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ANELASTIC BEHAVIOUR OF MATERIALS UNDER MULTI-AXIAL STRAINS

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Abstract. A theory of anelasticity is presented, which involves the relaxation of all the elastic compliances as a function of orientation. It is shown that, in addition to the usual relaxations of Young's and shear moduli, anelastic phenomena related to the relaxation of Poisson's ratio should be considered. Finally, the theory is applied to anelastic data in Zircaloy-4 and AISI-1080 steel, obtained in longitudinal excitations.

Introduction

The anelastic behaviour of materials is usually described by the standard anelastic solid (SAS) model (1). The anelastic properties of such a solid can be expressed by the linear differential equation

$$\epsilon+\tau_\sigma\dot{\epsilon}=J_R\sigma+J_U\tau_\sigma\dot{\sigma} \qquad (1)$$

where σ is the applied stress, ϵ the strain, J_U and J_R are the unrelaxed and the relaxed compliances, respectively, and τ_σ is the relaxation time at constant stress. The dot indicates a derivative with respect to the time. The dynamic response of the SAS is very well known and leads to the so called Debye equations for the dynamic modulus and the internal friction (1). These equations are generally used to represent the anelastic behaviour of specimens excited under simple situations, like longitudinally or in torsion. In these situations, only the relaxation of Young's and shear moduli are measured. Furthermore, for single crystals only the orientation dependence of the relaxation of these two moduli are generally obtained.
It is the purpose of this paper to extend the formalism to all the elastic compliances, as a function of orientation, both for cubic and hexagonal symmetries. This will allow the determination of the relaxed and unrelaxed Poisson's ratios, in two orthogonal directions located in the plane perpendicular to the direction of the applied stress. With this information, it is possible to study the relaxation behaviour in directions perpendicular to the excitation. Finally, some points defect symmetries and particular orientations will be considered, as special situations of the general formalism presented in the paper. Furthermore, the results will be applied to actual experimental data obtained in Zircaloy-4 and AISI-1080 steel.

Theory

Generalized Hooke's law can be expressed, in terms of the commonly used single index notation, as (2)

$$\epsilon_i=\sum_{j=1}^{6} s_{ij}\sigma_{ij} \qquad i,j=1,\ldots,6 \qquad (2)$$

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where s_{ij} are the elastic compliances. Eq. (2) can be simplified for crystals if, instead of the usual components of stress and strain, six independent linear combinations of these are chosen which possess certain fundamental symmetry properties associated with the crystal considered (3). These combinations are called symmetry coordinates of stress and strain or symmetrized stress and strain. Furthermore, whenever a symmetrized stress is decoupled from all the symmetrized strains, except the one which corresponds to it, Hooke's law reverts to the simple form

$$\epsilon_\gamma = S_\gamma \sigma_\gamma \quad (3)$$

where γ denotes the symmetry designation and S_γ is the appropriate symmetrized compliance. In order to generalize the equations of elasticity of crystals to allow for time-dependent effects, the validity of the SAS model will be accepted, for each symmetrized coordinate decoupled one from another. In this case, a generalization of Eq. (1) is obtained

$$\epsilon_\gamma + \tau_{\sigma\gamma}\dot{\epsilon}_\gamma = S_{\gamma R}\sigma_\gamma + \tau_{\sigma\gamma} S_{\gamma U}\dot{\sigma}_\gamma \quad (4)$$

where R and U denote relaxed and unrelaxed compliances, respectively. Furthermore, there are two types of symmetrized strains, Type I and Type II. The special feature of strains of Type I is that a crystal subjected to such a strain is not lowered in symmetry by the deformation. On the other hand, a crystal under Type II strain is lowered in symmetry. It will be assumed in the theoretical development that follows that defects of only one single species are present, and only compliances of Type II may under go relaxation (1).

Cubic symmetry. Longitudinal stress

For cubic symmetry and an extension along <100> and a contraction along <010> or <001>, under a sinusoidal applied stress, Eq. (4) leads to (4)

$$\epsilon_1' = \{[s_{11} + (s_{11} - \tfrac{2}{3}\,\delta)\omega^2\tau^2 - i\omega\tau\,\tfrac{2}{3}\,\delta]/(1+\omega^2\tau^2)\}\sigma_1' \quad (5)$$

$$\epsilon_2' = \{[s_{12} + (s_{12} + \tfrac{1}{3}\,\delta)\omega^2\tau^2 + i\omega\tau\,\tfrac{1}{3}\delta]/(1+\omega^2\tau^2)\}\sigma_1' \quad (6)$$

where δ represents the intensity and τ the time, at constant stress, for the relaxation of $(s_{11}-s_{12})$. $\sigma_1'=\sigma_{01}'\exp(i\omega t)$ is the applied stress, ϵ_1' gives the strain in the direction of the applied stress and ϵ_2' the strain in the perpendicular directions. It should be pointed out that more complicate expressions are obtained if the stress is applied in any direction, since the orientation factors appropriate for the cubic symmetry will be involved. Moreover, the complex Poisson's ratio is given by

$$\nu_{12} = \nu_1 - i\nu_2 = -\epsilon_2'/\epsilon_1' \quad (7)$$

with

$$\nu_1 = [-s_{11}s_{12} + (s_{12} + \tfrac{\delta}{3})(s_{11} - \tfrac{2}{3}\,\delta)\omega^2\tau^2]/[s_{11}^2 + (s_{11} - \tfrac{2}{3}\,\delta)^2\omega^2\tau^2] \quad (8)$$

and

$$\nu_2 = [\omega\tau\delta(s_{11} + 2s_{12})]/[s_{11}^2 + (s_{11} - \tfrac{2}{3}\,\delta)^2\omega^2\tau^2] \quad (9)$$

It is easy to see that ν_2 has a maximum at $\omega\tau=1$, as for a Debye curve and ν_1 has an inflection point at $\omega\tau=(1/3)^{1/2}$, slightly displaced with respect to the classical value for Young's and shear moduli, as shown in Fig. 1. Furthermore, the internal friction due to each strain mode is given by (4)

$$\tan\phi_1 = \Delta E_{11}/4E_{11} = s_{11}^i/s_{11}^r \quad (10) \quad ; \quad \tan\phi_2 = \Delta E_{12}/4E_{12} = s_{12}^i/s_{12}^r \quad (11)$$

where the superscripts r and i indicate the real and the imaginary component, respectively. ϕ_1 is the phase lag between the longitudinal stress and the longitudinal strain and ϕ_2 between the stress and the transversal strain. The phase lags are given by

$$\tan\phi_1 = [-\tfrac{2}{3}\,\omega\tau\delta]/[s_{11} + (s_{11} - \tfrac{2}{3}\delta)\omega^2\tau^2] \quad (12)$$

$$\tan\phi_2=[\frac{1}{3}\omega\tau\delta]/[s_{12}+(s_{12}+\frac{\delta}{3})\omega^2\tau^2] \quad (13)$$

The phase lag between longitudinal and transversal strain is

$$\tan(\phi_2-\phi_1)=\nu_2/\nu_1=-[\frac{\delta}{3}\omega\tau(s_{11}+2s_{12})]/[s_{11}s_{12}+(s_{11}-\frac{2}{3}\delta)(s_{12}+\frac{\delta}{3})\omega^2\tau^2] \quad (14)$$

Eq. (14) is shown in Fig. 1 and some characteristic points are given in Table 1.

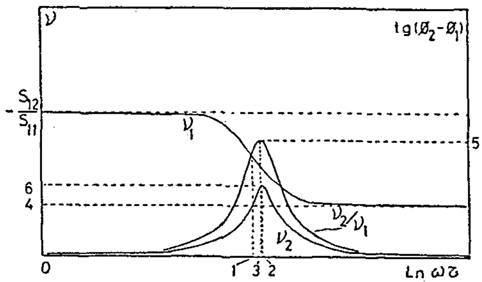

Fig. 1. Complex Poisson's ratio and tan $(\phi_2-\phi_1)$ versus $\ln\omega\tau$. Some characteristic values indicated from 1 to 6 are given in Table 1.

Table 1
Characteristic values for the points indicated from 1 to 6 in Fig. 1.

Position	Value
1	$s_{11}/3^{1/2}(s_{11}-\frac{2}{3}\delta)$
2	$s_{11}/(s_{11}-\frac{2}{3}\delta)$
3	$[s_{11}s_{12}/(s_{11}-\frac{2}{3}\delta)(s_{12}+\frac{\delta}{3})]^{1/2}$
4	$-(s_{12}+\frac{\delta}{3})/(s_{11}-\frac{2}{3}\delta)$
5	$[s_{11}s_{12}/(s_{11}-\frac{2}{3}\delta)(s_{12}+\frac{\delta}{3})]^{1/2}[(s_{11}+2s_{12})/2(s_{11}+s_{12})]$
6	$[\delta(s_{11}+2s_{12})]/[6s_{11}(s_{11}-\frac{2}{3}\delta)]$

Hexagonal symmetry. Longitudinal stress

If the angle θ formed between the direction of the applied sinusoidal stress and the <c>-axis of the hexagonal crystal is $\pi/2$, that is, if the excitation is perpendicular to this axis, the strains produced are (4)

$$\epsilon_3'=[s_{11}+(s_{11}-\frac{\delta}{2})\omega^2\tau^2-i\omega\tau\frac{\delta}{2}]/(1+\omega^2\tau^2) \quad (15)$$

$$\epsilon_1'=[(s_{12}\cos^2\psi+s_{13}\sin^2\psi)+(s_{12}\cos^2\psi+s_{13}\sin^2\psi+\frac{\delta}{2}\cos^2\psi)\omega^2\tau^2+i\omega\tau\frac{\delta}{2}\cos^2\psi]/(1+\omega^2\tau^2) \quad (16)$$

where ϵ_3' is the strain in the direction of the applied stress and ϵ_1' in a perpendicular direction. δ represents the intensity and τ the time, at constant stress, for the relaxation of $(s_{11}-s_{12})$. ψ indicates the direction in the plane whose normal is in the direction of the applied stress. θ and ψ are the two Eulerian angles that define the orientation of the excitation with respect to the coordinate system referred to the hexagonal cell (5,6). For ψ=O the complex Poisson's ratio is given by

$$\nu_o^* = -\epsilon_1^*/\epsilon_3^* = \nu_o^1 - i\nu_o^2 = -[s_{11}s_{12} + (s_{11}-\frac{\delta}{2})(s_{12}+\frac{\delta}{2})\omega^2\tau^2]/[s_{11}^2 + (s_{11}-\frac{\delta}{2})^2\omega^2\tau^2]$$

$$- i\omega\tau\delta/[s_{11}^2 + (s_{11}-\frac{\delta}{2})^2\omega^2\tau^2] \quad (17)$$

It is easy to show from Eq. (17) that

$$\nu_o^1(\omega\tau\to 0) = -s_{12}/s_{11} \quad \text{and} \quad \nu_o^1(\omega\tau\to\infty) = -(s_{12}+\frac{\delta}{2})/(s_{11}-\frac{\delta}{2}) \quad (18)$$

In addition, the curve of ν_0^1 versus $\omega\tau$ has an inflection point at

$$\omega\tau = [s_{11}/(s_{11}-\frac{\delta}{2})]/3^{1/2} \quad (19)$$

For $\psi=\pi/2$ the complex Poisson's ratio is given by:

$$\nu_{\pi/2} = -\epsilon_1^*/\epsilon_3^* = \nu_{\pi/2}^1 - i\,\nu_{\pi/2}^2 = -s_{13}[s_{11} + (s_{11}-\frac{\delta}{2})\omega^2\tau^2]/[s_{11}^2 + (s_{11}-\frac{\delta}{2})^2\omega^2\tau^2]$$

$$- i\omega\tau\delta/2[s_{11}^2 + (s_{11}-\frac{\delta}{2})^2\omega^2\tau^2] \quad (20)$$

Eq. (20) leads to

$$\nu_{\pi/2}^1(\omega\tau\to 0) = -s_{13}/s_{11} \quad \text{and} \quad \nu_{\pi/2}^1(\omega\tau\to\infty) = -s_{13}/(s_{11}-\frac{\delta}{2}) \quad (21)$$

The inflection point is given by Eq. (19) also in this case. ν^1 and ν^2 lead to curves similar to those shown in Fig. 1 and, in addition, ν_0^1 decreases and $\nu_{\pi/2}^1$ increases as $\omega\tau$ increases.

Applications

Measurements of ν, as a function of temperature, were performed by using the "free-free" or floating beam resonant method. The equipment and the measuring technique have been described elsewhere (7). ν is obtained from the measured resonant frequencies, at two different harmonics j and j_o, through the relationship

$$\nu = [(j_o f_j - j f_{jo})/(j_o^3 f_j - j^3 f_{jo})]^{1/2}[2L/\pi(r_i^2 + r_o^2)^{1/2}] \quad (22)$$

where L is the length of the cylindrical specimen, r_i and r_o are the inner and outer diameter (r_i=o for a cylindrical rod), respectively, and f_j is the resonant frequency where j=1, 3, 5, gives the order of the harmonic for a specimen suspended in the center. The fundamental resonant frequency used was of the order of 17 kHz. Fig. 2 shows data obtained in a tubular specimen of Zircaloy-4 (hexagonal symmetry). The pair of harmonics used to calculate ν with Eq. (22) are indicated on each curve.

Curve 1-3 shows the typical behaviour expected for $\nu_{\pi/2}^1$ (Eq. (21)) and curve 3-5 the general trend expected ν_0^1 (Eq. (18)). The different behaviour at different harmonics can be explained by the texture present in the tube, leading to coupling between longitudinal and transversal vibrations, which depends on the frequency (8).

Fig. 3 shows data obtained in a cyclindrical specimen of AISI-1080 steel (cubic symmetry). The pair of harmonics used to calculate ν with Eq. (22) is indicated on each curve. For the classical texture obtained in a rod of material with cubic symmetry, like the specimen used, the tensile stress is applied mainly along <110> and the contraction occurs in a (110) plane. In these conditions and for a tetragonal defect, it can be shown (9) that ν_1 has a behaviour similar to the one shown in Fig. 1, but increases with $\omega\tau$. This is the general trend observed in Fig. 3. The decrease observed in all the curves, above about 500 K, is due to a desordering transition (9).

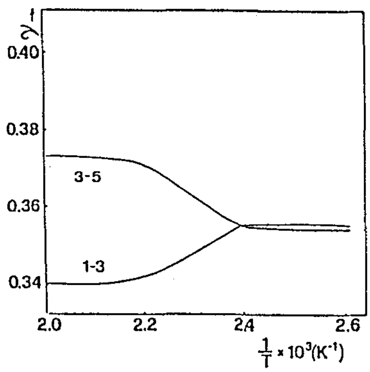

Fig. 2. Poisson's ratio as a function of the reciprocal of the absolute temperature for a Zircaloy-4 tube.

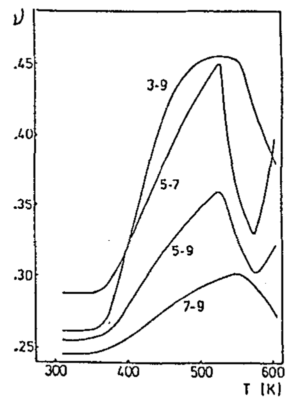

Fig. 3. Poisson's ratio as a function of temperature for a rod of AISI-1080 steel.

Conclusion

A theory of anelasticity produced by the relaxation of Poisson's ratio has been presented, which takes into account the loss produced by other strains, in addition to the one in the direction of the applied stress. Due to the higher elastic anisotropy of ν in single crystals, than both Young's and shear moduli, a similar behaviour should be expected for time dependent events. Therefore, a more substantial information can be obtained from measurements of the dynamical behaviour of ν than from those of Young's or shear moduli. Finally, the concepts developed have been applied to actual experimental data obtained in Zircaloy-4 and AISI-1080 steel.

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