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To cite this version:

HAL Id: jpa-00226751
https://hal.archives-ouvertes.fr/jpa-00226751
Submitted on 1 Jan 1987

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ASYMMETRIC SHUBNIKOV-DE HAAS OSCILLATIONS AND HALL PLATEAUS OF HETEROJUNCTIONS IN THE QUANTUM HALL REGIME

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Nous avons observé, sur des heterojuctions GaAs-AlGaAs dopées en Be et en Si, des déplacements importants et de sens opposé de la position des plateaux de la résistance de Hall. Nous présentons un calcul de transport au niveau microscopique, basé sur l'approximation self-consistent de la matrice T, qui explique les déplacements en terme d'interaction electron-impureté non-Bornienne.

We present experimental results on Be-doped and Si-doped GaAs/AlGaAs heterostructures which show strong and opposite shifts of the quantum Hall plateaus. We also present a microscopic transport calculation based on the self-consistent T-matrix approximation, which explains these shifts in terms of strong, non-Bornian scattering of electrons by individual impurities.

The concentration and distribution of impurities determines the low temperature transport properties of a two-dimensional electron gas (2DEG) in a semiconductor. Even the quantum Hall effect (QHE) [1], which is famous for its universality, is strongly influenced by impurities. Although the observed plateau values of the Hall resistance are sample-independent, the very occurrence and the width of the plateaus are attributed to localized electron states in the fluctuating potential due to randomly distributed impurities. Plots of the Hall resistance \( p_{xy} \) versus magnetic field \( B \) show, that not only the width of the plateaus depends on the impurities in the sample, but also their position with respect to the classical free electron result \( p_{xy}^0 = \frac{e}{m^* n_s} \), where \( e \) is the elementary charge and \( n_s \) the area density of the 2DEG.

However, only a few systematic investigations of the effects of the impurity distribution on density of states (DOS) and magnetotransport properties of the 2DEG have been published. For Si-MOSFETs with driftable Na\(^+\) ions in the oxide, the dependence of width and position of the Hall plateaus on the oxide charge was studied [2]. For high mobility GaAl/AlGaAs heterostructures, Haug et al. [3] investigated the effect of a backgate voltage, which changes the mean position of the 2DEG and thus the effective interaction with donors in the AlGaAs and acceptors in the GaAs. In both cases the experimental results could be qualitatively explained by the assumption that attractive impurities lead to an asymmetric broadening of Landau levels (LLs), with localized states on the low-energy side [2], whereas repulsive impurities tend to produce localized states on the high-energy side. The assumption of an asymmetric level broadening is consistent with exact results for the effect of repulsive 6-potentials on the lowest LL [4]. Exact results about localization of these states and a reliable magnetotransport theory including localization effects and covering both the plateau regimes and the dissipative regions between plateaus do, however, not exist.

In the present paper we report on new experimental results for selectively doped GaAs/AlGaAs heterostructures. The drastic and opposite effects obtained for doping with donors and with acceptors, respectively, are explained by a microscopic transport calculation within the self-consistent T-matrix approximation (STMA) [5] which is equivalent with the so-called single-site approximation [6,7]. This calculation neglects coherent multi-center scattering processes, which may lead to localization [8], but treats exactly the scattering by individual impurities.

The investigated samples were prepared from modulation-doped heterostructures grown by molecular beam epitaxy. On top of 2 \( \mu \)m nominally undoped GaAs, a 21 nm...
undoped AlGaAs spacer layer was grown, followed by 40 nm Si-doped AlGaAs. During the growth process a single layer in the GaAs, 2 nm below its interface with the spacer, was selectively doped with either Si which acts as donor, or Be which acts as acceptor. In the following we mean the doping in this layer, if we refer to the Si-doped or Be-doped samples. The highest doping was with an impurity density of \( n_i = 4 \times 10^{10} \) cm\(^{-2}\), leading to a mobility of \( 3 \times 10^5 \) cm\(^2\)/Vs at 4 K, more than an order of magnitude less than the mobility of a reference sample without the layer doping. Hall bridge specimen of 100 µm width were used to measure longitudinal resistivity \( \rho_{xx} \) and Hall resistance \( \rho_{xy} \) for different temperatures down to 1.4 K in a magnetic field perpendicular to the 2DEG. Fig. 1 shows results for a Be-doped sample. The thin straight line indicates the classical result for the Hall effect, \( \rho_{xy}^0 = B/\rho_0 \), for \( n_s = 2.1 \times 10^{11} \) cm\(^{-2}\) as determined from the minima of \( \rho_{xx}(B) \) at even-integer filling factors \( \nu = h n_s/eB > 2 \). We want to emphasize the remarkable result that for large values of the magnetic field, the measured \( \rho_{xy} \) values fall below the classical line so that the plateau with the value \( \rho_{xy} = h/\nu eB = 25,813 \) Ω does not cross the classical line, which assumes this value for \( \nu = 1 \), corresponding to \( B = 8.5 \) T. The filling factor is less than unity in the whole plateau region and becomes \( \nu = 0.83 \) near its center, where \( \rho_{xx} \) for the lowest temperatures has a minimum owing to the resolved spin-splitting of the lowest LL. Fig. 2 shows the corresponding results for a Si-doped sample, where similar shifts to smaller \( B \)-values, i.e., larger filling factors are observed.

Since the observed shifts are different for attractive (Si\(^+\)) and repulsive (Be\(^-\)) impurities and persist at high temperatures where the QHE is no longer observed, a theoretical explanation must certainly go beyond the lowest order Born approximation, but a sophisticated treatment of localization seems not important for a qualitative understanding of the effect. We thus performed model calculations within the STMA. We replaced the Coulomb potentials in a distance \( Z_u \) from the 2DEG by a two-dimensional \( \delta \)-potential with strength \( U_\delta \) adjusted to an appropriate expectation value of the Coulomb potential. For a given distance \( Z_u \), the potentials are assumed randomly distributed with area density \( n_\mu \) and described by scattering amplitudes \([5,9]\)

\[
\begin{align*}
t_\mu(E) &= \left| t_\mu(E) \right| \exp[i\phi_\mu(E)] = (1/U_\mu) (F(E))^{-1} \tag{1} \\
F(E) &= (eBh)^2 \sum_i \left[ E - \epsilon_i - \Sigma(E) \right]^{-1}, \tag{2}
\end{align*}
\]

where \( \epsilon_i = \hbar \omega_c (N + \frac{1}{2}) \) is a Landau energy. The self-energy of the averaged one particle Green's function is

\[
\Sigma(E) = \Delta - \frac{1}{2} \left[ \Gamma(E) = \sum_\mu n_\mu t_\mu(E) \right] \tag{3}
\]

Eqs. (1)-(3) are solved self-consistently for a suitable set of parameters \( (Z_u, n_\mu) \) modelling the impurity distribution. Isolated bands are obtained which lead to a strongly asymmetric broadening of the Landau levels. For \( \delta \)-potentials, the Kubo formulas for the conductivities can be written as \([5]\)

\[
\sigma_{\mu \nu} = \int dE \sigma_{\mu \nu}(E) (-df/dE), \tag{4}
\]

with \( f(E) \) the Fermifunction,

\[
\sigma_{XX}(E) = \frac{e^2}{h} \cdot \frac{1}{2\pi} \cdot \frac{1}{(\hbar \omega_c \Gamma)^2 + 1} \int \frac{dE}{N} \frac{(E-E')n_{\mu}(E')}{(E-E')^2 + \frac{1}{4} \Gamma^2} \tag{5}
\]

and

\[
\sigma_{yx}(E) = -\frac{e^2}{\hbar \omega_c} n(E) - \frac{RI}{\hbar \omega_c} \sigma_{xx}(E) + \delta \sigma_{yx}(E), \tag{6}
\]

where

\[
n(E) = -\frac{1}{\pi} \int_{-\infty}^{E} dE' \text{Im} f(E') \tag{7}
\]

is the integrated density of states. The correction

\[
\delta \sigma_{yx}(E) = \frac{e^2}{\hbar \omega_c} \int \frac{1}{\pi} n_\mu \left[ \phi_\mu(-\infty) - \phi_\mu(E) + \frac{1}{2} \sin 2 \phi_\mu(E) \right] \tag{8}
\]
Fig. 1 Measured longitudinal resistivity $\rho_{xx}$ and Hall resistance $\rho_{xy}$ versus magnetic field for a Be-doped ($n_i = 4 \times 10^{11}$ cm$^{-2}$) sample at three temperatures as indicated. The thin straight line indicates $\rho_{xy} = B/e^2 n_s$ for $n_s = 2.1 \times 10^{11}$ cm$^{-2}$. At $B = 8.5$ T the filling factor is $\nu = 1$.

Fig. 2 As in fig. 1 but for a Si-doped sample with $n_i = 4 \times 10^{10}$ cm$^{-2}$ and $n_s = 3.4 \times 10^{11}$ cm$^{-2}$. At $B = 7$ T the filling factor is $\nu = 2$.

Fig. 3 Calculated conductivities $\sigma_{xx}$ (heavy broken line) and $\sigma_{xy}$ (heavy solid line) in units of $e^2/h$ vs energy in the region of the $N=1$ Landau level for $B=3.8$ T. The thin broken line gives the filling factor $\nu(E)=(h/eB)n(E)$ and the thin solid line the density of states in arbitrary units. A donor-doped sample is considered.

Fig. 4 Calculated resistivity vs magnetic field for two model samples with $n_s = 2.5 \times 10^{11}$ cm$^{-2}$ which differ only by the additional doping with attractive (solid lines) or repulsive (dash-dotted lines) potentials of density $n_i = 2 \times 10^{11}$ cm$^{-2}$. The thin dashed line indicates $\rho_{xy}$. 
to the Drude-type relation between \( o_{xx} \) and \( o_{xy} \) depends on the exact scattering phases and vanishes in the lowest order Born approximation. With \( E-\Delta(E)=e_n \) and \( \Gamma(E)<\hbar \omega_c \), eq. (5) yields for the maxima of \( o_{xx}(E) \) the values \( (e^2/h)(2n+1)/n \). If \( E \) is in a gap between impurity bands and LLs, and satisfies \( e_{N-1} < E < e_N \), then the equations \( \Gamma(E)-o_{xx}(E)=0 \) and \( o_{xx}(E)=N (E)-N e^2/h \) hold exactly [7].

Numerical results for the DOS, \( n(E) \), \( o_{xx}(E) \) and \( o_{xy}(E) \) for one spin in the region of the LL with \( N=1 \) are shown in fig. 3. Doping with donors of density \( n_d=2 \times 10^{10} \text{cm}^{-2} \) produces the impurity band at low energies. The main LL is broadened and shifted below the expected 9.9 meV by remote donors assumed in two layers beyond the spacer, with total density \( 4.5 \times 10^{11} \text{cm}^{-2} \). Residual acceptors of total density \( 1 \times 10^{11} \text{cm}^{-2} \) are assumed in two planes. Those of one plane lead to the small impurity band near 10.3 meV. This merges with the impurity band due to doping, if we dope with acceptors instead of donors and keep all other parameters fixed. In order to compare with the experiments, we have calculated the conductivities for fixed density \( n_d \) as functions of the magnetic field. Furthermore, we have included the enhancement of spin-splitting by many-body effects [10] by the simple interpolation formula [11]

\[
g^*(\mathbf{B}) = g \mu_B B + E_{xc} (n_+ - n_-)/(n_+ + n_-)^2, \quad (9)
\]

for the effective Landé factor \( g^* \), where \( \mu_B \) and \( a_B \) are the effective Bohr magneton and radius, respectively, \( g = 0.4 \) and \( E_{xc} = 12 \text{ meV} \) is a fit parameter. Since the density \( n_+ (n_-) \) of spin up (down) electrons depends on \( g^* \), spin splitting is calculated self-consistently with the DOS for one spin calculated from the STMA.

The two calculations presented in fig. 4 differ only by the sign of the impurity potentials used to describe the intentional doping. For a realistic density of impurities \( (2 \times 10^{10} \text{cm}^{-2}) \) shifts are obtained in qualitative agreement with experiments. The fine structure of the curves is determined by the impurity bands. For the acceptor (donor) doped case, the peak of \( \rho_{xx} \) with maximum at \( B = 5.3 \text{ T} \) \( (4.7 \text{ T}) \) is due to impurity states at the Fermi level. Both spin directions contribute to these peaks, although the spin-splitting of the main LLs is resolved in the calculation. If localization could be taken into account in the transport calculation, one would expect the states in the region of impurity bands to be localized. Then \( \rho_{xx} \) would vanish where the STMA yields the peaks owing to impurity band states. Ignoring details of the impurity band structure and taking the impurity bands as a rough indication for the position of localized states, we can explain the essential features of the experiments. The asymmetric broadening of the LLs due to the strong non-Bornian scattering leads to the observed shifts. Near filling factor \( v=1 \) \( (B=10.3 \text{ T}) \), extended states of one spin direction overlap with "localized" states of the other spin direction. This is the reason why in both cases considered the plateaus do not reach the classical straight line.

We also performed calculations for higher temperatures, where the QHE is no longer observed. In agreement with experiments, we found that the shift of the \( \rho_{xy} \)-versus-\( B \)-curves with respect to the classical result persists. This indicates that an exact treatment of localization is not essential for the understanding of the asymmetries and supports our approach.

References