ON SYSTEMATIC ERRORS IN CHARACTERIZING CHAOS

F. Mitschke, M. Möller, W. Lange

JOURNAL DE PHYSIQUE
Colloque C2, Supplément au n°6, Tome 49, juin 1988

ON SYSTEMATIC ERRORS IN CHARACTERIZING CHAOS

F. MITSCHKE, M. MÖLLER and W. LANGE

Institut für Quantenoptik, Universität Hannover, D-3000 Hannover 1, F.R.G.

Résumé - Les erreurs systématiques dans la détermination de la dimension et de l'entropie des attracteurs chaotiques ont été étudiés. La limitation de bande passante conduit à une surestimation, la limitation de résolution à une sousestimation de la dimension. Dans l'un et l'autre cas l'entropie ne semble pas affectée.

Abstract - Systematic errors in the determination of dimension and entropy of chaotic attractors are investigated. Bandwidth limitation leads to an overestimate, resolution limitation to an underestimate of dimension. The entropy seems unaffected in either case.

INTRODUCTION

An important step in the characterization of chaotic dynamics is the determination of the dimension of the underlying attractor. Many laboratories have adopted a method originally proposed by Grassberger and Procaccia /1/, that requires only a single-variable time series and yields the correlation dimension D_2 and the order-2 entropy K_2 /2/. The method is reasonably robust in the presence of noise and still works with relatively small data sets /3/. It is not immediately obvious, however, whether spectral filtering or digitizing error, as encountered in a realistic data acquisition, are tolerated.

Badii and Politi pointed out recently /4/, that a low-pass filtering of the signal, e.g. due to insufficient bandwidth of the measuring apparatus, results in an overestimate for the dimension. Very recently they tested their prediction by simulating spectral filtering on existing experimental data /8/. Independently we pursued the question how well this theory would apply to an actual experimental situation, with the additional restraints of small data sets with low digitizing resolution. We find the predicted influence of low-pass filtering on the dimension /9/ and in addition we present evidence that low resolution can lead to an underestimate of the dimension. The entropy seems unaffected from either error.

To understand the effect of filtering, consider the case of a chaotic signal $x(t)$, passed through a single-pole (first order) low pass filter as modeled by

$$\dot{z} = - \eta \cdot z + x$$

with the "filter output" $z(t)$. If the chaos-generating process has a phase space of dimension d, the phase space of the combined system (including the filter) is increased to d+1, and so is the number of Lyapunov exponents. The additional exponent, $-\eta$, is negative, and η is equal to the filter roll-off frequency. Assuming the validity of the Kaplan-Yorke conjecture /5/, η will increase the information dimension D_1 determined from signals at the filter output (i.e. from a time series of z values), provided that $0 \geq -\eta > \lambda_-$. Here λ_- denotes the negative Lyapunov exponent that appears in the denominator of the Kaplan-Yorke formula. The lower the filter roll-off frequency, the more pronounced is this increase. In the limit $\eta \rightarrow 0$, the filter becomes an integrator, and the increase is predicted to be as much as one over the unfiltered original : $D_1(\eta = 0) = D_1(\eta = \infty) + 1$.

Since only the negative Lyapunov exponents are affected, we expect that the positive ones and thus the entropy K_1 will remain unchanged in the filtering process. With K_1 being an upper bound for K_2 /6/, it is reasonable to assume that K_2 will not depend strongly on η.

EXPERIMENT ON A TIME-CONTINUOUS SYSTEM

For our experiment, we used the system described in /7/ as a source of chaotic signals. It consists of an electronic circuit simulating a hybrid optical device and is described very well by the differential equation

$$\dddot{U} + a\cdot\ddot{U} + b\cdot\dot{U} + c\cdot U = d\cdot(U - e)^2 .$$

with a,b,c,d,e constant coefficients depending on component values and parameter choice. For our present purpose, we picked a set of parameters such as to obtain mildly chaotic behaviour /9/.

We passed the output signal U of the electronic circuit through a low-pass filter of selectable RC time constant. At the filter output, we measure with a LeCroy 9400 transient digitizer which has 8-bit resolution and can store up to 32000 data points in a row. Strictly speaking, its bandwidth of 125 MHz contributes a second pole to the overall frequency response, with $\eta \approx 10^4$. This value is high enough to be safely considered infinite. The intrinsic noise in the circuit (-85 dB) is negligible at 8-bit resolution.

From the acquired data, D_2 and K_2 were evaluated with a Grassberger-Procaccia type program. Typically, the calculations were done with 5000 data points and embedding dimensions up to 20; the length scales used in the evaluation (the "scaling region") range from 2^{-5} to $2^{-2.7}$.

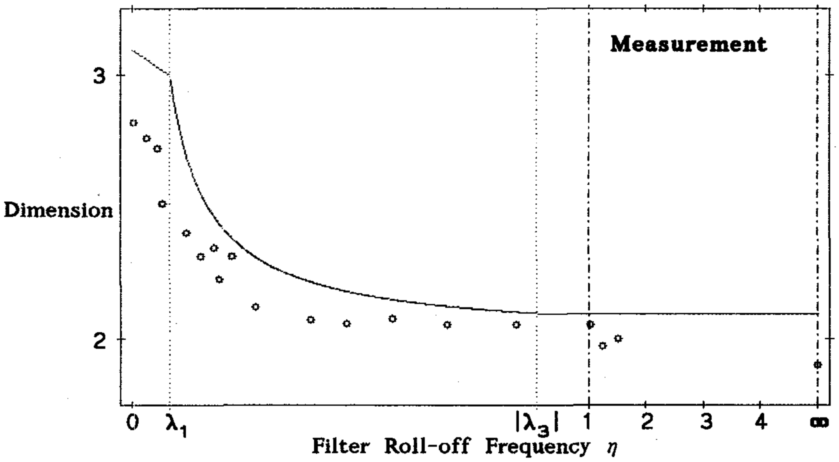

FIG. 1. D_2 values for low-pass filtering with different roll-off frequencies η. The solid line shows the predicted $D_1(\eta)$. Note change of scale at the dashed lines.

We find K_2 = 0.067, independent on η. As it must be, this value is smaller than K_1 = 0.081, which was previously determined independently. The results for $D_2(\eta)$ are shown in Fig. 1, together with the prediction according to /4/ (solid line). It is perfectly reasonable that all data points lie below the solid line, because D_1 is an upper bound for D_2. However, the points clearly follow the trend of the solid line, and for η -> 0, D_2 indeed tends to $D_2(\eta = 0) = D_2(\eta = \infty) + 1$. Obviously,. the result of Badii and Politi is confirmed.

NUMERICAL EXPERIMENT ON A TIME-DISCRETE SYSTEM

Filtering can also be applied to iterative maps. For a test we used the well-known Hénon map with an added filter equation:

$$X_{n+1} = 1 - A \cdot X_n^2 + Y_n$$

$$Y_{n+1} = B \cdot X_n$$

$$Z_{n+1} = \exp(-\eta) Z_n + X_n .$$

with the standard values A=1.4 and B=0.3.

From several thousand iterations done with 32-bit precision for several values of η, we stored the Z values as "mother" files. As we intended to use the same procedure as for the experimental data, we created files of 8-bit resolution from the "mother" files and we determined K_2 and D_2 from these "daughter" files the same way as above. Again K_2 is found to be independent of η. The result for D_2 is shown in Fig. 2, in comparison with the predicted $D_1 = D_1(\eta)$ (solid line). There is the same trend as above, and the theory by Badii and Politi is confirmed.

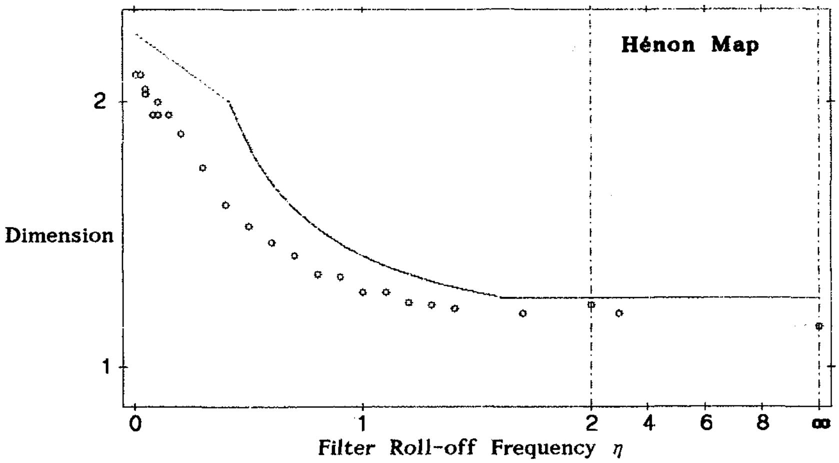

FIG. 2. Same as FIG. 1, but for the Hénon map with simulated low-pass filtering.

Comparison of our D_2 value for the unfiltered Hénon map with the known literature value /6/ showed that it was underestimated. In order to find out whether this was due to quantization error, we calculated D_2 and K_2 from "daughter" files of n-bit resolution, with n ranging from 6 to 12, that had been created from the same "mother" file. To avoid possible problems related to the attractor lacunarity, we kept the same scaling region throughout. We found that the dimension seems to depend systematically on n, with an error increasing with decreasing resolution. This is demonstrated for $\eta = \infty$ in Fig. 3.

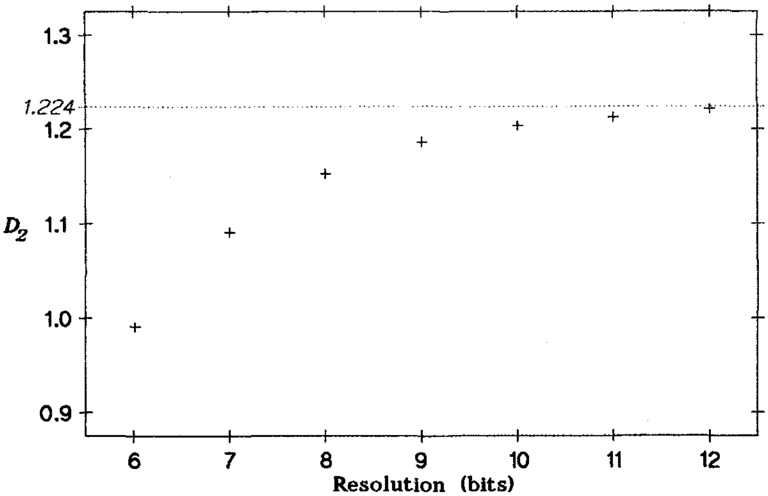

FIG. 3. D_2 values for the (unfiltered) Hénon map as a function of bit resolution. The dotted line indicates the literature value of 1.224 for D_2 evaluated with high precision.

CONCLUSION

Given that any measuring device has finite bandwidth, data acquisition involves low-pass filtering by necessity. A filtered chaotic signal, however, corresponds to a more or less distorted attractor /9/. Evaluating such a signal may lead to an overestimate for the dimension. Moreover, any digitizing equipment has a limited resolution. This introduces a quantization error which may lead to an underestimate of D_2. It follows that for an accurate determination of dimension of chaotic attractors, it is important to have sufficient bandwidth and resolution. On the other hand we observe that the entropy is not affected by either systematic error; it may thus give a much better criterion to distinguish chaos from noise.

We thank N. B. Abraham for valuable discussions.

REFERENCES

/1/ P. Grassberger, I. Procaccia, Physica 9D, 189 (1983)
/2/ P. Grassberger, I. Procaccia, Phys. Rev. A 28, 2591 (1983)
/3/ N. B. Abraham et. al., Phys. Lett. 114A, 217 (1986)
/4/ R. Badii, A. Politi, in "*Dimensions and Entropies in Chaotic Systems*", G. Mayer-Kress (Editor), Springer Verlag 1986
/5/ see, e.g., R. Badii, A. Politi, Phys. Rev. A 35, 1288 (1987) and references therein
/6/ P. Grassberger, I. Procaccia, Physica 13D, 34 (1984)
/7/ F. Mitschke, N. Flüggen, Appl. Phys. B35, 59 (1984)
/8/ R. Badii, G.Broggi, B. Derighetti, M. Ravani, S. Ciliberto. A. Politi, M.A. Rubio, Phys. Rev. Lett. (in press)
/9/ F. Mitschke, M. Möller, W. Lange, Phys. Rev. A (in press)