# CROSSOVER FUNCTIONS UND EFFECTIVE EXPONENTS OF DISORDERED FERROMAGNETS

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JOURNAL DE PHYSIQUE
Colloque C8, Supplément au n° 12, Tome 49, décembre 1988



# CROSSOVER FUNCTIONS UND EFFECTIVE EXPONENTS OF DISORDERED FERROMAGNETS

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**Abstract.** – We present crossover calculations for the susceptibility and specific heat of disordered ferromagnets. We show that the critical behaviour in the experimental temperature range is governed by the unstable fixed point of strongly diluted magnets. The asymptotic behaviour is found only in a narrow region around $T_c$.

We investigate the critical behaviour of disordered spin systems with short-ranged interactions $J_{ij}$ between $n$-component spins $\mathbf{S}_i$ and $\mathbf{S}_j$. The spins are assumed to be distributed independently with concentration $p$ on a regular lattice. Introducing occupation numbers $K_i$, which take the value 1 if site $i$ is occupied by a spin and which are 0 otherwise, the disordered ferromagnet is described by the Hamiltonian

$$\mathcal{H}(K,\mathbf{S}) = \sum_i J_{ij} K_i \cdot \mathbf{S}_i \cdot K_j \cdot \mathbf{S}_j + H_\mathrm{m} \sum_i K_i \cdot S_i^1. \quad (1)$$

It has been shown in previous works [1-3] that the configurational average of the free energy over all $K$-distributions [4] can be avoided by the quasi equilibrium method which treats the occupation numbers like normal degrees of freedom. To compensate for that, generalized chemical potentials $g$, $B$, ... are introduced in such a way that the moments $\langle K_i \rangle = p$, $\langle K_i . K_j \rangle = p(1-p)$ $(i \neq j)$, ... are fixed. In an $\varepsilon$-expansion ($\varepsilon = 4-d$) to $O(\varepsilon)$ the resulting field theoretic Hamiltonian was calculated in [1]:

with the coupling parameters $r \sim t = (T - T_c)/T_c$ and $H$, $h \sim H_\mathrm{m}$. $R$ and $X = R^2/B$ respectively as well as $u$ [1] depend on the concentration $p$.

To perform the crossover calculations it is convenient to reduce the Hamiltonian to an effective spin Hamiltonian. Since $1/B = p(1-p) = O(1)$ [1] we eliminate the occupation numbers by integrations in the free energy and we obtain the resulting effective GLW-Hamiltonian

$$\mathcal{H}_\mathrm{eff} = -\frac{1}{2} \cdot \int \mathbf{S}_\mathbf{q} \cdot \mathbf{S}_{-\mathbf{q}} \left(\tilde{r} + q^2\right) - -\tilde{u} \cdot \int_{\mathbf{q}_1} \int_{\mathbf{q}_2} \int_{\mathbf{q}_3} \mathbf{S}_{\mathbf{q}_1} \cdot \mathbf{S}_{\mathbf{q}_2} \mathbf{S}_{\mathbf{q}_3} \cdot \mathbf{S}_{-(\mathbf{q}_1+\mathbf{q}_2+\mathbf{q}_3)} + \tilde{H} \cdot S_0^1 \quad (2)$$

which is characterized by the new coupling parameters $\tilde{r} = r - 2 \cdot \frac{gR}{B}$, $\tilde{u} = u - \frac{R^2}{2B}$ and $\tilde{H} = H$. The free energy is given in terms of $\mathcal{H}_\mathrm{eff}$ (2) by

$$\tilde{F} = -\ln \int_\mathbf{S} \exp \mathcal{H}_\mathrm{eff} - \frac{g^2}{2B} + \frac{K_4}{2} \int_\mathbf{q} \ln B. \quad (3)$$

The concentration $\langle K_0 \rangle$ and variance $\langle K_\mathbf{q} K_{-\mathbf{q}} \rangle$ are calculated from by a Feynman graph expansion or using $g$ and $B$ as a source terms:

$$\langle K_0 \rangle = \frac{g}{B} + 2 \cdot \frac{R}{B} \cdot \tilde{E}\left(\tilde{t},\ \tilde{u}\right) = 0 \quad (4)$$

$$\langle K_\mathbf{q} \cdot K_{-\mathbf{q}} \rangle = \frac{1}{B} + \frac{R^2}{2B} \cdot \tilde{C}\left(\tilde{t},\ \tilde{u}\right) = p(1-p) \quad (5)$$

$\tilde{E}$ and $\tilde{C}$ are the energy and specific heat of the effective spin system (2). The equations (4, 5) connect the effective Hamiltonian (2) to the disordered ferromagnet to be described. Thus, the crossover in disordered ferromagnets consists of two elements. A crossover in the effective spin Hamiltonian (2) between the tricritical fixed point ($\tilde{u} = 0$), which will describe strong dilution, and the critical fixed point ($\tilde{u} = u_c$), which describes weak dilution. Secondly, there is a crossover-like transformation between the coupling parameters which results from (4) and (5).

We apply the trajectory integral method [5] to calculate the crossover functions and effective exponents to $O(\varepsilon)$ of the effective spin Hamiltonian (2). The susceptibility $\tilde{\chi}$ of the spin Hamiltonian (2) is obtained in scaling form with nonlinear critical scaling fields [6].

It describes the crossover between the singularity at the critical fixed point and at the tricritical fixed point ($\gamma_t = 1$). One gets an impression of the crossover in the susceptibility $\tilde{\chi}$ of the effective spin Hamiltonian (2), if one calculates the effective exponent from [7]:

$$\tilde{\gamma}_\mathrm{eff} = - \left.\frac{\partial \ln \tilde{\chi}}{\partial \ln \tilde{t}}\right|_{\tilde{H},\ \tilde{u}} = \frac{\gamma_c}{1 - \sigma_c \dfrac{\tilde{c}_c^*}{1 + \tilde{c}_c^*}}. \quad (6)$$

In (6) we have introduced the local scaling variable $\tilde{c}_c^*$ which is calculated from the matching condition of the trajectory integral method [7].

The free energy of the effective spin Hamiltonian (2) is obtained by the trajectory integral method as [7]

$$\tilde{F} = -A_t \cdot \tilde{g}_{1t}^{2-\alpha_t} \tilde{F}_0\left(\tilde{c}_t^*\right) \quad (7)$$

with the crossover function

$$F_0\left(\tilde{c}_t^*\right) = \frac{\left(1 + \tilde{c}_t^*\right)^{\frac{\alpha_c}{\alpha_t}} - 1}{\frac{\alpha_c}{\alpha_t} \cdot \tilde{c}_t^*} \quad (8)$$

and the well-known critical and tricritical exponent of the specific heat $\alpha_c = \frac{4-n}{n+8} \frac{\varepsilon}{2}$ and $\alpha_t = \frac{\varepsilon}{2}$. The en-

Article published online by EDP Sciences and available at http://dx.doi.org/10.1051/jphyscol:19888575

ergy and the specific heat are calculated from (7, 8) as temperature $(\tilde{t}-)$ derivatives:

$$\tilde{E} = \left.\frac{\partial \tilde{F}}{\partial \tilde{t}}\right|_{\tilde{H},\ \tilde{u}} = 2\frac{\tilde{F}}{\tilde{t}} \qquad (9)$$

$$\tilde{C} = \left.\frac{\partial^2 F}{\partial \tilde{t}}\right|_{\tilde{H},\ \tilde{u}} = \frac{\tilde{E}}{\tilde{t}}. \qquad (10)$$

and the effective exponent of the specific heat is given by [7]

$$\tilde{\alpha}_{\text{eff}} = -\left.\frac{\partial \ln \tilde{C}}{\partial \ln \tilde{t}}\right|_{\tilde{H},\ \tilde{u}} = \alpha_t \cdot (1+\tilde{c}_t^*)^{-\left(1-\frac{\alpha_c}{\alpha_t}\right)} F_0^{-1}(\tilde{c}_t^*). \qquad (11)$$

The energy $\tilde{E}$ and the specific heat $\tilde{C}$ are also important for the second part of our crossover theory.

Our first condition of quenched disorder (4) leads to the transformation $t = \tilde{t} - 4X.\tilde{E}(\tilde{t},\ \tilde{u})$ between the temperature $t = (T - T_c)/T_c$ of the disordered ferromagnet and the temperature $\tilde{t}$ of the effective spin system. Together with our result for the energy $\tilde{E}$, we obtain [7]

$$t = \tilde{t}\left(1 - \frac{nK_4X}{\alpha_t} \cdot \frac{1-(1-\tilde{\mu}_2)^{\frac{\alpha_c}{\alpha_t}}}{\frac{\alpha_c}{\alpha_t}\tilde{\mu}_2}\right) + + \frac{nK_4X}{\alpha_t}\frac{\tilde{g}_{1t}^{2-\alpha_t}}{\tilde{t}} F_0(\tilde{c}_t^*) \qquad (12).$$

This equation shows that $t$ and $\tilde{t}$ are related by a Fisher-renormalization in the critical temperature range $t,\ \tilde{t} \ll 1$ where the singular term dominates. The second condition (5) leads to $1/B = p(1-p)$ in $O(\varepsilon)$. This is well-known from previous works [1, 3]. In $O(\varepsilon^2)$ we expect quantitative changes from the second term in (5).

The susceptibility $\chi$ of the disordered ferromagnet with the temperature $t$ and the concentration-dependent coupling parameters $u$ and $X$ can be calculated from the susceptibility $\tilde{\chi}$ and the transformation (12) as

$$\chi(t,\ u,\ X) = \tilde{\chi}\left(\tilde{t}(t,\ u,\ X),\ \tilde{u}(u,\ X)\right) \qquad (17)$$

Likewise, the specific heat is calculated from (7, 8, 10) and (12) as [7]

$$C(t,\ u,\ X) = \tilde{C}\left(\tilde{t},\ \tilde{u}\right) \cdot \frac{\tilde{t}^2}{t^2}. \qquad (18)$$

The variation of the critical singularities as functions of $t$ can be seen from the effective exponents. Defining the logarithmic temperature-derivative

$$\left.\frac{\partial \ln t}{\partial \ln \tilde{t}}\right|_{\tilde{u},\ X} = 1 - \tau \qquad (19)$$

we obtain the effective exponents from (6, 11) and (12):

$$\gamma_{\text{eff}} = -\left.\frac{\partial \ln \chi}{\partial \ln t}\right|_{H,\ u,\ X} = \frac{\tilde{\gamma}_{\text{eff}}}{1-\tau} \qquad (20)$$

$$\alpha_{\text{eff}} = -\left.\frac{\partial \ln C}{\partial \ln t}\right|_{H,\ u,\ X} = \frac{\tilde{\alpha}_{\text{eff}} - 2\tau}{1-\tau} \qquad (21)$$

which are plotted in figures 1 and 2 for disordered ferromagnets with different concentrations and coupling parameters $\tilde{\mu}_2 = \tilde{u}/u_c$ and $\mu_X = nK_4X/\alpha_t$ respectively. The experimental temperature region is characterized by temperature and concentration-dependent exponents with the extreme values of Fisher-renormalized tricritical exponents $\gamma_{\text{eff}} = 2$ and $\alpha_{\text{eff}} = -1$ for strongly diluted systems. The asymptotic behaviour should be observable only for weakly diluted ferromagnets.

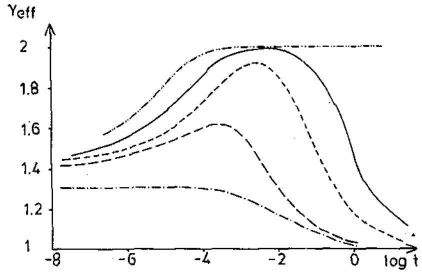


Fig. 1. – Effective exponent $\gamma_{\text{eff}}$ as a function of $t = (T - T_c)/T_c$ for disordered ferromagnets with different concentrations and coupling parameters $\tilde{\mu}_2 = \tilde{u}/u_c$ and $\mu_X = nK_4X/\alpha_t$ respectively: (-·-·-) $\tilde{\mu}_2 = 10^{-5}$, $\mu_X = 1$ ; (—) $\tilde{\mu}_2 = 10^{-4}$, $\mu_X = 0.5$ ; (- - -) $\tilde{\mu}_2 = 10^{-3}$, $\mu_X = 0.05$ ; (- -) $\tilde{\mu}_2 = 10^{-2}$, $\mu_X = 10^{-2}$ ; (-·-) $\tilde{\mu}_2 = 0.1$, $\mu_X = 10^{-3}$.

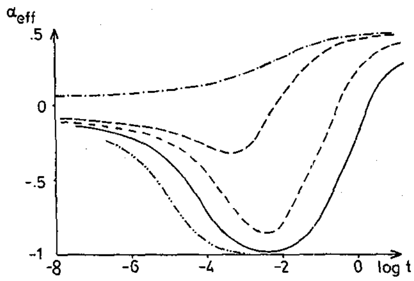


Fig. 2. – Effective exponent $\alpha_{\text{eff}}$ as a function of $t = (T - T_c)/T_c$ for disordered ferromagnets with the same coupling parameters as in figure 1.


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