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ISING MODEL IN CORRELATED RANDOM FIELDS ON THE SQUARE LATTICE

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Abstract. We investigate the Ising model on the square lattice with correlated random fields which take the values \( \pm h_0 \) and 0. The random fields are assumed to vary from row to row but not from column to column. In the limit \( h \to 0 \), the system shows the phase transition, where \( h \) is the uniform external field with \( h \geq h_0 \).

The quenched random fields alter the nature of the phase transition [1, 2]. The most interesting point is the value of the lower critical dimensionality, \( d_{cL} \), below which long-range order cannot exist. The result from experimental and theoretical investigations are not yet conclusive for this problem. These situations are described, for example, in the papers by Birgeneau et al. [3, 4] for the experimental aspects and in the papers by Imry [5] and by Imbrie [6] for the theoretical aspects. \( d_{cL} = 2 \) given by Imry and Ma is strongly supported by Fisher et al. [7] and Imbrie [6].

Recently, the lower critical dimensionality has been discussed for the Ising model on the \( d \)-dimensional lattice with random fields which are independently random at sites only in \( (d - \ell) \)-dimensional subspace with \( 1 \leq \ell \leq d \) [8]. The domain wall argument gives that the value \( d_{cL} \) is equal to 1, by taking into account that there exist \( L^{d-\ell} \) independent random variables in each domain. Here \( L \) is the linear size of domain in lattice constant units. This result is supported by two examples: the Ising models on the square lattice and on the linear chain, with a random field applied to all the spins.

In this paper, we consider the Ising model on the square lattice with random fields which vary independently from row to row and take \( \pm h_0 \) and 0, but do not vary from column to column. We also apply a uniform field \( h \) to the system. Keeping \( h \geq h_0 \) and taking \( h \to 0 \), we show that the bulk spontaneous magnetization shows the critical behaviour.

A similar model has been investigated by Forgacs et al. [9], recently. In their model, the random fields take the values \( \pm \infty \) and 0 along a row but vary randomly from row to row. The free energy is expressed in terms of a summation of the free energy of strips with finite width.

We consider the Ising model on the square lattice with random fields which are translationally invariant in one of the lattice direction. The Hamiltonian of the system, following McCoy and Wu [10] notation is given by

\[
H = H_0 - \sum_{j=1}^{2M} \sum_{k=-N+1}^{N} (h_j + \bar{h}) \sigma_{jk},
\]

where

\[
H_0 = -J_1 \sum_{j=1}^{2M} \sum_{k=-N+1}^{N} \sigma_{jk} \sigma_{j+k} - J_2 \sum_{j=1}^{2M-1} \sum_{k=-N+1}^{N} \sigma_{jk} \sigma_{j+k+1}.
\]

Cyclic boundary conditions are imposed along the horizontal direction. \( J_1 > 0 \) and \( J_2 > 0 \) are exchange integrals and \( h_j \) is a random field which takes \( h_0 > 0 \) with probability \( p \), \(-h_0 \) with \( q \) and 0 with \( r \). \( \bar{h} \) is the uniform external field. We assume that \( h \geq h_0 \).

According to McCoy and Wu [10], we consider “the boundary magnetization” defined by

\[
m_{b0}(\bar{h}) = \langle \sigma_{1k} \rangle_{H'},
\]

where

\[
\langle \sigma_{1k} \rangle_{H'} = \sum_{\{\sigma\}} \sigma_{1k} \exp (-\beta H') / \sum_{\{\sigma\}} \exp (-\beta H').
\]

Here \( \beta = 1/kT \) as usual and \( H' \) is given by

\[
H' = H_0 - (h_1 + \bar{h}) \sum_{k=1-N}^{N} \sigma_{1k}.
\]

The angular brackets without any subscript mean the configurational average over the random variables. Under the condition \( \bar{h} \geq h_0 \), we use the Griffith inequality [11] and obtain the following inequality:

\[
m_{b0}(\bar{h}) \leq \langle \sigma_{jk} \rangle_{H'},
\]
where the angular brackets with subscript $H$ mean the canonical average under the Hamiltonian $H$. On the other hand, according to Falk and Gehring [12] and Jedrzejewski [13], we have

$$\langle \sigma_{jk} \rangle_H \leq \langle \sigma_{jk} \rangle_{(H)}$$

(7)

where the angular brackets with subscript $(H)$ mean the canonical average under the Hamiltonian $(H)$:

$$\langle (H) \rangle = H_0 - \langle \hat{h} + (p-q) h_0 \rangle \sum_{j=1}^{2M} \sum_{k=-N+1}^{N} \sigma_{jk}.$$  

(8)

Thus in the limit $\hat{h} \to 0$, we have

$$m_{b0} \leq m_{bu} \leq m_s,$$

(9)

where

$$m_{b0} = \lim_{\hat{h} \to 0} \lim_{N,M \to \infty} m_{b0} (\hat{h}),$$

(10)

$$m_{bu} = \lim_{\hat{h} \to 0} \lim_{N,M \to \infty} \langle \sigma_{jk} \rangle_H,$$

(11)

and $m_s$ is the spontaneous magnetization on the Onsager lattice.

In the previous paper, [8] the present authors obtained $m_{b0}$ as follow:

$$m_{b0} = (p - q + r) m_0,$$

(12)

where

$$m_0 = \left( \frac{\cosh 2\beta J_2 - \coth 2\beta J_1}{\cosh 2\beta J_2 - 1} \right)^{-1/2}$$

(13)

$m_0$ shows a critical behaviour as $(T_c - T)^{1/2}$, where $T_c$ is the critical temperature of the Onsager lattice. Thus we conclude that the spontaneous bulk magnetization shows a critical behaviour as

$$m_{bu} = (T_c - T)^{\beta_r},$$

(14)

with $1/8 \leq \beta_r \leq 1/2$.

Even when $p = q$, we have the bulk spontaneous magnetization if $r \neq 0$. This is consistent with $d_{lc} = 1$. For the case $r = 0$ and $p = p$, we cannot say anything definitely for the bulk spontaneous magnetization.