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STRUCTURE FACTOR OF THE RANDOM FIELD ISING MODEL

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Abstract. – Using Monte Carlo simulations, we studied the structure factor \( S(q) \) of the two-dimensional (2D) random field Ising model (RFIM) with a binary probability distribution. We observe that \( S(q) \) changes its lineshape from Lorentzian to a Lorentzian-three half form, \((LRZ)^{3/2}\) as the random field and temperature are lowered. The time dependent structure factor \( S(q,t) \) after a quench from a paramagnetic phase to low temperatures is also studied. These results can provide a better understanding of diffraction scattering experiments performed on RFIM.

The study of the effects of disorder on physical systems has been in the forefront of research activity [1-6] for the last fifteen years. Some general characteristics are now well-established; the presence of multiple local minima in the free energy surface [3] that lead to a slow relaxation towards equilibrium and the absence of a well-ordered ground state. A specific model that incorporates disorder is the random field Ising model (RFIM), where the Ising nearest neighbor interaction is frustrated by the local random field. It has been established [4] that the lower critical dimensionality of this model is \( d_c = 2 \). The minimum free energy state in \( d = 2 \) is the so-called domain state. This state is realized if the system is cooled in the presence of a field from a paramagnetic state. This leads to characteristic changes of the structure factor \( S(q) \)

\[
S(q) = \frac{1}{N} \left| \sum_i S_i e^{i\mathbf{q}\cdot\mathbf{r}_i} \right|^2.
\]  

Diffraction experiments [7, 8] have achieved sufficient instrumental resolution to probe quantitative changes of \( S(q) \). The main purpose of this paper is to establish this connection by using Monte Carlo simulations [9] to calculate \( S(q) \) directly and to compare the phase boundary in the \( H-T \) plane identified from the lineshape analysis and the one deduced from the free energy surface (analysis) [3]. The low temperature domain state can be well identified from \( S(q) \) measurements. Besides these equilibrium measurements which for disorder systems might require long relaxation times to be realized, diffraction measurements of \( S(q) \) can be used dynamically as the system is evolving in time towards its final equilibrium state. We have used both the increase of the peak of \( S(q) \) [10] and the time evolution of the whole lineshape.

Experimentally, there are two types of systems that are realizations of the 2D RFIM. Dilute antiferromagnets [2] in the presence of a field with weak coupling in the \( z \) direction can be treated effectively as two-dimensional. Neutron scattering experiments have documented the loss of long-range order. They have also suggested that the \( S(q) \) lineshape can be fitted with a Lorentzian-to-3/2 (LRZ)\(^{3/2}\) lineshape. More recently [7, 11], surface overlayers in the presence of impurities have been used as a direct realization of the RFIM. It is well-known that the Ising Hamiltonian can be mapped into a lattice gas model describing a surface overlayer with the spin variables \( \pm 1 \) corresponding to the occupation variables \( 0,1 \) of the overlayer. All the results applicable to the magnetic system are extended to the lattice gas model.

The simulations were performed on the magnetic Ising Hamiltonian

\[
H = -J \sum_{\langle ij \rangle} S_i S_j + h \sum_i S_i, \tag{2}
\]

with \( \langle ij \rangle \) denoting nearest neighbors and the random field \( h \) given by a binary probability function with \( \langle h \rangle = 0 \). Henceforth, \( T \) and \( h \) will be measured in units of \( J \). Initially, the system was prepared in an infinite temperature paramagnetic state. The standard Monte Carlo algorithm was used [9]. The system was then cooled in the presence of a field to achieve the field cooled (FC) state.

Lattices of size \( N \times N \) with \( N = 40 \) to 60 were used. At least 100 independent runs with different \( h \) distributions were employed. The statistical error was less than 2\% and we saw no lattice size dependence for \( S(q) \). A large number of independent runs is especially important for the non-equilibrium kinetics [12]. The structure factor \( S(q) \) was calculated by circularly averaging all the \( q \) values satisfying \( \frac{2\pi}{N} (j-1) \leq q \leq \frac{2\pi}{N} j \).

In general, three well resolvable lineshapes were identified. If \( h \) and \( T \) were sufficiently small, sharp Bragg peaks were produced when the domain size in the domain region exceeded our sample size. For moderate values of \( T \) and \( h \), the \((LRZ)^{3/2}\) lineshape was found characterizing the domain state. It is possible that a \((LRZ)^{3/2}\) lineshape can equally fit this region. Here, irreversible effects are present due to the multiple minima of the free energy surface [3]. Finally, if the temperature is sufficiently high, the para-

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magnetic (LRZ) lineshape results independently of the value of the field. The phase diagram obtained from this lineshape analysis is shown in figure 1. Solid lines were drawn as phase boundaries to identify the three different states. This phase diagram agrees very well with the one suggested previously using mean field theory analysis of the free energy minima [3].

![Phase diagram of the 2D RFIM for a binary random field distribution of strength ± A. The shaded region indicates the unstable long-range order where the lineshape is reached quite early, while the average domain size is quite small.](image)

In summary, we have shown how the structure factor lineshape in the RFIM can be used to identify the equilibrium states at different temperatures and fields. The resulting phase diagram agrees very well with previous work based on the analysis of the free energy surface. For non-equilibrium growth from the paramagnetic state, the average domain site grows slower than the classic $t^{1/2}$ growth law. The $(LRZ)^{3/2}$ lineshape is reached quite early, while the average domain size is evolving.

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