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To cite this version:
H.-O. Heuer. GOLDSTONE SINGULARITIES IN ISOTROPIC FERROMAGNETS. Journal de Physique Colloques, 1988, 49 (C8), pp.C8-1561-C8-1562. <10.1051/jphyscol:19888715>. <jpa-00228953>

HAL Id: jpa-00228953
https://hal.archives-ouvertes.fr/jpa-00228953
Submitted on 1 Jan 1988

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GOLDSTONE SINGULARITIES IN ISOTROPIC FERROMAGNETS

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Abstract. - The equation of state and the susceptibilities of isotropic magnets are calculated in exponentiated scaling form to $O(\varepsilon)$ by repetitive use of the trajectory integral method. The resulting coexistence-curve singularities do not depend on the dimension for $d \geq 3$ since the Goldstone modes show Fisher-renormalized classical tricritical behaviour.

It is well-established [1, 2] that the coexistence-curve of isotropic ferromagnets is a line of critical points where the transverse susceptibility diverges. It ends at the ordinary critical point, which is a kind of bicritical point where longitudinal and transverse fluctuations become critical. In this paper, I apply the trajectory integral method [3] to calculate the crossover between the ordinary critical behaviour and the coexistence behaviour of isotropic magnets in $O(\varepsilon)$ ($\varepsilon = 4 - d$). My starting point is the usual Ginzburg-Landau-Wilson-Hamiltonian for an $n$-component isotropic spin system:

$$\mathcal{H} = -\frac{1}{2} \sum_q \mathbf{S}_q \cdot \mathbf{S}_{-q} \left( r + q^2 \right) - u \sum_{q_1} \sum_{q_2} \sum_{q_3} \mathbf{S}_{q_1} \cdot \mathbf{S}_{q_2} \cdot \mathbf{S}_{q_3} \cdot \mathbf{S}_{-q_1-q_2+q_3} + \mathcal{H}_0$$

Separating longitudinal and transverse modes by a shift $\sigma_q = S_q - M \cdot \delta(q)$, one obtains the Hamiltonian

$$\tilde{\mathcal{H}}(\sigma, S_\perp) = -\frac{1}{2} \tau_1 \sigma^2 - \frac{1}{2} \tau_T |S_\perp|^2 - u_1 |S_\perp|^2 - u_2 \sigma^4 - u_3 \sigma^2 |S_\perp|^2 + \tilde{H} \sigma_0$$

with the coupling parameters $\tau_L = r + 12uM^2$, $\tau_T = r + 4uM^2$, $w_1 = w_2 = 4uM$, $u_1 = u_2 = u$, $u_3 = 2u$ and $\tilde{H} = H - rM - 4uM^3$. $M$ is the magnetization given by the condition $\langle \sigma_0 \rangle = 0$. It is the idea of the trajectory integral method to renormalize the system into the noncritical region $\tau(\varepsilon^*) = O(1)$ and to match it with Landau theory plus fluctuation corrections. Mean field theory shows that this concept fails near the coexistence-curve: if one chooses $\varepsilon^*$ in such a way that the longitudinal fluctuations are noncritical ($\tau_L(\varepsilon^*) = O(1)$), the transverse fluctuations are still critical ($\tau_T(\varepsilon^*) \approx 0$). These considerations led to a parquet graph summation of the susceptibilities in Nelsons work [1]. In this paper I apply the trajectory integral method throughout. The idea is to continue the renormalization of the system $\tilde{\mathcal{H}}(\varepsilon^*)$ [1, 3] until the critical Goldstone modes are noncritical at some matching point $\varepsilon^*$. Before, it is necessary to eliminate the noncritical longitudinal modes in the renormalized free energy

$$\tilde{F}(\varepsilon^*) = -\frac{1}{V(\varepsilon^*)} \ln \int_{\mathcal{S}_{\perp}} \exp \tilde{\mathcal{H}}(\sigma, S_\perp; \varepsilon^*)$$

by integration over $\sigma$. The Feynman-graph expansion leads to

$$\tilde{F}(\varepsilon^*) = -\frac{1}{V(\varepsilon^*)} \ln \int_{\mathcal{S}_{\perp}} e^{\tilde{\mathcal{H}}(S_\perp)} - v(\varepsilon^*) \Delta F_\sigma = \tilde{F} + \Delta F_\sigma$$

$\Delta F_\sigma$ is the integrated free energy of the longitudinal modes in $\tilde{\mathcal{H}}(\sigma, S_\perp; \varepsilon^*)$. Since these modes are noncritical, $\Delta F_\sigma$ can be calculated by Landau theory plus leading fluctuation corrections:

$$\Delta F_\sigma = -\frac{\tilde{H}^2(\varepsilon^*)}{2\tau_L(\varepsilon^*)} + \frac{1}{2} \int_p G_L^{-1}(\varepsilon^*) +$$

$$+ 3 \frac{u_2(\varepsilon^*)}{r_L(\varepsilon^*)} \int_p G_L(\varepsilon^*) + O\left( u_2(\varepsilon^*), u_2^2(\varepsilon^*) \right).$$

The effective Hamiltonian

$$\tilde{\mathcal{H}}(S_\perp) = -\frac{1}{2} \tilde{\tau} |S_\perp|^2 - \tilde{u} |S_\perp|^4$$

in (4) describes the critical Goldstone modes coupled to the noncritical longitudinal modes. The coupling parameters $\tilde{\tau}$ and $\tilde{u}$ are calculated to $O(u_1(\varepsilon^*), u_1^2(\varepsilon^*))$ as

$$\tilde{\tau} = r_T(\varepsilon^*) + 2 \frac{u_1(\varepsilon^*)}{r_L(\varepsilon^*)} \tilde{H}(\varepsilon^*) + 2u_3(\varepsilon^*) \int_p G_L(p) -$$

$$-6 \frac{w_1(\varepsilon^*)}{r_L(\varepsilon^*)} \int_p G_L(p)$$

and

$$\tilde{u} = u_1(\varepsilon^*) - \frac{w_1^2(\varepsilon^*)}{2} \int_p G_L(q_1 + q_2 + p).$$

$G_L$ in (5, 7, 8) is the longitudinal propagator at $\varepsilon^*$: $G_L = (r_L(\varepsilon^*) + p^2)^{-1}$. Before I work out the new coupling parameters $\tilde{\tau}$ and $\tilde{u}$ in terms of the original pa-
rameters of $H$ it is necessary first to calculate the susceptibilities and the equation of state from the condition $\langle \sigma_0 \rangle = 0$. The usual Feynman graph expansion would lead to logarithmic terms in $\hat{t}$. A better way is to make use of (2) which shows that $H$ is the source term for $\sigma$:

$$\begin{align*}
H(\ell^*) - \tau(\ell^*) M(\ell^*) - 4u(\ell^*) M^3(\ell^*) &= 12u(\ell^*) \\
M(\ell^*) \int_q G_q(\tau_q) &= 12u(\ell^*) M(\ell^*) E(\hat{t}, \hat{u})
\end{align*}$$

(9)

where

$$\hat{E} := \frac{\partial E}{\partial \hat{t}} = \frac{1}{2} \int_q \langle S \cdot q \cdot S \cdot \overrightarrow{q} \rangle = \frac{\partial E}{\partial \hat{t}}$$

(10)

is the energy of the $(n - 1)$-dimensional Goldstone system which has been calculated in exponentiated scaling form [4, 5]. It is sensible to introduce the abbreviations

$$\begin{align*}
T_L(\ell^*) := t(\ell^*) + 12u(\ell^*) M^2(\ell^*) &= r_L(\ell^*) + O(\ell^*) \\
T_T(\ell^*) := t(\ell^*) + 4u(\ell^*) M^2(\ell^*) &= r_T(\ell^*) + O(\ell^*)
\end{align*}$$

(11)

where $t(\ell^*) = \tau(\ell^*) + A/2u(\ell^*)$ is the renormalized temperature scaling field [3]. $T_L(\ell^*) \simeq \mid - 2t(\ell^*) \mid$ is the renormalized temperature distance, whereas $T_T(\ell^*)$ is the renormalized distance from the coexistence-curve. Choosing $T_L(\ell^*) = 1$ as the matching condition, the effective temperature $\hat{t} := \hat{t} + \hat{A}/2\hat{u}$ of the Goldstone system follows from (7) as

$$\hat{t} = T_T(\ell^*) + (1 - T_T(\ell^*)) \cdot 8u(\ell^*) E(\hat{t}, \hat{u}) .$$

(12)

The effective interaction $\hat{u}(8)$ is momentum independent in $O(\varepsilon)$, given by

$$\hat{u} = u(\ell^*) \cdot \frac{T_T(\ell^*)}{T_L(\ell^*)} .$$

(13)

The equation of state (9) and the susceptibilities which follow from (9) depend on the effective temperature $\hat{t}$ (12) and $\hat{u}$ (13) via the energy $E(\hat{t}, \hat{u})$. $\hat{t}$ and $\hat{u}$ itself depend on the equation of state via $T_T(\ell^*)$. Thus, the magnetic properties for the whole $M - T$ - phase diagram result from the coupled equations (9, 12, 13) [6].

In this paper I present the results for the coexistence-curve only setting the $\hat{A}^d$-coupling to its critical value $u = u^c$. The behaviour at the coexistence-curve is obtained from the general results (9, 12, 13) in the limit $T_T(\ell^*) \ll 1$. Inserting the energy of the Goldstone system [5, 6]

$$\hat{E}(\hat{t}, \hat{u}) = \frac{(n - 1) K_4}{4\alpha_t} \hat{t} - \frac{(n - 1) K_4}{4\alpha_t} \hat{t}^{1 - \alpha_t} F_0(\hat{u})$$

(14)

in tricritical scaling fields into (12), leads to

$$T_T(\ell^*) \simeq \frac{9}{n + 8} \hat{t}^2 + \frac{n - 1}{n + 8} \hat{t}^{2 - \alpha_t} F_0(\hat{u}) .$$

(15)

This equation explicitly shows that the critical distance $T_T(\ell^*)$ is related to the temperature $\hat{t}$ of the Goldstone system by a Fisher-renormalization [7] since the nonanalytic term $\hat{t}^{2 - \alpha_t}$ with $\alpha_t = \alpha_t/2$ dominates near the coexistence-curve. The physical reason for this Fisher-renormalization is the coupling of the critically fluctuating transverse modes to the longitudinal modes in the Goldstone regime. The equation of state and the transverse susceptibility near the coexistence-curve follow from (9) and (15) as

$$\begin{align*}
\frac{h}{m^{\alpha_t}} &= L^{-2} \hat{t} \\
\frac{x_T}{m^{1 - \alpha_t}} &= L^2 \hat{t}^{-1}
\end{align*}$$

(16, 17)

The longitudinal susceptibility is evaluated in the same way observing that the term $E/\hat{t}$ dominates:

$$\frac{x_L}{m^{1 - \alpha_t}} \sim \left[ \frac{9}{n + 8} + \frac{n - 1}{n + 8} \hat{t}^{2 - \alpha_t} F_0(\hat{u}) \right] .$$

(18)

Note that the equation of state and the transverse susceptibility near the coexistence-curve have the very simple structure of a system with temperature $\hat{t}$ and vanishing interaction $\hat{u}$. This point is verified from (13), which leads to

$$\hat{u} \sim \left[ \frac{h}{m^{\alpha_t}} \right]^{1 - \alpha_t} .$$

(19)

Working out the matching condition $T_L(\ell^*) = 1$ and $T_T(\ell^*)$ in (15), one obtains the equation of state near the coexistence-curve

$$\frac{h}{m^{\alpha_t}} = \left[ \frac{1}{m^{1/\alpha_t}} + 1 \right]^{1 - \alpha_t} .$$

(20)

using the usual normalization conditions $f(-1) = 0$ and $f(0) = 1$. The longitudinal susceptibility is given by

$$\frac{x_L}{m^{1 - \alpha_t}} \sim \left( \frac{h}{m^{\alpha_t}} \right)^{1 - \alpha_t}$$

(21)

ear the coexistence-curve. Note that the results (20, 21) for the functional form of the equation of state and the susceptibilities near the coexistence-curve are exact for $d \geq 3$ since the specific heat exponent $\alpha_t = \frac{1}{2}$ is classical tricritical for $d \geq 3$. 