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MACROSCOPIC DESCRIPTION OF PAIRING

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Abstract:

A classical description of pairing as vibrations and rotations in the number of particles as a function of the gauge angle is outlined and applied to a calculation of the pair transfer by the pair field in a collision between two superfluid nuclei.

I. Surface Vibrations and Rotations

The motivation for taking up the subject of a macroscopic description of pairing is the success of the macroscopic or classical description of the excitation of surface vibrations in heavy ion collisions. It is well-known that one may talk about pairing in much the same way [1]. Pairing vibrations and pairing rotations are excited in two-nucleon transfer reactions, and the question is, whether a similar classical description of these phenomena may also be useful. Most of what I am going to say will be as well-known to many of you, who have been working in nuclear structure as it was to Ricardo Broglia who has contributed a great deal to the subject [2].

I shall start out by a short summary of the macroscopic description of surface deformations, where we envisage that the nuclear density ρ can perform vibrations around an equilibrium density $\rho^{(0)}$

$$\rho = \rho^{(0)} + \delta\rho \tag{1}$$

The quantity $\rho^{(0)}$ can be parametrized as

$$\rho^{(0)} = \rho_0/(1 + \exp((r-R)/a)) \tag{2}$$

and surface vibrations can be described by varying the nuclear radius R as function of direction

$$R = R^{(0)}(1 + \Sigma_{\lambda\mu}\, \alpha_{\lambda\mu}\, Y^*_{\lambda\mu}(\hat{r})) \tag{3}$$

The corresponding change in density is

$$\delta\rho(\vec{r}) = R^{(0)}\, \frac{\partial\rho}{\partial R}\, \Sigma_{\lambda\mu}\alpha_{\lambda\mu}\, Y^*_{\lambda\mu}(\hat{r}) \tag{4}$$

The deformation amplitudes $\alpha_{\lambda\mu}$ are the dynamical variables describing the collective modes. A deformation of the density gives rise to a change in the average field

$$U(\vec{r}) = \int V_{12}(r-r')\, \rho^{(0)}(\vec{r})\, d^3r' \tag{5}$$

by the amount

$$\delta U = \int V_{12}\, \delta\rho = R^{(0)} \frac{\partial U}{\partial R} \Sigma_{\lambda\mu}\, \alpha_{\lambda\mu}\, Y_{\lambda\mu}(\hat{r}) \tag{6}$$

For small distortions the density and the field perform selfconsistent harmonic oscillations described by the Hamiltonian

$$H_0 = \Sigma_{\lambda\mu} \frac{1}{2D_\lambda} |\pi_{\lambda\mu}|^2 + \frac{1}{2} C_\lambda |\alpha_{\lambda\mu}|^2 \tag{7}$$

where $\pi_{\lambda\mu}$ is the momentum to conjugate to $\alpha_{\lambda\mu}$

The Hamiltonian (7) leads to a harmonic spectrum of the collective modes. The matrix elements of $\alpha_{\lambda\mu}$

$$\langle 1_{\lambda\mu} | \alpha_{\lambda\mu} | 0 \rangle = \alpha_\lambda^{(0)} \tag{8}$$

are, independent of μ , the socalled zero point amplitudes.

In some cases, as we know, even the equilibrium density is deformed and thè dynamical deformation parameters are then more appropriately given by the intrinsic deformations $\alpha'_{\lambda\mu}$ and the orientation angles θ_i

$$\alpha_{\lambda\mu} = \Sigma_{\mu'} \alpha'_{\lambda\mu'} D^{\lambda}_{\mu'\mu}(\theta_i) \tag{9}$$

The Hamiltonian is then

$$H_0 = \frac{\vec{L}^2}{2J} + \frac{\pi'^2}{2D} + \frac{1}{2} C \alpha'^2 \tag{10}$$

where the angular momentum components L are the variables conjugate to the orientation angles θ_i

In a heavy ion collision an external field from the projectile, a , interacts with the surface of the target, A . The interaction energy may be written

$$\begin{aligned} H_{int} &= \int \rho_a(\vec{r}) V(\vec{r}-\vec{r}') \rho_A(\vec{r}') d^3r d^3r' \\ &= \int \rho_a^{(0)} V_{12} \rho_A^{(0)} + \int \rho_a^{(0)} V_{12} \delta\rho_A + \int \delta\rho_a V_{12} \rho_A^{(0)} + \int \delta\rho_a V_{12} \delta\rho_A \end{aligned} \tag{11}$$

The first term indicates the diagonal part of H_{int}

$$\langle 0 | H_{int} | 0 \rangle = \int \rho_a^{(0)} V_{12}\, \rho_A^{(0)} = U_{aA} \tag{12}$$

which is the folding potential for the ion-ion interaction. The second term give rise to target excitations i.e.

$$\begin{aligned} \langle 1^{(A)}_{\lambda\mu} | H_{int} | 0 \rangle &= \int \rho_a^{(0)} V_{12} \langle 1^{(A)}_{\lambda\mu} | \delta\rho_A | 0 \rangle \\ &\approx R_A^{(0)} \frac{\partial}{\partial R_A} U_{nA}\, \alpha_\lambda^{(0)}(A)\, Y^*_{\lambda\mu}(\hat{R}) \end{aligned} \tag{13}$$

The third term similarly gives rise to excitations of the projectile while the last term describes the mutual excitation where the deformed field of the target excites the projectile and target at the same time.

II. Non Local Densities

The above derivations are not quite correct because we know that the antisymmetrization implies that we should include the exchange interaction, i.e.

$$H_{int} = \int \rho_a(\vec{r}_1\vec{r}_1') \; U_A(\vec{r}_1\vec{r}_1')d^3r_1 d^3r_1' - \int \rho_a(\vec{r}_1\vec{r}_1') \; U_A^{ex}(\vec{r}_1\vec{r}_1')d^3r_1 d^3r_1' \tag{14}$$

where we defined the nonlocal density

$$\rho(\vec{r}\vec{r}') = \langle\psi|a^+(\vec{r}') \; a(\vec{r})|\psi\rangle \tag{15}$$

and the mean field

$$U_A(\vec{r}_1\vec{r}_1') = \int V_{12}(\vec{r}_1\vec{r}_2\vec{r}_1'\vec{r}_2') \; \rho_A(\vec{r}_2\vec{r}_2')d^3r_2 d^3r_2' \tag{16}$$

while

$$U_A^{ex}(\vec{r}_1\vec{r}_1') = \int V_{12}(\vec{r}_1\vec{r}_2\vec{r}_2'\vec{r}_1')\rho_A(\vec{r}_2\vec{r}_2') \; d^3r_2 d^3r_2' \tag{17}$$

is the exchange field.

For a local interaction

$$V_{12}(\vec{r}_1\vec{r}_2\vec{r}_1'\vec{r}_2') = \delta(\vec{r}_1-\vec{r}_1') \; \delta(\vec{r}_2-\vec{r}_2') \; V_{12}(\vec{r}_1-\vec{r}_2) \tag{18}$$

we find that U_A is local

$$U_A(\vec{r}_1\vec{r}_1') = \delta(\vec{r}_1-\vec{r}_1') \int V_{12}(\vec{r}_1-\vec{r}_2) \; \rho_A(\vec{r}_2\vec{r}_2)d^3r_2 \tag{19}$$

and only depends on the local density $\rho(r) = \rho(r,r)$. On the other hand the exchange potential

$$U_A^{ex}(\vec{r}_1\vec{r}_1') = V_{12}(\vec{r}_1-\vec{r}_1') \; \rho_A(\vec{r}_1\vec{r}_1') \tag{20}$$

is essentiallly nonlocal even for local interactions.

The nonlocal density matrix $\rho(r_1r_1')$ also has a classical interpretation. It carries in fact information about the single particle momentum distribution. This can be seen by performing a Wigner transformation

$$\bar{\rho}(\vec{p},\vec{q}) = (2\pi\hbar)^{-3}\int \rho(\vec{q}+\vec{\xi}/2, \; \vec{q}-\vec{\xi}/2)e^{-i\vec{p}\cdot\vec{\xi}/\hbar} \; d^3\xi \tag{21}$$

where we introduced

$$\vec{q} = (\vec{r} + \vec{r}')½ \quad \text{and} \quad \vec{\xi} = \vec{r} - \vec{r}' \tag{22}$$

The quantity $\bar{\rho}(\vec{p},\vec{q})$ is equivalent to the classical single particle distribution in phase space, and the local density that we talked about in the previous section is given by the classical expression

$$\rho(\vec{q}) = \rho(\vec{q},\vec{q}) = \int \bar{\rho}(\vec{p},\vec{q}) \; d^3p \tag{23}$$

in terms of $\bar{\rho}(\vec{p},\vec{q})$.

Another quantal feature is the spin. In fact the full density matrix (15) should depend also on the spin coordinates. We shall in the following only consider the component

$$\rho_0(\vec{r},\vec{r}') = 1/2 \; (a_+^+(\vec{r}') \; a_+(\vec{r}) + a_-^+(\vec{r}') \; a_-(\vec{r})) \tag{24}$$

where the subindex indicates the value of σ .

The idea of mean field theories is to write the interaction in the form

$$
\begin{aligned}
&\langle \int V_{12}(1234)\, a^{+}(1)\, a^{+}(2)\, a(4)\, a(3)\rangle \\
&\approx \int \langle a^{+}(1)\, a(3)\rangle\, V_{12}(1234) \langle a^{+}(2)\, a(4)\rangle \qquad \text{(Hartree)} \\
&- \int \langle a^{+}(1)\, a(4)\rangle\, V_{12}(1234) \langle a^{+}(2)\, a(3)\rangle \qquad \text{(Fock)} \\
&+ 1/2 \int \langle a^{+}(1)\, a^{+}(2)\rangle\, V_{12}(1234) \langle a(4)\, a(3)\rangle \\
&+ 1/2 \int \langle a(4)\, a(3)\rangle\, V_{12}(1234) \langle a^{+}(1)\, a^{+}(2)\rangle \qquad \text{(Bogoljubov)}
\end{aligned}
\tag{25}
$$

$$
= \int \rho(11')\, U(1'1) - \int \rho(11') U^{ex}(1'1) + 1/2 \int (\rho_2(11')\Delta_{-2}(1'1) + \rho_{-2}(11')\Delta_{+2}(1'1)).
$$

When we make variations of the density we should therefore vary the nonlocal equilibrium density and we should include variations that imply components of the type of $\rho_{\pm 2}$. In varying the nonlocal density and in taking pairing into account one should in principle also include terms proportional to the velocity of the amplitudes. We shall neglect such variations in the following.

III. Pair Densities

The quantities ρ_2 and ρ_{-2} in (25) are generalized single particle densities defined by

$$
\rho_2(\vec{r}\vec{r}') = a_{+}^{+}(\vec{r}')\, a_{-}(\vec{r}) \quad , \quad \rho_{-2}(\vec{r}\vec{r}') = a_{-}(\vec{r}')\, a_{+}(\vec{r}) \tag{26}
$$

while the mean field associated with these quantities are the pair fields

$$
\Delta_{\pm 2}(\vec{r}\vec{r}') = \int V_{12}\, \hat{\rho}_{\pm 2} = V_{12}(\vec{r}-\vec{r}')\, \hat{\rho}_{\pm 2}(\vec{r},\vec{r}') \tag{27}
$$

The latter equation holds for a local two body interaction.

A formal connection between the traditional densities and the pair densities is obtained by defining the pair moments

$$
M_{\pm 2} = \int \rho_{\pm 2}(\vec{r},\vec{r})\, d^3r
$$

and (28)

$$
M_0 = \int \rho_0(\vec{r},\vec{r})\, d^3r = \hat{N}
$$

The latter being the total particle number operator.

The three quantities (28) have properties in common with the spherical components of the total angular momentum. Defining $M_{\pm 2} = M_x \pm i M_y$, $M_0 = M_z$ we find

$$
[M_x, M_y] = i\, M_z \ , \text{ etc.} \tag{29}
$$

Like the total angular momentum they generate rotations among the components of ρ defined by

$$
\rho_{\pm 2}(\vec{r},\vec{r}') = \rho_x \pm i\, \rho_y \quad \text{and} \quad \rho_0 = \rho_z \tag{30}
$$

as e.g.

$$
[\rho_0(\vec{r},\vec{r}') \ , \ M_{+2}] = \rho_{+2}(\vec{r},\vec{r}') \tag{31}
$$

The space in which the rotations take place is called quasispin-space. By rotating ρ_0 through a finite angle around the y-axis one obtains mixtures of ρ_0 and ρ_x and the result is equivalent to a

Bogoliubov-Valatin transformation. Rotations around the z-axis leaves the Hamiltonian invariant and the corresponding conserved quantity is M_0 i.e. the total number of particles. This particular transformation is called a gauge transformation and the angle of rotation the gauge angle.

The relations between ρ_0 and $\rho_{\pm 2}$ emphasizes that $\rho_{\pm 2}$ are single particle densities although the quantity

$$\langle A+2 | \rho_{+2}(rr') | A \rangle = \psi(r,r') \tag{32}$$

is related to two particles. If the state $|A+2\rangle$ of the nucleus with $A+2$ particles were described by a Tamm-Dancoff approximation it would in fact be the two particle wavefunction. In the random phase approximation (RPA) it is not.

It is wellknown that the quantity $\langle A\pm 2 | M_{\pm 2} | A \rangle$ measures the strength of two particle transfer reactions on nucleus $|A\rangle$. If, as often is the case, most of this cross section is concentrated on the ground states of the neighbouring nuclei $|A\pm 2\rangle$ we may from the commutation relation (32) conclude that

$$\begin{aligned} &\langle A+2 | [\hat{\rho}_0, M_2] | A \rangle \\ &\approx \langle \hat{A}+2 | M_2 | A \rangle \{ \langle A+2 | \rho_0(rr') | A+2 \rangle - \langle A | \rho_0(rr') | A \rangle \} \end{aligned} \tag{33}$$

i.e.

$$\langle A+2 | \rho_2(rr') | A \rangle \approx \langle \rho(rr') \rangle_{A+2} - \langle \rho(rr') \rangle_A$$

IV. Pairing Vibrations

The relation (33) gives the basis for describing a pairing vibration quite analogously to surface vibrations. We thus assume that in a pairing vibration there is a deformation in particle number

$$A = A^{(0)} + \sum_\alpha \alpha_\alpha \, e^{-i\frac{\alpha}{2}\phi} \tag{34}$$

where $\alpha = \pm 2$, $A^{(0)}$ being the equilibrium particle number. We then find

$$\delta\rho = \frac{\partial \rho(\vec{r}\vec{r}')}{\partial A} \sum_\alpha \alpha_\alpha \, e^{-i\alpha\phi/2} \tag{35}$$

In order to estimate the derivative of the nonlocal density we may use the Thomas Fermi approximation for the density in phase space

$$\bar{\rho}(\vec{p},\vec{q}) = (2\pi\hbar)^{-3} \, \Theta(p_F - p) \tag{36}$$

and the local density

$$\rho(\vec{q}) = \int \bar{\rho}(\vec{p}\vec{q}) \, d^3p = k_F^3/(3\pi^2) \tag{37}$$

where $k_F = p_F/\hbar$ is the Fermi momentum. The derivative of ρ is

$$\begin{aligned} \frac{\partial \bar{\rho}(\vec{p}\vec{q})}{\partial A} &= (2\pi\hbar)^{-3} \, \delta(p_F - p) \, \frac{\partial p_F}{\partial A} \\ &= \frac{\delta(p_F - p)}{4\pi \, p_F^2} \, \frac{\partial \rho(\vec{q})}{\partial A} \end{aligned} \tag{38}$$

Using the Wigner transformation this implies that the nonlocal pair density is

$$\delta\rho(\vec{r}\vec{r}') = \frac{\partial\rho(q)}{\partial A} \cdot \frac{\sin k_F(q)|\vec{r}-\vec{r}'|}{k_F(q)|\vec{r}-\vec{r}'|} \sum_{\alpha} \alpha_{\alpha} e^{-i\alpha\phi/2} \qquad (39)$$

with $q = (\vec{r}+\vec{r}')/2$.

In comparing with (30) and noticing that

$$\int \frac{\partial\rho(q)}{\partial A} d^3q = \frac{\partial A}{\partial A} = 1 \qquad (40)$$

we see that the amplitudes $\alpha_{\pm 2}$ of the oscillation can be identified with the pair moment

$$M_{\pm 2} = \alpha_{\pm 2} \qquad (41)$$

In Fig. 1 the result (39) is compared to the transition density connecting the ground state of ^{210}Pb with ^{208}Pb as calculated in the random phase approximation. In fact the expression (39) gives a rather accurate parametrization of the RPA result (cf.ref. [2]).

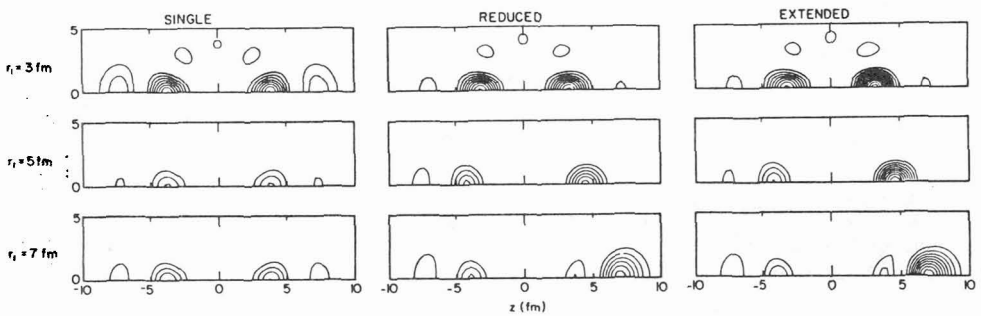

Fig. 1. Contour levels of the square of the two-particle transition density $\rho_2(r_1,r_2)$ in the x-z phase fixing r_1 along the z-axis. The first, second and third columns correspond to the single, reduced and extended configurations space, respectively. The first, second and third row correspond to r_1 = 3, 5 and 7 fm, respectively. The contours indicate the values of the function in integral units starting from 1 x 10^{-6} *Harmonic oscillator wave functions were used.*

We can write down the Hamiltonian for the pairing vibrations in the harmonic approximation and find

$$H_0 = \frac{1}{D} \pi_{-2}\pi_2 + C\, \alpha_{-2}\alpha_2 + i\omega\, (\alpha_2\pi_2 - \alpha_{-2}\pi_{-2}) \qquad (42)$$

The last term is proportional to the angular momentum around the z-axis in the quasispin space, i.e. the particle number. The occurrence of this term is associated with the ambiguity in the definition of Fermi level, λ . It is in fact equal to $2\lambda N$ where N is the number of pairs.

In a collision between two heavy ions the interaction energy will contain terms

$$\int \delta\rho^{(a)}_{\pm 2}\, V_{12}\, \rho_A + \int \rho_a\, V_{12}\, \rho^{(A)}_{\pm 2}$$

where the matrix elements are zero because of the overall particle conservation. The only non-vanishing term in first order is

$$H_{int} = \int \delta\rho^{(a)}\, V_{12}\, \delta\rho^{A} = \int \delta\rho^{(a)}\, \Delta^{(A)} = \int \Delta^{(a)}\, \delta\rho^{(A)} \qquad (43)$$

which causes mutual excitation in which the pair field of the projectile or of the target excites both nuclei.

With the parametrization (39), H_{int} is readily evaluated to give

$$H_{int} = \sum_{\alpha\alpha'} f(R)\ \alpha^{(a)}_{\alpha'}\ \alpha^{(A)}_{\alpha}\ e^{-i(\alpha+\alpha')\phi/2} \tag{44}$$

where

$$f(R) = \int d^3q\ \frac{\partial\rho^a(\vec{q}+\vec{R})}{\partial A} \cdot \frac{\partial\rho^A(\vec{q})}{\partial A} \int d^3\xi\ \frac{\sin k_F^{(a)}\xi}{k_F^a\xi}\ \frac{\sin k_F^{(A)}\xi}{k_F^A\xi}\ V_{12}(\xi) \tag{45}$$

is a function of the distance R between the two ions. The matrix elements of H_{int} lead to the particle conservation $\alpha = -\alpha'$ and is proportional to the product of the deformation parameters.

In the classical description where the ions move on classical trajectories, f(R) is a given function of time and the classical equations of motion with the Hamiltonian

$$H = H_0^a + H_0^A + H_{int}\ (R(t)) \tag{46}$$

can be solved explicitly.

V. Pairing Rotation

In solving the general mean field equations that follow from (27) one usually finds that the ground state is deformed in ordinary space as well as in quasispin space. This violation of symmetry, i.e. of conservation of angular momentum as well as of particle number is interpreted by assuming that the deformed mean field is the field in an intrinsic frame. In order to have a complete theory we must specify the frequency with which the field rotates around the three axes in space as well as around the z-axis in quasispin space. Such rotations give rise to additional Coriolis terms in the mean field equations and the frequencies of rotation can then be determined selfconsistently by specifying the angular momentum and the particle number, which are the conjugate variables to the orientation angles.

For the deformed pair density we may still use the estimate given above because the deformations are never very large. One should however use the deformation parameters referred to the intrinsic frame, i.e. one should change to the variables $\alpha = \alpha_0 + \alpha'$ and ϕ' where α_0 is the equilibrium deformation, i.e.

$$\alpha_{\pm 2} = (\alpha_0 + \alpha')\ e^{\pm i\phi} \tag{47}$$

With these new variables the Hamiltonian becomes

$$H_0 = \frac{\hbar^2 N'^2}{2D\ \alpha_0'} + 2\lambda N' + \frac{\pi'^2}{4D} + 4C\ \alpha'^2 \tag{48}$$

The number of pairs N times $\hbar$ is the conjugate variable to ϕ' while π' is the conjugate momentum to the vibration amplitude α' around the deformed shape specified by α_0. The first two terms is the rotational part of the Hamiltonian, the last two describe the vibrations around α_0. Neglecting these vibrations the deformed density is according to (35)

$$\begin{aligned}\delta\rho &= \sum_{\alpha} \frac{\partial\rho}{\partial A} \cdot \alpha_0\ e^{i\alpha/2(\phi'-\phi)} \\ &= 2\alpha_0 \frac{\partial\rho}{\partial A} \cos(\phi-\phi')\end{aligned} \tag{49}$$

In a collision between two pairing deformed nuclei the particle conserving total Hamiltonian is according to (45) and (47)

$$H = \frac{\hbar^2 N_A'^2}{2J_A} + 2\lambda_A N_A' + \frac{\hbar^2 N_a'^2}{2J_a} + 2\lambda_a N_a' + f(R)\,\alpha_0^{(0)}\,\alpha_0^{(A)}\cos(\phi_a' - \phi_A') \tag{50}$$

The classical equations of motion are

$$\frac{\partial H}{\partial \phi_a'} = -\hbar \dot{N}_a' \qquad \frac{\partial H}{\partial \phi_A'} = -\hbar \dot{N}_A'$$

$$\frac{\partial H}{\partial N_a'} = \hbar \dot{\phi}_a' \qquad \frac{\partial H}{\partial N_A'} = \hbar \dot{\phi}_A \tag{51}$$

These equations lead to

$$\dot{N}_a' + \dot{N}_A' = 0 \quad \text{i.e.} \quad N_A' + N_a' = \text{constant} \tag{52}$$

and

$$\dot{\phi}_A = \frac{\hbar N_A'}{J_A} + \frac{2\lambda_A}{\hbar}$$

$$\dot{\phi}_a' = \frac{\hbar N_a'}{J_a} + \frac{2\lambda_a}{\hbar} \tag{53}$$

If we can neglect the first term, we find

$$\phi_A' - \phi_a' = \frac{2}{\hbar}(\lambda_A - \lambda_a)t + \delta \tag{54}$$

where δ is an integration constant indicating the relative orientation of the two nuclei at $t = 0$. The equation for N_A is then

$$\dot{N}_A = -\frac{1}{\hbar} f(R(t)) \sin\left(\frac{2}{\hbar}(\lambda_A - \lambda_a)t + \delta\right) \alpha_0^{(a)}\,\alpha_0^{(A)} \tag{55}$$

In the static limit where $f(R)$ is constant this equation describes the Josephson junction between two superconductors. The number of pairs in A oscillates with the frequency $\omega = 2(\lambda_A - \lambda_a)/\hbar$. For a heavy ion collision the number of transferred pairs is

$$\Delta N_A = N_A(\infty) - N_A(-\infty) = -\frac{1}{\hbar}\int_{-\infty}^{\infty} f(R(t))\sin(\omega t + \delta)\,dt\;\alpha_0^{(0)}\alpha_0^{(A)}$$

$$= -\sin\delta\,\frac{1}{\hbar}\int_{-\infty}^{\infty} f(R(t))\cos\omega t\,dt \cdot \alpha_0^{(a)}\alpha_0^{(A)} \tag{56}$$

where in the last equation we assumed a trajectory symmetric in time. For an actual collision the Josephson oscillations only act to reduce ΔN_A

The number of transferred particles (56) depends on the relative orientation δ at $t=0$ such that it is maximum for $\delta = \pi/2$ and 0 for $\delta = 0$ and π. For an isotropic initial distribution of the orientation angle one may calculate the distribution in the number of transferred pairs. It is indicated by a dashed curve in Fig. 2 as a function of $f = \Delta N/\Delta N_{max}$.

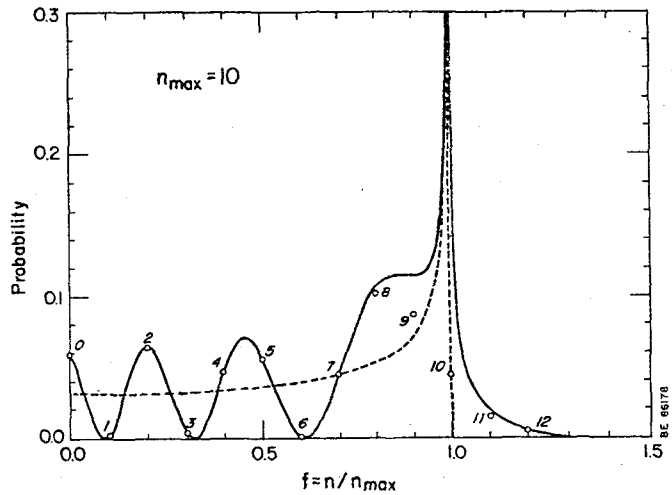

Fig. 2. The probability for transfer of n pairs in a collision with $n_{max} = 10$.

In the same figure is also indicated by circles the results of a quantal calculation (for $\Delta N_{max} = 10$) of the same process, which was made more than 10 years ago by Klaus Dietrich and Hara [3].

The quantal calculations show clear oscillations which were tentatively associated with the Josephson alternating current. The above derivation show however that this can not be the case. The oscillations can be understood however as a quantal interference effect of the same type as has been observed in the corresponding excitation of rotational states of shape-deformed nuclei. There are for any given value of $\Delta N < \Delta N_{max}$ two values of the relative orientation δ that lead to this final result. In order to calculate the probability we must add the amplitudes, i.e.

$$P(\Delta N) = |\sqrt{P_1(\Delta N)}\, e^{iS_1} + \sqrt{P_2(\Delta N)}\, e^{iS_2}|^2 \qquad (57)$$

where S_1 and S_2 are the action integrals for the two initial conditions of $\delta = \delta_1$ and $\delta = \delta_2$, respectively, that lead to the result ΔN. The result of using (57) is given by the full drawn curve in Fig. 2.

VI. Conclusion

In the present lecture I have tried to give a presentation of the broader understanding of pairing that one obtains through the classical macroscopic description. I also indicated some of the more practical results that may be obtained through this exercise. It should be said however that unfortunately the two nucleon transfer is not at all well described by this theory. This is because the transfer of two nucleons is always dominated by the second order process where each nucleon is transferred to the target A by the normal mean field U_A. Still, the correlation in space between the two nucleons give an enhancement of the successive transfer that is essentially proportional to the zero point amplitudes $\propto$. Carlos Dasso, Pollarolo, Vitturi and others have recently tried to introduce effective two nucleon transfer potentials which retain some of the simplicity of the macroscopic theory [4].

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