ELASTIC-PLASTIC COMBINED STRESS WAVES UNDER CONDITION OF PLANE STRESS

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Des ondes de contraintes élasto-plastiques dans une plaque sont étudiées en utilisant la théorie incrémentale de la plasticité et la théorie caractéristique de la propagation des ondes. Quelques nouveaux et intéressants phénomènes sont discutés.

Abstract--Elastic-plastic combined stress waves in a plate are studied by using the incremental theory of plasticity and the characteristic theory of wave propagation. Some new and interesting phenomena are discussed.

1 - INTRODUCTION

So far much attention has been given to the problem of elastic-plastic combined stress waves by many authors. Clifton/1/ presented a treatment of axial and torsional impact on a thin-walled tube. Bleich and Nelson/2/, Ting and Han/3/, Goel and Malvern/4/ discussed the combined compressive and shear stress waves in a half space. At an earlier time Rakhmatulin/5/ and Cristescu /6/ studied the elastic-plastic combined stress waves in a plate subjected to compressive and shear dynamic loadings in its plane by using the elastic-plastic deformation theory based on proportional loading. However, even in the case of pure longitudinal waves the condition of proportional loading won't hold /8/. In this paper we will give a new study of the elastic-plastic combined stress waves under condition of plane stress by using the incremental plastic theory and show some new and interesting results.

2 - BASIC EQUATIONS

Consider a thin plate in the x-y plane subjected to combined compressive and shear dynamic loadings on its boundary $x=0$. Assume the plate to be infinite in y-direction and consider plane waves propagating in

x-axis. Let u, v denote the particle velocities in x and y -directions, σ, s and τ the normal stresses in x, y -directions and the shear stress in the x - y plane, ϵ and γ the normal strain in x -direction and the shear strain in x - y plane. All these quantities are functions of x and t only. The Mises yield condition can be written as

$$f = \tau^2 + (\sigma^2 + s^2 - \sigma s) / 3 = k^2 (w^p) \quad (1)$$

where the hardening parameter k is a function of the plastic work w^p . If the total strain rates are assumed to be a sum of elastic and plastic strain rates, then the plastic potential theory and the flow rules associated with the Mises yield condition give the following constitutive equation:

$$\dot{\underline{\epsilon}} = \underline{\underline{G}} \dot{\underline{\sigma}} \quad (2)$$

where a dot means the time derivative, the vectors $\underline{\sigma}$ and $\underline{\epsilon}$ are

$$\underline{\sigma}^T = (\sigma, \tau, s), \quad \underline{\epsilon}^T = (\epsilon, \gamma, 0) \quad (3)$$

The compliance tensor $\underline{\underline{G}}$ in Eq. (2) has the form:

$$\underline{\underline{G}} = \begin{bmatrix} 1/E + H(2\sigma - s)^2/12 & H\tau(2\sigma - s)/2 & -v/E + H(2\sigma - s)(2s - \sigma)/12 \\ H\tau(2\sigma - s)/2 & 1/G + 3H\tau^2 & H\tau(2s - \sigma)/2 \\ -v/E + H(2\sigma - s)(2s - \sigma)/12 & H\tau(2s - \sigma)/2 & 1/E + H(2s - \sigma)^2/12 \end{bmatrix} \quad (4)$$

where

$$H = (1/g(\sqrt{3}k) - 1/E)/k^2 \quad (5)$$

and g is the tangential modulus of the stress-strain curve of simple tension, E and ν the Young's modulus and Poisson's ratio, respectively. Substitution of continuity equations

$$\dot{\epsilon} = \partial u / \partial x, \quad \dot{\gamma} = \partial v / \partial x \quad (6)$$

into Eq. (2), together with the equations of motion

$$\rho \dot{u} = \partial \sigma / \partial x, \quad \rho \dot{v} = \partial \tau / \partial x \quad (\rho = \text{mass density}) \quad (7)$$

gives the basic equations of the problem:

$$\underline{\underline{A}} \underline{\underline{W}}_t + \underline{\underline{B}} \underline{\underline{W}}_x = 0 \quad (8)$$

where

$$\underline{\underline{W}} = \begin{bmatrix} \underline{v} \\ \underline{\sigma} \end{bmatrix}, \quad \underline{\underline{A}} = \begin{bmatrix} \rho \underline{\underline{I}} & 0 \\ 0 & \underline{\underline{G}} \end{bmatrix}, \quad \underline{\underline{B}} = \begin{bmatrix} 0 & -\underline{\underline{M}} \\ \underline{\underline{N}} & 0 \end{bmatrix}, \quad \underline{v} = \begin{bmatrix} u \\ v \end{bmatrix}, \quad \underline{\sigma} = \begin{bmatrix} \sigma \\ \tau \\ s \end{bmatrix} \quad (9)$$

$$\underline{\underline{I}} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \quad \underline{\underline{M}} = \underline{\underline{N}}^T = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$$

$\underline{\underline{W}}_t$ and $\underline{\underline{W}}_x$ represents the partial derivatives with respect to t and x , respectively. The eigenvalues c are determined by the characteristic equation, which can be simplified as the following form:

$$\|\underline{\underline{B}} - c\underline{\underline{A}}\| = \|\underline{\underline{N}}\underline{\underline{M}} - \rho c^2 \underline{\underline{G}}\| = -p(ap^2 + bp + d) = 0 \quad (10)$$

where

$$p = \rho c^2, \quad a = \|\underline{\underline{G}}\|, \quad b = -G_{33} (G_{11} + G_{22}) - G_{13} G_{31} - G_{23} G_{32}, \quad d = G_{33} \quad (11)$$

Besides the double roots $c_0 = 0$ we have two characteristic wave speeds $c_f \geq c_s \geq 0$, which are called the fast and slow wave speeds respectively. It is easy to show that

$$0 \leq c_s \leq c_2 = \sqrt{G/\rho} \leq c_f \leq c_1 = \sqrt{E/(\rho(1-\nu^2))} \quad (12)$$

The characteristic relation along a characteristic curve $dx/dt=c$ is given by

$$\underline{L}^T \underline{B} d\underline{W} = 0 \quad (\text{for } c \neq 0) \quad \text{or} \quad \underline{L}^T \underline{A} d\underline{W} = 0 \quad (\text{for any } c) \quad (13)$$

where $\underline{L}$ is the left null vector of $\underline{B}-c\underline{A}$,

$$\underline{L}^T (\underline{B}-c\underline{A}) = \underline{0}^T \quad (14)$$

It's easy to show that the Eqs. (13) can be written as

$$\underline{L}^T \underline{N} (d\underline{y} - \underline{M} d\underline{g}/\rho c) = 0 \quad (\text{when } dx/dt = \pm c_f, \pm c_s) \quad (15)$$

$$G_{31} d\sigma + G_{32} d\tau + G_{33} ds = 0 \quad (\text{when } dx/dt = 0) \quad (16)$$

where $\underline{L}$ is the left null vector of $\underline{NM}-\rho c^2 \underline{G}$. In our case the left and the right null vectors are identical because of symmetry of the matrices concerned. Eqs. (15) and (16) can be used to integrate numerically the initial and boundary value problems along the characteristic curves.

3 - SIMPLE WAVE SOLUTIONS

Simple wave solutions are the particular solutions of (8) in which $\underline{W}$ is a constant vector along one set of characteristics. In a simple wave region $d\underline{W}$ is proportional to the right null vectors $\underline{R}$ of $\underline{B}-c\underline{A}$:

$$d\underline{W} = q \underline{R} \quad (17)$$

$$(\underline{B}-c\underline{A}) \underline{R} = \underline{0} \quad (18)$$

where q is an arbitrary scalar factor. It is easy to show that the simple wave solution (17) can be written as

$$d\sigma/r_1 = d\tau/r_2 = ds/r_3, \quad du = -d\sigma/\rho c, \quad dv = -d\tau/\rho c \quad (19)$$

where $\underline{r}$ is the right null vector of $\underline{NM}-\rho c^2 \underline{G}$, which depends on stress only:

$$(\underline{NM}-\rho c^2 \underline{G}) \underline{r} = \underline{0} \quad (20)$$

$$r_1 = ((2\sigma-s) + \nu(2s-\sigma)) c_1^2 / (c^2 - c_1^2) \quad (21)$$

$$r_2 = 6\tau c_1^2 / (c^2 - c_1^2), \quad r_3 = 3\nu r_1 + (2s-\sigma)E/(\rho c^2)$$

For $c=c_f$ or $c=c_s$ the first two equations of (19) can be integrated to give a two-parameter family of curves in the stress space (σ, τ, s) , which are called the stress paths for the fast and slow simple waves, respectively. We have drawn the projections of these stress paths on (σ, τ) and (s, τ) planes, as shown in Figs. 1 and 2. The dashed lines and the solid lines represent the stress paths for the fast simple waves and those for the slow simple waves, respectively. The point $(\sigma^*, 0, s^*)$ in the stress space as shown in Figs. 1 and 2 is a singular point, at which $c_f = c_s = c_2$ [7]. Thus in the case of pure compressive impact the longitudinal wave is the fast wave for $\sigma < \sigma^*$, but the slow wave for $\sigma > \sigma^*$. This result is similar to the conclusion of Clifton [1/

and Ting /7/. In the elastic regions the stress paths for $c_1 = c_2$ and those for $c_1 \neq c_2$ are horizontal and vertical lines, respectively. The arrowheads in the figures indicate the directions along which the simple wave speeds decrease. Any reasonable simple wave combination must correspond to an admissible stress path along which the wave speed is nonincreasing. This idea will help us to construct the right simple wave solutions for some initial and boundary value problems. For example, the path oaij in Figs. 1 and 2 is an admissible stress path corresponding to a simple wave solution of the problem, the initial condition of which is stress free, point o, the boundary condition of which is a combined compressive and shear step load, point j. Likewise, oYbg and oY σ^e are also admissible stress paths. The above three simple wave solutions are shown in Figs. 3(a), 3(b), and 3(c), respectively.

Assume the plate to be presheared to a plastic state τ^a and subjected at $t=0$ to a combined step load $\sigma = \sigma^c$, $\tau = \tau^c > \tau^a$ on its boundary, Figs. 4 and 5, where ab and bc are the fast simple wave stress path through point a and the slow simple wave stress path through point c, respectively. It is easy to see that we will have a simple wave solution in Fig. 6. Here we have an interesting phenomenon: Although the boundary shear load τ^c is greater than the presheared stress τ^a , the shear stress in the plate will be first decreased by a fast simple wave followed then by a slow simple wave to change the stress state to the boundary condition.

If the plate is precompressed to a plastic state σ^a , the lateral normal stress s^a can be determined /8/. The succeeding yield surface through point $(\sigma^a, 0, s^a)$

$$\sigma^2 + s^2 - \sigma s + 3\tau^2 = (\sigma^a)^2 + (s^a)^2 - \sigma^a s^a \quad (22)$$

and the unloading plane from point $(\sigma^a, 0, s^a)$

$$s - s^a = \nu (\sigma - \sigma^a) \quad (23)$$

have an intersection curve, the projections of which to (σ, τ) plane and (s, τ) plane are shown in Figs. 7 and 8. If the boundary load is represented by point d, Figs. 7 and 8, we will have the simple wave solution shown in Fig. 9. Here we have an even more interesting and unexpected phenomenon: A longitudinal unloading shock occurs first, a shear shock propagates then, followed by a slow simple wave to load the plate to the boundary condition.

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Fig. 1

Fig. 2

Fig. 3

Fig. 4

Fig. 5

Fig. 6

Fig. 7

Fig. 8

Fig. 9