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A NEW DISORDERED PHASE AND ITS PHYSICAL CONTENTS OF THE BLUME-EMERY-GRiffiths MODEL

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Abstract. — Thermodynamic properties of the anisotropic Blume-Emery-Griffiths model are discussed in the new type of effective-field theory. A new disordered phase which may correspond to the staggered quadrupolar phase predicted by the Monte Carlo simulation is found and its physical contents are investigated.

The anisotropic Blume-Emery-Griffiths (BEG) model is a spin-one Ising model with bilinear and biquadratic nearest-neighbor pair interactions in which a single-ion anisotropy parameter is included [1]. The BEG model has been investigated theoretically by many authors in connection with the experimental results on magnetic phase transitions in some compounds and phase separation of a binary fluid.

The Hamiltonian of the model is given by

$$\mathcal{H} = - \sum_{i, j} J_{ij} s_i^z s_j^z - \sum_{i, j} J'_{ij} \left( s_i^z \right)^2 \left( s_j^z \right)^2 - D \sum_i \left( s_i^z \right)^2 - H \sum_i s_i^z, \quad (1)$$

where $J_{ij}, J'_{ij}$ and $D$ are the bilinear, biquadratic and anisotropy parameters, respectively. $H$ is the applied magnetic field. Each $s_i^z$ can take the values ±1 and 0, and the summation is carried out over all pairs of nearest-neighbor spins.

Recently, the existence of a new disordered phase, namely the staggered quadrupolar phase, has been predicted from the Monte Carlo simulation, for when the conditions of $J + J' < 0$ and $D > 0$ are satisfied in the BEG model [2]. In the preceding studies [3, 4] we have found a new disordered phase which may correspond to the staggered quadrupolar phase by the use of the effective-field theory with correlations (EFT). The purpose of this work is to clarify the physical contents of the new disordered phase in the BEG model.

Following the formulation by Fittipaldi and Siqueira (F-S) [5] for the BEG model, the statistical mechanical quantities $m = \langle s_i^z \rangle$ and $q = \langle (s_i^z)^2 \rangle$ for a honeycomb lattice may be evaluated from the following set of equations, within the framework of the EFT,

$$m = \left[ 1 + m \sinh (k \nabla x) + q \cosh (k \nabla x) - 1 \right]^3$$

$$\times \left[ 1 + q \left\{ \exp (K' \nabla y) - 1 \right\} \right]^3 f_H (x, y) \mid_{x=0, y=0} \quad (2)$$

and

$$q = \left[ 1 + m \sinh (k \nabla x) + q \cosh (K \nabla x) - 1 \right]^3$$

$$\times \left[ 1 + q \left\{ \exp (K' \nabla y) - 1 \right\} \right]^3 g_H (x, y) \mid_{x=0, y=0'} \quad (3)$$

where $\nabla \mu = \frac{\partial}{\partial \mu}$ ($\mu = x, y$) gives the two differential operators, and the functions $f_H (x, y)$ and $g_H (x, y)$ are defined by

$$f_H (x, y) = \frac{2 e^y \sinh (x + h)}{2 e^y \cosh (x + h) + \exp (-\beta D)} \quad (4)$$

$$g_H (x, y) = \frac{2 e^y \cosh (x + h)}{2 e^y \cosh (x + h) + \exp (-\beta D)} \quad (5)$$

with $\beta = 1/k_B T$ and $h = \beta H$, where $K$ and $K'$ are defined by $K = \beta J$ and $K' = \beta J'$.

On the other hand, the initial susceptibility per site is defined by

$$\chi = \left( \frac{\partial m}{\partial H} \right)_{H=0}, \quad (6)$$

which is given by a complex equation, after differentiating (2) with $H$ and substituting $\frac{\partial q}{\partial H}$ into it. However, the paramagnetic susceptibility has a simple form, which is given by

$$\chi_{para} = \frac{1}{k_B T} \frac{c}{1 - a} \quad (7)$$

with

$$a = 3 \sinh (k \nabla x) \left[ 1 + q \left\{ \exp (K' \nabla y) - 1 \right\} \right]^3$$

$$\times \left[ 1 + q \left\{ \cosh (K \nabla x) - 1 \right\} \right]^3 f (x, y) \mid_{x=0, y=0} \quad (8)$$

and

$$c = \left[ 1 + q \left\{ \exp (K' \nabla y) - 1 \right\} \right]^3$$

$$\times \left[ 1 + q \left\{ \cosh (K \nabla x) - 1 \right\} \right]^3 f_1 (x, y) \mid_{x=0, y=0}, \quad (9)$$

where the functions $f (x, y)$ and $f_1 (x, y)$ are defined by

$$f (x, y) = f_H=0 (x, y)$$

and

$$f_1 (x, y) = \left[ \frac{\partial}{\partial H} f_H (x, y) \right]_{H=0},$$

respectively. The parameters $a$ and $c$ can be easily calculated by using a mathematical relation $e^{a \nabla \mu} \phi (\mu) = \phi (\mu + a)$. 

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The second-order phase transition line is then determined by \( I = a \), where the parameter \( q_0 \) is the solution of (3) for \( m = 0 \) and \( H = 0 \). For \( H = 0 \), it is well-known in the BEG model that there appears a tricritical point, when \( D \) takes a large negative value. By performing a straightforward calculation for (2 and 3), we have, as discussed in the previous works,

\[
m^2 = \frac{1 - a}{b} \quad \text{for} \quad H = 0.
\]

The tricritical point is obtained from the condition \( a = 1 \) and \( b = 0 \).

In figure 1, we present the results of \( \alpha = 1 \) and the tricritical condition in the \((T, \frac{D}{J})\) space for a honeycomb lattice. In the figure, the black points denote the tricritical points. Until a value of \( \alpha = -1.0 \left( \alpha = \frac{J'}{J} \right) \), the critical frontiers in the space are equivalent to those of F-S, except the values of the tricritical point for which F-S could not be evaluated. As is seen from the figure, when the value of \( \alpha \) further decreases, the \( T_c \) curve may separate into two parts, illustrated by the curve with \( \alpha = -1.8 \). The \( T_c \) value could not be obtained in a restricted region of \( D > 0 \), namely \( 0 < \frac{D}{J} < 0.79 \), for the curve of \( \alpha = -1.8 \).

The region in which \( T_c \) reduces to zero becomes wider with a decrease of \( \alpha \). Thus, the new disordered phase at which \( T_c \) reduces to zero occurs in the region of \( J + J' < 0 \) and \( D > 0 \), which may correspond to the staggered quadrupolar phase found from the Monte Carlo simulation of this model.

As shown in figure 1, it is interesting to investigate the behavior of \( \chi_{\text{para}}^{-1} \) for the system with a fixed value \( D = 0.2 \, J \), changing the value of \( \alpha \). In figure 2, the plots are given. With the decrease of \( \alpha \), the inverse paramagnetic susceptibility follows the usual Curie-Weiss law until \( \alpha = -1.5 \). However, when the new disordered phase appears, the behavior changes dramatically, as is seen from the curve labeled \( \alpha = -1.8 \).

On the other hand, in the previous work [4] we have investigated the magnetization process for the system with \( \alpha = -1.8 \) and \( D \) in the disordered phase. The above and previous results suggest that the new disordered phase found in figure 1 (or the region of \( 0 < \frac{D}{J} < 0.79 \) in the curve of \( \alpha = -1.8 \)) consists of two interpenetrating local sublattices randomly distributed in the system; one local sublattice has sites occupied by \( s_i^z = 0 \) and the other local sublattice has sites occupied randomly by \( s_i^z = \pm 1 \).

[1] Blume, M., Emery, V. J. and Griffiths, R. B., 


