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AN ANALYTICAL MODEL OF HEAT FLUX IN LASER SURFACE TREATMENT

P. MAGNIN and H. MAYER
Ecole Polytechnique Fédérale de Lausanne, CTML-DMX, CH-1015 Lausanne, Switzerland

A new analytical model of heat flux during laser surface treatment is presented. Prior knowledge of the absorption coefficient is precluded and can be later calculated by giving special consideration to the peak surface temperature which is easily obtained from an optical pyrometer. The results are in excellent agreement with the experimental values measured during hardening and remelting treatments using variable laser conditions.

INTRODUCTION

The laser surface treatment of metals is of considerable technological interest in solving problems of wear and corrosion. Surface processes such as hardening and remelting are basically governed by a heat flux mechanism. It is, therefore, important to be able to calculate the thermal cycle induced by the scanning laser beam, as it determines the thickness and properties of the treated layer.

A number of papers have been published on this subject, giving analytical [1 to 4] as well as numerical method [5, 6] solutions. However, all these models involve knowledge of an important parameter: the absorption coefficient of the laser energy by the treated area. There are techniques which allow rather easy measurement of the absorption on small samples [7], but it is difficult to establish this parameter during treatment of, for example, large scale production pieces. The degree of absorption depends strongly on the surface finish [7], which can change between a test sample and the actual piece, especially when an absorbing coating is used whose properties (thickness, density, ...) are not well controlled. Furthermore, since the absorption coefficients of liquid and solid differ, the average absorption obtained during remelting treatments depends strongly on the relative amount of each beneath the laser beam, which, in turn, depends on the laser conditions (power, beam diameter, scanning velocity).

It is relatively easy to measure the maximum surface temperature, with a simple pyrometer. If a mathematical model allows calculation of the surface temperature from the absorption coefficient, it is then possible to reverse the process and obtain the absorption value from the measured surface temperature. This requires an analytical solution, as the numerical methods need too long a calculation time to allow the use of an iterative process for the absorption adjustment. Unfortunately the existing analytical models as far as they are simple to solve are not able to correctly predict the temperature field at the surface. The main difficulty is taking into account the extent of the beam, which can be neither neglected (Rosenthal point source [1]), nor considered as very large (infinite uniform source [2]). Ashby and Easterling [4] have proposed a method for solving this problem, which will be the starting point of the present model.

THE ANALYTICAL MODEL

Consider a laser beam of TEM\(_{0,0}\) (gaussian) energy distribution and total power \(P\), moving in the \(x\) direction with velocity \(V\). The closest beam geometry for which a simple analytical solution for the temperature field is available is a gaussian line source of finite width in the \(y\) direction, but infinitesimally thin in the \(x\) direction [3] (Fig. 1). In this case, the temperature at a coordinate point \(y,z\) in the \(x = 0\) plane (fixed frame of reference) is given as a function of time \(t\) (\(t = 0\) when the line source crosses the plane \(x = 0\)) as:
Here $T_0$ is the initial temperature, $a$ is the absorption coefficient, $\kappa$ is the thermal conductivity and $h$ the thermal diffusivity of the treated material. The constant $t_0$ represents the time taken for the heat to diffuse over the beam radius $r$ (defined as the radius at which the power density has fallen to $1/e$ of its peak value) and can be expressed as $t_0 = r^2 / 4h$. Eq. 1, however, leads to an infinite temperature at the surface because the line source geometry implies zero beam area (i.e. infinite power density) injected during a zero interaction time.

In order to take account of the beam thickness, Ashby and Easterling have proposed replacing the real source in the plane $z = 0$ by an apparent source in the plane $z = z_0$ above the surface, allowing the heat to diffuse in a virtual layer, thus simulating the non-zero beam area and interaction time $r_1$. The solution of the temperature field can then be written:

$$T = T_0 + \frac{a \cdot P}{2 \pi \kappa} \exp \left( \frac{1}{4h} \left( \frac{z^2 + y^2}{t} \right) \right) \left( z + z_0 \right)^2 + \frac{y^2}{t + t_0}$$

(2)

In order to calculate the depth of the molten (if any) and heat affected zones, one must find the depth at which the corresponding transformation temperature has been reached. One must then solve two problems: what is the time, $t^*$, at which the temperature reaches a maximum and what is the value of the parameter $z_0$.

The time $t^*$ of the peak temperature

The maximum temperature below the center of the beam track is reached at the time $t^*$ corresponding to $dT/dt = 0$, which leads to (4):

$$t^* \frac{2 t^* + t_0}{t^* + t_0} = \frac{(z + z_0)^2}{2h}$$

(3)

In order to simplify the calculations Ashby and Easterling consider two limiting cases, obtaining $t^*$ from eq. 3 with the approximations $t^* \ll t_0$ and $t^* \gg t_0$. The temperature field is then given by a kind of empirical mean between the two solutions obtained when these estimations of $t^*$ are used for the time $t$ in eq. 2. This solution is however not very precise as we expect $t^*$ to be of the order of magnitude of $t_0$, as in most practical cases the depth of the treated zone often approximates the beam radius.

The model proposed here relaxes this approximation. Firstly, note that eq. 3 can be written as a second degree equation for $t^*$, which can then be solved exactly. One obtains:

$$t^* = t_0 \frac{X - 1 + \sqrt{X^2 + 6X + 1}}{4}$$

with $X = \frac{(z + z_0)^2}{2h t_0} = 2 \frac{(z + z_0)^2}{t}$

(4)

Setting the time $t^*$ given by eq. 4 in eq. 2 gives the maximum temperature obtained below the center of the beam track ($y = 0$), as a function of the depth $z$.

Fig. 1 Geometry of the line source approximation (eq. 1)
It is, however, more convenient to find an expression which directly gives the depth $z$ as a function of the peak temperature $T_p$. This can be done noting that eq. 3 can be written as:

$$t^* = \frac{(z + z_0)^2}{2h\beta} \quad \text{where} \quad \beta = \frac{2\alpha + 1}{\alpha + 1} \quad \text{and} \quad \alpha = \frac{t^*}{t_0} \quad (5)$$

Using $t^*$ from eq. 5, eq. 2 for $y = 0$ becomes (recall that $h = \kappa/\rho c_p$ where $\rho$ is the density and $c_p$ the specific heat):

$$T_p = T_0 + \frac{a \cdot P}{\pi \sqrt{\rho c_p (z + z_0)^2}} F(\alpha)$$

with $F(\alpha) = \exp(\beta/2) / \beta \sqrt{1 + 1/\alpha}$

Notice that $\exp(\beta/2)/\beta$ is a weak function of $\beta$, which is a very weak function of $\alpha$. One can then make the approximation (leading to a maximum error of ± 3.5 %):

$$\exp(\beta/2)/\beta = s/2$$

Furthermore, the time $t^*$ for the peak temperature at the depth $z + z_0$ is related to the heat diffusion time over $z + z_0$, $t_z = (z + z_0)^2/4h$. The ratio $t^*/t_z$ can be shown to be approximately $s/2$. One can then write:

$$\alpha = \frac{t^*}{t_0} = s (z + z_0)^2$$

Substituting eqs. 7 and 8 in eq. 9 gives:

$$T_p = T_0 + \frac{a \cdot P}{\pi \sqrt{\rho c_p ((z + z_0)^2 + (z + z_0)^2 r^2/s^2)}}$$

The values of the peak temperatures given by eq. 9 are nearly indistinguishable from the exact values obtained by using $t^*$ from eq. 4 in eq. 2, independent of the material constants and beam parameters. The main advantage of eq. 9 is that it can be rearranged, giving:

$$z = \sqrt{-\frac{r^2 + \sqrt{r^4 + 4a \cdot P \pi \sqrt{\rho c_p ((T_p - T_0)^2)}}}{2s}} - z_0$$

The parameter $z_0$

In order to determine the value of the parameter $z_0$, Ashby and Easterling make the value of the peak temperature calculated by eq. 2 for $y = 0$, $z = 0$ equal to the surface peak temperature produced by an infinite stationary beam of the same power density, applied for a time equal to the interaction time of the scanning beam. The infinite uniform source model unfortunately gives surface temperature which are much too high because the heat fluxes, which determine all temperatures, are essentially non-planar. A model valid for a stationary gaussian beam is available, but it has been shown to be imprecise, since a moving source should not be approximated by a stationary pulse.

To solve the problem, it is necessary to have an understanding of the physical meaning of the parameter $z_0$. Now consider Fig. 2. In the case of the real beam, the injected heat has to diffuse along the $x$ axis over a distance $\gamma$ which corresponds in average to a fourth of the diffusion distance during the interaction time $t_i$. This diffusion distance does not exist in the line source model and must, therefore, be represented in some way by the $z_0$ value. Two limiting cases may be envisaged:

- $z_0$ can be considered as being simply equal to the constant $\gamma$:
The correction of the line source model can also be taken as a purely superficial effect, $z_0$ then becomes a function of the depth $z$, giving (see Fig. 2):

$$z_0 = \gamma = \frac{1}{4} \left(2 \sqrt{h \tau_l} - \sqrt{h \tau_l/2} \right)$$  \hspace{1cm} (11)$$

Substituting eq. 12 in eq. 10 gives:

$$z = \sqrt{\frac{r^2 + r^4 + 4 \left(\frac{a \cdot \rho}{\pi \cdot V \cdot c_p \cdot (T_p - T_0)}\right)^2}{2 \sqrt{2}}}$$  \hspace{1cm} (13)$$

The actual value must lie somewhere between the two models. As they are not very different (Fig. 3) one can simply take the mean value of eq. 10 (with $z_0$ given by eq. 11) and eq. 13. The final solution is then:

$$z = \left(\sqrt{A} - \gamma + \sqrt{A} - \gamma^2\right)/2$$  \hspace{1cm} (14)$$

with $A = \frac{r^2 + r^4 + 4 \left(\frac{a \cdot \rho}{\pi \cdot V \cdot c_p \cdot (T_p - T_0)}\right)^2}{2 \sqrt{2}}$ and $\gamma = \sqrt{h \tau_l/2}$

Setting $z = 0$ in eq. 14 yields the surface peak temperature:

$$T_S = T_0 + \frac{a \cdot \rho}{\pi \cdot V \cdot c_p \sqrt{\sqrt{4} + r^2 \gamma^2/\sqrt{2}}}$$  \hspace{1cm} (15)$$

If the surface temperature is known, the absorption coefficient is given by

$$a = \frac{\sqrt{2} \cdot \pi \cdot V \cdot c_p \cdot \sqrt{4} + r^2 \gamma^2/\sqrt{2}}{p} (T_S - T_0)$$  \hspace{1cm} (16)$$

Simultaneously measuring the surface temperature and the absorption coefficient allows to check the value of the constant $\gamma$:

$$\gamma = \sqrt{\frac{- r^2 + r^4 + 4 \left(\frac{a \cdot \rho}{\pi \cdot V \cdot c_p \cdot (T_S - T_0)}\right)^2}{2 \sqrt{2}}}$$  \hspace{1cm} (17)$$

As can be seen, eqs. 14 and 16 yields the depth of an isotherm using only the measured surface temperature, whereas eqs. 14 and 17 require $T_S$ and the measured absorption to calculate the depth.
The latent heat correction

During remelting treatments, a part of the energy is consumed as latent heat through formation of the molten pool. Although this energy is later released, it is temporarily not available to heat the material. Ashby and Easterling have proposed to take account of this by replacing the beam power P with an apparent beam power P*. Their estimation of the amount of this correction and of the volume melted per second was, unfortunately, not very good.

Let ε be defined as the ratio P*/P. If V_f is the volume melted per second and L the volume latent heat of the material, one can write:

$$\varepsilon = \frac{P^*}{P} = \frac{a \cdot P}{a \cdot P} - \frac{V_f \cdot L}{a \cdot P}$$  \hspace{1cm} (18)

To avoid an interactive process in solving eq. 18, it is proposed that the volume melted per second is nearly proportional to the absorbed power. The value of ε is then directly given by:

$$\varepsilon = \frac{1}{1 + \frac{V_f \cdot L}{a \cdot P}}$$  \hspace{1cm} (19)

where V_f = V_f/ε is the value of V_f calculated without the latent heat correction. One still has to calculate the value of V_f*, which can be written:

$$V_f^* = f \cdot \text{depth}^* \cdot \text{width}^* \cdot V$$  \hspace{1cm} (20)

where f is a geometrical factor whose value is always very close to 0.8, and the superscript * indicates that the corresponding values are calculated before the latent heat correction.

The width of the molten zone can be calculated following the same procedure as that used for the depth. The time t* corresponding to dT/dt = 0 on the surface (z = 0) is given by a third degree equation which can be solved analytically, but less easily than eq. 3, for the depth. In addition, this gives the peak temperature as a function of the width, however it was not possible to find an approximation which would allow rearrangement of this equation.

The width, however, does not need to be known precisely, as the latent heat factor represents only a small correction (a maximum of about 10%). One can easily see that, if there is no keyhole effect, the half-width of the molten zone should lie
somewhere between its depth and the sum of its depth and the beam radius. One can then write as a first approximation:

\[
\text{width} = 2 \cdot (\text{depth} + r \cdot e^{-\psi \cdot r/\text{depth}})
\]

(21)

where \(\psi\) is an adjustment constant which can be shown to be approximately 0.4. From the value of the depth calculated without the latent heat correction, eqs. 19 to 21 allow calculation of \(\varepsilon\). Replacing \(P\) by \(\varepsilon P\) in eq. 14 then gives the corrected value of the depth, without any further iteration. If the absorption coefficient is calculated from the measurement of the surface peak temperature, the value of \(\varepsilon\) has to be recalculated in order to take account of the new value obtained. This is:

\[
\varepsilon = \frac{1}{T_m - T_0 + \frac{T_S}{T_C - T_0} \frac{V^\circ f/a^0 \cdot P}{(T_m - T_0)}}
\]

(22)

where \(T_m\) and \(T_C\) are, respectively, the surface peak temperature measured and calculated from the initial estimation of the absorption \((a^0)\).

### COMPARISON WITH EXPERIMENT

A series of remelting treatments of small X165CrMoV12 steel samples with different scanning velocities and beam diameters have been done, using two different beam powers (1.5 kW CO₂ laser from ROFIN-SINAR providing a stable near top hat mode). The surface peak temperature was measured by optical pyrometry, and the absorption coefficient was obtained by calorimetry. Metallographic observation of transverse sections allowed measurement of the maximal depth of three different isothersms: the temperatures of the liquidus (1400 °C), the solidus (1230 °C) and the Ac1 temperature at a high heating rate (860 °C, measured in separate DTA experiments). Some hardening treatments using a much more defocalised beam were also done in order to test the validity of the mathematical model under completely different conditions. In the latter case, an absorbing graphite coating of about 6 μm thick was applied to the samples. Additionally, remelting treatments of pure Ti samples have been done in order to study the validity of the model for different material constants. In this case, only the \(\alpha/\beta\) transformation isotherm (882°C) was visible on the metallographic sections.

The experimental results are summarized in table 2 and compared with the calculated values, using the physical constants given in Table 1. The experimental depths \((\text{exp})\) are compared with the values calculated from the surface temperature and absorption coefficient measurements given by eqs. 14 and 17 \((a+T_S)\), and only from the surface temperature value given by eqs. 14 and 16 \((T_S)\).

It can be seen that the agreement between experimental and calculated values is very good for all temperatures at each laser setting, even when the absorption coefficient is not used. In the latter case, however, higher precision could be obtained if a more precise evaluation of the constant \(\gamma\) than the simple one given in eq. 11 were used, especially for the treatment of titanium.

### CONCLUSIONS

The model developed here is very simple and can easily be programmed on a pocket computer. It allows estimation of the depth of the molten or heat affected zone with

<table>
<thead>
<tr>
<th></th>
<th>(\rho) [kg/m³]</th>
<th>(C_p) [J/kg K]</th>
<th>(\mathcal{L}) [J/cm³]</th>
<th>(\kappa) [W/m K]</th>
</tr>
</thead>
<tbody>
<tr>
<td>X165CrMoV12</td>
<td>7300</td>
<td>650</td>
<td>1970</td>
<td>28</td>
</tr>
<tr>
<td>Ti</td>
<td>4300</td>
<td>700</td>
<td>1570</td>
<td>22</td>
</tr>
</tbody>
</table>

Table 1 Physical constants of the studied alloys
### Table 2
Comparison between measured (exp) and calculated values (from the absorption and surface temperature 
\(a + Ts\)) for the isotherms corresponding to the liquidus-, solidus- and HAZ- temperature.

<table>
<thead>
<tr>
<th>Mat.</th>
<th>P (W)</th>
<th>(g_{spec}) mm/min</th>
<th>V (cm/min)</th>
<th>Ts (°C)</th>
<th>(Z(T_{liq})) [mm]</th>
<th>(Z(T_{sol})) [mm]</th>
<th>(Z(T_{haz})) [mm]</th>
</tr>
</thead>
<tbody>
<tr>
<td>XI65</td>
<td>835</td>
<td>1</td>
<td>1500</td>
<td>1617</td>
<td>0.15</td>
<td>0.10</td>
<td>0.30</td>
</tr>
<tr>
<td></td>
<td>835</td>
<td>1</td>
<td>2000</td>
<td>1787</td>
<td>0.12</td>
<td>0.10</td>
<td>0.30</td>
</tr>
<tr>
<td></td>
<td>835</td>
<td>1</td>
<td>2000</td>
<td>1790</td>
<td>0.12</td>
<td>0.10</td>
<td>0.30</td>
</tr>
<tr>
<td>X165</td>
<td>835</td>
<td>1</td>
<td>1000</td>
<td>1910</td>
<td>0.16</td>
<td>0.13</td>
<td>0.34</td>
</tr>
<tr>
<td></td>
<td>835</td>
<td>1</td>
<td>1500</td>
<td>1935</td>
<td>0.10</td>
<td>0.13</td>
<td>0.34</td>
</tr>
<tr>
<td></td>
<td>835</td>
<td>2</td>
<td>2000</td>
<td>1620</td>
<td>0.10</td>
<td>0.10</td>
<td>0.30</td>
</tr>
</tbody>
</table>

Very good accuracy for any laser setting used for surface treatment. When the absorption coefficient is unknown, the model allows these depths to be estimated from the surface peak temperature, which can be easily obtained. This characteristic is particularly interesting for "on line" process control, which assumes a given treated thickness, independent of variations in the state of the surface. Further discussion of this model will be presented in a forthcoming paper.

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**References**