is the half width at half maximum of the scattering intensity when \( q \) is kept constant

\[
C(q, \omega_c, \xi) = \frac{1}{2} C(q, 0, \xi) .
\] (10)

The solution \( \omega_c \) is a homogeneous function, so that

\[
\omega_c = \frac{1}{2} \Gamma q^3 \Omega(x)
\] (11)

where \( \Omega \) is fixed at \( T_c \) to be \( \Omega(\infty) = 1 \). The homogeneity of the dynamic shape function \( \Phi \) was already anticipated in equation (9). In reference [8] \( \Omega \) and \( \Phi \) are given in terms of \( \Pi \).

At \( T_c \) \( \Phi \) reduces to the form used in reference [4], thus leaving unchanged the conclusions in references [5, 6]. For \( T > T_c \) \( \Omega(x) \) is similar to a mode coupling result obtained in reference [10]. Especially for small values of \( x \) there is less agreement with results found in Fe [11] or EuO [12], but his is due to dipolar forces (see below). In the shape \( \Phi \), particularly at \( T_c \), the small \( \omega \) behaviour is not as flat as found in mode coupling calculations [10]. A feature that is very sensitive to the model is a constant \( \omega \) scan. An example is shown in the figure for different values of \( \xi \). By increasing the distance from \( T_c \) (decreasing \( \xi \)) the peak position is shifted very slightly to higher values of \( q \). This weak variation, already expected in [13], matches very well to neutron scattering results as e.g. in Ni [14, 15] and EuS [16]. Further the equal intensity contours of \( C(q, \omega, \xi) \) are confirmed by an experiment on Fe\(_3\)Si [17]: in the \((q, \omega)\) domain considered a dynamic Lorentzian shape function is clearly excluded.

So the results presented above lead directly to the limitations of the model. Though dipolar interactions are small, due to their spin nonconserving property they are ultimately relevant very close to the critical point. In reference [18] it was shown that dipolar interactions can account for the scaling function \( \Omega \) found experimentally. In the shape function \( \Phi \) their influence becomes evident at \( T_c \) only at very small values of the wave vector \( q \) [19]. In conclusion one can say that equation (2) together with (8) describes the critical behaviour very well but very close to the dynamical critical point the spin nonconserving interactions become important. The onset of this influence depends on the quantity considered.

Acknowledgment

This work was supported by the Fonds zur Förderung der wissenschaftlichen Forschung.

[9] The condition

\[
\int_{-\infty}^{\infty} \frac{d\omega}{2\pi} C(q, \omega, \xi) = \chi(q, \xi)
\]

is satisfied by \( \Pi(x, i\omega) \) as given by equation (8) in a nontrivial manner.