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To cite this version:

H.-O. Heuer. GOLDSTONE SINGULARITIES IN ISOTROPIC FERROMAGNETS. Journal de Physique Colloques, 1988, 49 (C8), pp.C8-1561-C8-1562. <10.1051/jphyscol:19888715>. <jpa-00228953>

HAL Id: jpa-00228953
https://hal.archives-ouvertes.fr/jpa-00228953
Submitted on 1 Jan 1988

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GOLDSTONE SINGULARITIES IN ISOTROPIC FERROMAGNETS

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Abstract. - The equation of state and the susceptibilities of isotropic magnets are calculated in exponentiated scaling form to $O(\epsilon)$ by repetitive use of the trajectory integral method. The resulting coexistence-curve singularities do not depend on the dimension for $d \geq 3$ since the Goldstone modes show Fisher-renormalized classical tricritical behaviour.

It is well-established [1, 2] that the coexistence-curve of isotropic ferromagnets is a line of critical points where the transverse susceptibility diverges. It ends at the ordinary critical point, which is a kind of birectical point where longitudinal and transverse fluctuations become critical. In this paper, I apply the trajectory integral method [3] to calculate the crossover between the ordinary critical behaviour and the coexistence behaviour of isotropic magnets in $O(\epsilon)$ ($\epsilon = 4 - d$). My starting point is the usual Ginzburg-Landau-Wilson-Hamiltonian for an $n$-component isotropic spin system:

$$\mathcal{H} = -\frac{1}{2} \int \sum_{q} S_q \cdot S_{-q} (r + q^2) - u \int \int \sum_{q_1} S_{q_1} \cdot S_{q_2} \cdot S_{-q_1+q_2+q_3} + H S_0^2 \quad (1)$$

Seperating longitudinal and transverse modes by a shift $\sigma = S_{\mathbf{q}} - M \delta (\mathbf{q})$, one obtains the Hamiltonian

$$\tilde{\mathcal{H}} (\sigma, S_{\perp}) = -\frac{1}{2} \tau_L \sigma^2 - \frac{1}{2} \tau_T |S_{\perp}|^2 - u_1 \sigma |S_{\perp}|^2 - u_2 \sigma^3 - u_3 \sigma^4 - u_4 |S_{\perp}|^4 + \tilde{H} \sigma_0 \quad (2)$$

with the coupling parameters $\tau_L = r + 12uM^2$, $\tau_T = r + 4uM^2$, $u_1 = u_2 = 4uM$, $u_3 = u_4 = 2u$ and $\tilde{H} = H - rM - 4uM^3$. $M$ is the magnetization given by the condition $\langle \sigma_0 \rangle = 0$. It is the idea of the trajectory integral method to renormalize the system into the noncritical region $r(\epsilon') = O(1)$ and to match it with Landau theory plus fluctuation corrections. Mean field theory shows that this concept fails near the coexistence-curve: if one chooses $\epsilon'$ in such a way that the longitudinal fluctuations are noncritical ($\tau_L (\epsilon') = O(1)$), the transverse fluctuations are still critical ($\tau_T (\epsilon') \approx 0$). These considerations led to a parquet graph summation of the susceptibilities in Nelsos work [1]. In this paper I apply the trajectory integral method throughout. The idea is to continue the renormalization of the system $\tilde{\mathcal{H}} (\epsilon')$ [1, 3] until the critical Goldstone modes are noncritical at some matching point $\tilde{\epsilon}$. Before, it is necessary to eliminate the noncritical longitudinal modes in the renormalized free energy

$$\tilde{F} (\epsilon') = -\frac{1}{V (\epsilon')} \ln \int \mathcal{S}_{\perp} \exp \tilde{\mathcal{H}} (\sigma, S_{\perp}; \epsilon') \quad (3)$$

by integration over $\sigma$. The Feynman-graph expansion leads to

$$\tilde{F} (\epsilon') = -\frac{1}{V (\epsilon')} \ln \int \mathcal{S}_{\perp} e^{\tilde{\mathcal{H}} (S_{\perp}) - \mathcal{V} (\epsilon')} \Delta F_{\sigma} =: \tilde{F} + \Delta F_{\sigma} \quad (4)$$

$\Delta F_{\sigma}$ is the integrated free energy of the longitudinal modes in $\tilde{\mathcal{H}} (\sigma, S_{\perp}; \epsilon')$. Since these modes are noncritical, $\Delta F_{\sigma}$ can be calculated by Landau theory plus leading fluctuation corrections:

$$\Delta F_{\sigma} = -\frac{H^2 (\epsilon')}{2 \tau_L (\epsilon')} + \frac{1}{2} \int P \ln G_{\perp}^{-1} (\epsilon') + \frac{3}{2} \frac{w_2 (\epsilon') \tilde{H} (\epsilon')}{\tau_L (\epsilon')} \int P G_L (\epsilon') + O (w_2 (\epsilon'), w_3^2 (\epsilon')) \quad (5)$$

The effective Hamiltonian

$$\tilde{\mathcal{H}} (S_{\perp}) = -\frac{1}{2} \tilde{r} |S_{\perp}|^2 - \tilde{u} |S_{\perp}|^4 \quad (6)$$

in (4) describes the critical Goldstone modes coupled to the noncritical longitudinal modes. The coupling parameters $\tilde{r}$ and $\tilde{u}$ are calculated to $O(w_4 (\epsilon'), w_5 (\epsilon'))$ as

$$\tilde{r} = \tau_T (\epsilon') + 2 \frac{w_1 (\epsilon') \tilde{H} (\epsilon')}{\tau_L (\epsilon')} + 2u_3 (\epsilon') \int G_L (p) - 6 \frac{w_1 (\epsilon') w_2 (\epsilon')}{\tau_L (\epsilon')} \int G_L (p) \quad (7)$$

and

$$\tilde{u} = w_1 (\epsilon') - \frac{w_2^2 (\epsilon')}{2} \int G_L (q_1 + q_2 + p) \quad (8)$$

$G_L$ in (5, 7, 8) is the longitudinal propagator at $\epsilon'$: $G_L = (\tau_L (\epsilon') + p^2)^{-1}$. Before I work out the new coupling parameters $\tilde{r}$ and $\tilde{u}$ in terms of the original pa-
rameters of $\mathcal{H}$ it is neccesary first to calculate the susceptibilities and the equation of state from the condition $\langle \sigma_0 \rangle = 0$. The usual Feynman graph expansion would lead to logarithmic terms in $\tilde{r}$. A better way is to make use of (2) which shows that $\tilde{H}$ is the source term for $\sigma$.

$$H(\ell^*) - r(\ell^*) M(\ell^*) - 4u(\ell^*) M^3(\ell^*) = 12 u(\ell^*)$$

$$M(\ell^*) \int_q G_L(r_L(\ell^*)) + 8 u(\ell^*) M(\ell^*) \tilde{E}(\tilde{t}, \tilde{u})$$

where

$$\tilde{E} := \frac{\partial \tilde{F}}{\partial \tilde{r}} = \frac{1}{2} \int_q \langle S \cdot \tilde{q} \cdot S \cdot \tilde{-} \tilde{q} \rangle = \frac{\partial \tilde{F}}{\partial \tilde{t}}$$

is the energy of the $(n-1)$-dimensional Goldstone system which has been calculated in exponentiated scaling form [4, 5]. It is sensible to introduce the abbreviations

$$T_L(\ell^*) := t(\ell^*) + 12 u(\ell^*) M^2(\ell^*) = r_L(\ell^*) + O(u(\ell^*))$$

$$T_T(\ell^*) := t(\ell^*) + 4 u(\ell^*) M^2(\ell^*) = r_T(\ell^*) + O(u(\ell^*))$$

is the energy of the Goldstone system by a Fisher-renormalization [7] since the nonanalytic term $\tilde{t}^{\alpha_1-\alpha}$ with \(\alpha_1 = \epsilon/2\) dominates near the coexistence-curve. The physical reason for this Fisher-renormalization is the coupling of the critically fluctuating transverse modes to the longitudinal modes in the Goldstone regime. The equation of state and the transverse susceptibility near the coexistence-curve follow from (9) and (15) as

$$\frac{h}{m_{\ell^*}} = L^{-2} \tilde{t}$$

$$\frac{x_T}{m^{1-\alpha}} = L^2 \tilde{t}^{-1}. \tilde{t}$$

The longitudinal susceptibility is evaluated in the same way observing that the term $\tilde{E}/\tilde{t}$ dominates:

$$\frac{x_L}{m^{1-\alpha}} = \left[ \frac{9}{n + 8} + \frac{n - 1}{n + 8} \tilde{t}^{-\alpha} \cdot F_0(\alpha_t) \right].$$

Note that the equation of state and the transverse susceptibility near the coexistence-curve have the very simple structure of a system with temperature $\tilde{t}$ and vanishing interaction $\tilde{u}$. This point is verified from (13), which leads to

$$\tilde{u} \sim \left[ \frac{h}{m_{\ell^*}} \right]^{1-\alpha}$$

Working out the matching condition $T_L(\ell^*) = 1$ and $T_T(\ell^*)$ in (15), one obtains the equation of state near the coexistence-curve

$$\frac{h}{m^{1-\alpha}} = \left[ \frac{1}{m^{1-\alpha}} + 1 \right]^{1-\alpha} \cdot F_0(\alpha_t)$$

using the usual normalization conditions $f(-1) = 0$ and $f(0) = 1$. The longitudinal susceptibility is given by

$$\frac{x_L}{m^{1-\alpha}} \sim \left( \frac{h}{m_{\ell^*}} \right)^{-\alpha}$$

near the coexistence-curve. Note that the results (20, 21) for the functional form of the equation of state and the susceptibilities near the coexistence-curve are exact for $d \geq 3$ since the specific heat exponent $\alpha_t = 1/2$ is classical tricritical for $d \geq 3$.