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NOTE ON ENERGY FORMULATION OF THE FMR RESONANCE CONDITION

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Abstract. – As is known the classical FMR resonance condition by Smit and Beljers cannot be directly used zero angle between the magnetization and polar axis. This difficulty can be avoided using the new form of resonance condition which has been derived by the transformation from the rectangular coordinate system.

Ferromagnetic resonance (FMR) can be phenomenologically described by the equation of motion

\[ \dot{M} = -\gamma M \times H_{\text{eff}} \]  

with the effective magnetic field given by

\[ H_{\text{eff}} = -\frac{\partial F}{\partial M} = H_0 - \frac{\partial F_{\text{anis}}}{\partial M}. \]  

In these equations \( M \) denotes the magnetization, \( \gamma \) the gyromagnetic ratio, \( F \) the total free energy density of ferromagnet, \( H_0 \) the applied static magnetic field, \( F_{\text{anis}} \) the energy contribution due to the magnetocrystalline and shape anisotropy. The latter term is usually expressed in the coordinate system of the magnetocrystalline anisotropy we usually prefer the relation (4) expressed in terms of the magnetization factors \( N_{1\',1'}^{\text{eff}}, N_{2\',2\'}^{\text{eff}}, N_{3\',3\'}^{\text{eff}}, N_{1\',1''}^{\text{eff}} \). Secondly the condition by Smit and Beljers [4]

\[ \left( \frac{\omega}{\gamma} \right)^2 = \frac{1}{M_0^2 \sin^2 \theta_0} \left( F_{\theta \theta} F_{\phi \phi} - F_{\theta \phi}^2 \right) \]  

with \( F_{\theta \theta}, F_{\phi \phi}, F_{\theta \phi} \) denoting the second derivatives of \( F \) taken at equilibrium position of \( M \), i.e. for \( F_{\theta \theta} = F_{\phi \phi} = 0 \). This relation was derived in the spherical system \( \theta, \phi \) and may be directly used for arbitrary angles with the exception of the point \( \theta_0 = 0 \). The latter angle can be included if we evaluate the finite ratio \( \sin \theta / \sin \theta' \) for \( \theta' \to 0 \) (\( \theta \) is the polar angle of \( H_0 \)) making use of the equilibrium conditions. For particular cases of practical interest the appropriate formulas, obtained in this way, were already published in the past, e.g. [5]. The general form of the resonance condition i.e. including \( \theta_0 = 0 \) has been found recently in the work [6] starting from the model which involves the demagnetizing and cubic anisotropy energy. The purpose of the present note is to show that this condition can be also derived quite generally without specifying the form of \( F(\theta, \phi) \).

As the first step we shall write the resonance condition in the rectangular system \( 1', 2', 3' \) [6]

\[ \left( \frac{\omega}{\gamma} \right)^2 = \left( M_0 F_{M_1'M_1'} - F_{M_2'} \right) \times \]  

\[ \times \left( M_0 F_{M_3'M_3'} - F_{M_2'} \right) - \left( M_0 F_{M_1'M_1'} \right)^2. \]  

We notice that equation (5) represents an analogy to the relation (3) where the field \( H_0 \) and the effective demagnetization factors are replaced by the derivatives of \( F \) with respect to the components of \( M \) in the \( 1', 2', 3' \) system. "The rectangular method" has an advantage that the conditions (3) and (5) may be used for the arbitrary direction of the vector \( M_0 \).

On the other hand when calculating the contribution of the magnetocrystalline anisotropy we usually prefer the relation (4) expressed in terms of the spherical coordinates. In order to combine the advantages of both the methods we shall start from the condition (5), transform the derivatives \( F_{M_1'M_1'} F_{M_2'M_2'} F_{M_3'M_3'} F_{M_4'M_4'} \) into the coordinate system \( 1, 2, 3 \) and write them as.
functions of the variables \( \theta, \varphi \). The components of the vector \( \mathbf{M} \) are transformed as

\[
M'_i = \sum_{j=1}^{3} A_{ij} M_j; \quad M_j = \sum_{i=1}^{3} A_{ij} M'_i,
\]

(6)

where the nine elements \( A_{ij} \) of the corresponding matrix are functions of three independent parameters. It is seen when the transformation is described by three successive rotations about the axes 3, \( \rho \) (the intersection of the planes 12 and \( 1'2' \)), \( 3' \) through Euler's angles \( \psi, \theta_0, \Phi \) respectively [7]. If the angle \( \varphi_0 = \psi - \pi / 2 \) is used instead of \( \psi \) then the elements \( A_{ij} \) will be given by

\[
A_{11} = -\sin \varphi_0 \cos \Phi - \cos \theta_0 \cos \varphi_0 \sin \Phi;
A_{12} = \cos \varphi_0 \cos \phi - \cos \theta_0 \sin \varphi_0 \sin \Phi
\]

\[
A_{13} = \sin \theta_0 \sin \Phi;
A_{21} = \sin \varphi_0 \sin \phi - \cos \theta_0 \cos \varphi_0 \cos \Phi
\]

\[
A_{22} = -\cos \varphi_0 \sin \phi - \cos \theta_0 \sin \varphi_0 \cos \Phi
\]

\[
A_{23} = \sin \theta_0 \cos \Phi.
A_{31} = \sin \theta_0 \cos \varphi_0
A_{32} = \sin \theta_0 \sin \varphi_0
\]

\[
A_{33} = \cos \theta_0.
\]

According to the chain rule for partial differentiation we have

\[
\frac{\partial F}{\partial M'_i} = \sum_{j=1}^{3} \frac{\partial F}{\partial M_j} \frac{\partial M_j}{\partial M'_i}, \quad (i = 1, 2, 3)
\]

(7)

where

\[
\frac{\partial \theta}{\partial M_1} = \frac{\cos \theta \cos \varphi}{M_0}; \quad \frac{\partial \theta}{\partial M_2} = \frac{-\sin \varphi}{M_0}; \quad \frac{\partial \theta}{\partial M_3} = 0
\]

and the derivatives \( \partial M_j / \partial M'_i \) follow from the relations (6). Using equation (7) and setting \( \theta = \theta_0, \varphi = \varphi_0 \) we get

\[
F_{00}' = 0
\]

(8)

\[
M_{00}' F_{00}' M_0' = a_{11} F_{00} + a_{22} F_{00} + a_{12} F_{00} + a_{11} F_{00} + a_{22} F_{00} + a_{12} F_{00}
\]

\[
M_{00}' F_{00}' M_0' = b_{11} F_{00} + b_{22} F_{00} + b_{12} F_{00} + b_{11} F_{00} + b_{22} F_{00} + b_{12} F_{00}
\]

\[
M_{00}' F_{00}' M_0' = c_{11} F_{00} + c_{22} F_{00} + c_{12} F_{00} + c_{11} F_{00} + c_{22} F_{00} + c_{12} F_{00}
\]

(9)

The relation (8) means that the effective field \((- F_{00}' \)) expressed in the coordinate system \( \theta, \varphi \) equals to zero, which is valid for arbitrary values of \( F_\theta F_\varphi \). (Let us remark that in the absence of the anisotropy, when \( F = - \mathbf{M} \cdot \mathbf{H}_0 \), we can use the coordinate system \( 1', 2', 3' \) and insert into equation (5) the nonzero effective field \(- F_{00}' = H_0 \).) We know that at equilibrium \( F_\theta = 0, F_\varphi = 0 \) but the relations (9) are, at this stage, written with the general symbols \( F_\theta, F_\varphi \). This is useful with respect to the special case \( \theta_0 \to 0 \). Substituting now the expressions (8), (9) for the derivatives into equation (5) we obtain finally

\[
\left( \frac{\omega}{\gamma} \right)^2 = \frac{1}{M_0^2} \left[ F_{00} \left( \frac{F_{00}}{\sin^2 \theta_0} + \cos \theta_0 F_\theta \right) - \left( \frac{F_{00}}{\sin \theta_0} - \frac{\cos \theta_0 F_\varphi}{\sin \theta_0 \sin \theta_0} \right)^2 \right], \quad (10)
\]

We see that for \( \theta_0 \neq 0 \) this equation is identical with that derived by Smit and Belgers \((F_\theta = F_\varphi = 0)\). The condition (10) can be however directly used for \( \theta_0 = 0 \), since in the limit \( \theta_0 \to 0 \) the parentheses containing \( F_\theta, F_\varphi \) remain finite, even if the above equilibrium conditions \( F_\theta = F_\varphi = 0 \) are fulfilled. For details and examples we refer to the work [6].