CRITICAL DYNAMICS OF THE ISING MODEL WITH ISING MACHINE
N. Ito, M. Taiji, M. Suzuki

To cite this version:
N. Ito, M. Taiji, M. Suzuki. CRITICAL DYNAMICS OF THE ISING MODEL WITH ISING MACHINE. Journal de Physique Colloques, 1988, 49 (C8), pp.C8-1397-C8-1398. <10.1051/jphyscol:19888641>. <jpa-00228870>

HAL Id: jpa-00228870
https://hal.archives-ouvertes.fr/jpa-00228870
Submitted on 1 Jan 1988

HAL is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers.

L’archive ouverte pluridisciplinaire HAL, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d’enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.
CRITICAL DYNAMICS OF THE ISING MODEL WITH ISING MACHINE

N. Ito, M. Taiji and M. Suzuki
Department of Physics, Faculty of Science, University of Tokyo, Bunkyo-ku, Tokyo, 113, Japan

Abstract. — The dynamical critical exponent $z$ of the two-dimensional Ising model with sequential-flip heat-bath dynamics is numerically estimated to be 2.2, which agrees with that of Metropolis dynamics. The dedicated machine “m-TIS” designed and constructed by the present authors is used for the simulations.

The Monte Carlo simulation is one of the most powerful methods in studying cooperative phenomena [1, 2]. The recent developments of digital computers make large scale simulations possible. There are two approaches to large scale simulations. One is to use vector processors. In the following, we discuss the Monte Carlo simulations of the Ising models. The fastest simulation of the three-dimensional ferromagnetic-Ising model up to now records 0.85 spin-trials per second on the HITAC S820/80 by Ito and Kanada [3]. The details of the use of vector processors are described in reference [3]. The other approach is to design, construct and use a dedicated (or special purpose) machine [4, 5].

Recently, the present authors have designed and constructed a special purpose machines for the Ising spin systems named “m-TIS” [5]. This m-TIS works as a subroutine in simulation programs. When the local spin configuration is transferred from the host computer, it returns the next spin configuration. In one operation, 16 spins are simulated. It takes one clock cycle for the simulation of one spin and the present host is EPSON PC-286, whose CPU is 10 MHz Intel 80286, and the data transfer is not fast enough to work the m-TIS at its peak speed. The present system can treat 2.2 M spins for square lattices and 2M spins for cubic lattices.

With this machine, we have studied the dynamical critical exponent $z$ of the square lattice Ising model with Metropolis sequential-flip dynamics [6]. The time correlation function of the magnetization $C_M(t)$ at temperatures higher than the critical point is defined by

$$C_M(t) = \lim_{N \to \infty} \left( \frac{1}{N} \sum_{i=1}^{N} M_i(t+\tau) \right),$$

where $M_t$ denotes the magnetization of the $t$-th sample of the Monte Carlo simulation. The $C_M(t)$ decays exponentially and the longest decay time is denoted by $\xi_t$. If the system approaches the critical temperature, this $\xi_t$ diverges like

$$\xi_t \sim (T - T_c)^{-\nu},$$

where $\nu$ denotes the critical exponent of the correlation length. This $z$ is called the dynamical critical exponent [7, 8]. Instead of varying the temperature, the system size is varied at the critical point of the infinite system and the finite-size-scaling analysis is tried. The result was $z = 2.132 \pm 0.008$. This $z$ is estimated by other authors. Rácz and Collins [9] have obtained $z = 2.125 \pm 0.01$ from the high temperature expansion. Mori and Tsuda [10] estimated $z = 2.076$ from simulations at temperatures near $T_c$. All these values are consistent with each other, although the details of the dynamics are not the same.

We have studied the difference of the dynamical nature between the heat-bath and the Metropolis dynamics. For this purpose, the correlation functions of the sequential flip dynamics with heat-bath algorithm are calculated.

Our dynamics is described in the following. The spin flip is tried sequentially from the $(1, 1)$-site to $(1, n)$-site and then from the $(2, 1)$-site to the $(2, n)$-site and so on.

The probability of the configuration $\{ \sigma \}$ at Monte Carlo step $t$, $P(\{ \sigma \}, t)$, is determined by

$$P(\{ \sigma \}, t+1) = L P(\{ \sigma \}, t),$$

where $L$ is defined by

$L = L_{(n,n)} \ldots L_{(n,1)} L_{(n-1,n)} \ldots L_{(2,n)} \ldots L_{(2,1)} L_{(1,n)} \ldots L_{(1,1)}$

and

$L_{(i,j)} = \left[ 1 + (F_{(i,j)} - 1) W_{(i,j)}(\{ \sigma \}) \right] f(\{ \sigma \}).$

That is,

$$F_{(i,j)} f(\{ ..., \sigma(i,j), ... \}) = f(\{ ..., -\sigma(i,j), ... \}).$$

The $F_{(i,j)}$ denotes the operator which will flips the spin on the $(i, j)$-site.

In the case of Metropolis dynamics, the transition probability $W_{(i,j)}$ is defined by

$$W_{(i,j)} = \min \{ 1, \exp \left( 2\beta E_{(i,j)} \right) \},$$

where

$$E_{(i,j)} = -J \sigma(i,j) \sum_{\{k,l\} \neq \{i,j\}} \sigma(k,l).$$
In the case of heat bath, it is defined by
\[
W_{(i,j)} = \frac{\exp \left( \beta E_{(i,j)} \right)}{\exp \left( -\beta E_{(i,j)} \right) + \exp \left( \beta E_{(i,j)} \right)}.
\] (9)

The results are shown in table I and figure 1. The obtained value of \( z \) is 2.2 and coincides with that of the Metropolis method. The ratio of \( \xi_t \) of the heat bath to that of the Metropolis is 2.4. Here, the time is defined by the Monte Carlo step (MCS) and 1 MCS means the step in which every spin in the system is tried to flip once.

It is easy to see that the value of \( z \) can depend on the dynamics of the system. For example, if the configuration at step \( t+1 \) is selected in the configuration space according to the Boltzmann probability without referring to the configuration at \( t \), the relaxation time of this dynamics is zero for all temperatures and the value of \( z \) is zero. In fact, the dynamics whose values of \( z \) are definitely different from the above results are discovered [11, 12].

The results suggest that the values of \( z \) of Metropolis dynamics and those of the heat bath dynamics are the same and consequently we have the dynamical universality [13] in the single spin flip dynamics.

Table I. - The correlation times of the square lattice Ising model at the critical point with heat bath dynamics.

<table>
<thead>
<tr>
<th>Size</th>
<th>( \xi_t )</th>
<th>Step / ( 10^8 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>16</td>
<td>824</td>
<td>1</td>
</tr>
<tr>
<td>32</td>
<td>4047</td>
<td>5</td>
</tr>
<tr>
<td>48</td>
<td>9990</td>
<td>18</td>
</tr>
<tr>
<td>64</td>
<td>16790</td>
<td>20</td>
</tr>
<tr>
<td>80</td>
<td>33830</td>
<td>40</td>
</tr>
<tr>
<td>96</td>
<td>42000</td>
<td>40</td>
</tr>
<tr>
<td>112</td>
<td>63100</td>
<td>50</td>
</tr>
</tbody>
</table>