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HAL Id: jpa-00226486
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Submitted on 1 Jan 1987

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MULTI-FRAGMENTATION DYNAMICS OF A SATURATING SYSTEM OF FERMIONS

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Résumé - L’ évolution d’ un système des Fermions, largement excité et sa fragmentation en plusieurs fragments est étudié dans une dynamique à un corp.

Abstract - The evolution of a highly excited system of Fermions and its fragmentation into many fragments is studied in a quantum one-body evolution dynamics.

1. Introduction

Two nuclei with a relative velocity close to the speed of light collide. On a time scale of about 10 fm/c a highly excited and compressed nuclear system is formed at a considerably high entropy. How does such a system expand and finally disassemble when passing through a regime of dynamical instabilities? While earlier disregarded, the interest in this question has grown recently. Quite a variety of models have been developed to discuss this question. Most of them, however, by-pass the dynamical evolution of the process and discuss the fragment formation in terms of a quasi-static concept: the freeze-out. These models range from the simple picture of coalescence in phase-space [1], percolation models [2], the thermo-chemical equilibrium among nuclei [3] or nuclear droplets (condensation model [4]) up to the discussion of nuclear matter at strained densities considering the phase coexistence of a nuclear fluid and a vapor phase [5]. Only few attempts have been undertaken to study the implications of the dynamics of the fragmentation process. They range from a quantum one-body evolution picture [6], over a description by means of a classical Vlasov equation [7], a Vlasov equation supplemented with collision term [8], up to an entirely classical model in the frame work of a molecular dynamics approach [9]. We like to report upon the first kind of model, the quantum one-body picture, which was actually the first one discussing these questions.

2. One-body Evolution at Finite Entropies

It was important to realize that mean-field approaches in the sense of treating the whole statistical ensemble by one mean field are principally not capable to treat the phenomenon of multifragmentation. Rather, the individual fluctuations of a certain state out of the statistical ensemble together with the instabilities implied by the saturating nature of the nuclear forces are the decisive factor for fragmentation.
As nuclear binding is of genuine wave mechanical nature (the Pauli-pressure determines the nuclear sizes, not the range of the forces) we consider a quasi-quantum mechanical model. It comprises three ingredients: a) the definition of the initial configuration at the onset of the expansion phase, b) the dynamics that cranks this configuration through the instability region, and c) a definition of the final stable fragments:

a: We assume that up to a certain time \( t = 0 \) which may be the moment of highest compression the reaction can be described by a macrodynamical approach like the cascade or the hydrodynamical models. This may provide sufficient information as to determine the initial state of the expansion phase, e.g. in form of the one-body density matrix \( \rho(r,p) \) at time \( t = 0 \). In line with the high entropy at this time we represent the system by a statistical ensemble of pure states (Slater determinants), such that the ensemble average reproduces \( \rho^0 \).

b: Given a Skyrme type of interaction with proper saturation properties each of these pure states (here to after called event) is then propagated by self-consistent one-body dynamics. The one-body field which governs the evolution of the event is deduced from the time-dependent one-body density of this Slater determinant wave function. In this way the dynamics has the knowledge on its proper one-body density and is capable to form fragments which ultimately remain stable due to the nonlinearity of the dynamical equations.

c: A fragment is defined as the connected area in space where the density exceeds a certain threshold value (at present taken as 0.1 of the saturation value \( \rho^0 \)).

So far the study is confined to a \( 2+1 \) dimensional model world (two space and one time dimension). In earlier presentations \([6]\) anti-symmetrization has been neglected, just considering a system of Boltzmann - Schrödinger particles. In this contribution we like to present the first results which properly account for the Fermion nature of nucleons. We calculated the evolution of a mass 40 system (40 single particle wave functions per event) on a physical theatre of 60 fm by 60 fm size. No additional symmetry is used. A zero range two- and three-body force (Skyrme - type) governs the dynamics. Note that these forces render the exchange term identical to the direct one, such that any spin-isospin component interacts only with the other components via a local force. Therefore the potential part of the energy density functional reads

\[
W(r) = t_0 \sum_{i \neq k} \rho_i(r) \rho_k(r) + t_3 \sum_{i \neq k \neq l \neq i} \rho_i(r) \rho_k(r) \rho_l(r) \tag{2.1}
\]

where \( i, k \) and \( l \) lable the four different spin-isospin components. For spin-isospin symmetric nuclear matter it reduces to the well known expression

\[
W(r) = \frac{3}{8} t_0 \rho(r)^2 + \frac{1}{16} t_3 \rho(r)^3. \tag{2.2}
\]

The two parameters \( t_0 \) and \( t_3 \) are chosen such that saturation is achieved at a Fermimomentum of \( k = 1.2 / \text{fm} \) (with the saturation density \( \rho^0 = k^2 \pi / 6 \)) at a binding energy of 16 MeV. Note, that this force together with the Pauli principle implies a symmetry energy favouring symmetric nuclear matter. The inclusion of a repulsive Coulomb force among the protons is in progress \([10]\).

For the present study we bypass the dynamics of the compression phase and and start with the description of the system at the onset of decompression. We assume that the stochastic properties of the system at this instant may be just be described by a few macroscopic properties expressed through its one-body Wigner density \( f^0(r,p) \). The essential point is to construct a representive ensemble of pure states which is in line with the one-body distribution \( f^0 \) thereby exploiting all possible fluctuations permitted by the statistics of the particles. While for Boltzmann-Schrödinger particles practically no constraints are imposed on \( f^0 \) (except for the uncertainty relation) and the construction of a representive set of states for the statistical ensemble is quite straight forward, the Pauli correlations impose quite some complication. Mere anti-symmetrisation of a constructed
product wave function, e.g. product of Gaussian single particle wave functions as used in refs. [6,7] strongly alters the original one-body properties of such a state. This was shown in ref. [7]. To overcome such difficulties we have restricted $f_0$ to such distributions which in the classical limit respect the Fermi-Dirac statistic, i.e.

$$f_0(r,p) = 1 / \{ 1 + \exp\left( (\epsilon(r,p) - \mu) / T \right) \},$$

(2.3)

where $\epsilon(r,p)$ is a judiciously chosen single particle energy. We employed those of the Harmonic oscillator varying the spring constant such that one achieves a desired compression relative to the saturation density $\rho_0$ at a given value of the temperature $T$. According to the distribution (2.3) we selected for each nucleon a classical coordinate $r_i$ and a classical momentum $p_i$. This is done for all $A$ nucleons to construct one pure $A$-body state (event). The single particle wave functions are taken as Gaussian wave functions with $r_i$ and $p_i$ as centroids in position and momenta (boost). The $A$-body wave function is then constructed as the anti-symmetrized product of the $A$ Gaussian wave functions. We realized that in this way the anti-symmetrization alters the one-body properties (like the mean square radius and momentum) of the event by not more than 20%. In order to be even further free from such modifications we took the above procedure at different temperatures in the range of 5 to 12 MeV and classified all the so generated events according to their excitation energy relative to the Hartree-Fock ground state energy.

A few comments are in order: While the above procedure to construct a representative ensemble of initial states works nicely at high excitation energies, difficulties appeared to construct states below an excitation energy of 8 MeV per nucleon. As compared to ordinary mean field theories at finite temperature which treat the whole statistical ensemble in a single common (i.e. ensemble averaged) mean field, here each individual state evolves in its proper one-body field. Thus we account for fluctuations in the one-body field, which in turn generate the various ways the system can granulate. Our picture implies that the evolution for $t \geq 0$ is isentropic.

![Multifragmentation dynamics of a hot and compressed nuclear system from model calculations in a configuration space of two dimensions.](image)

Fig. 1. Multifragmentation dynamics of a hot and compressed nuclear system from model calculations in a configuration space of two dimensions. The figure displays the time evolution of the density $p(r,t)$ of a certain event at four different time steps. The perspective plot shows vertically the density which initially exceeds the saturation value by a factor 2.5, at a temperature of $T = 50$ MeV.

3. Time Development of a Hot and Compressed Mass 40-System

Let us first consider the development of one pure state in time as displayed by fig. 1. The sequence shows the density distribution $p(r,t)$ at four different time steps $t=0, 24, 48,$
72 fm/c, for an initial value of $T=50$ MeV. Due to the internal stress caused by the large momentum spread the event immediately expands and overstresses around $t=24$ fm/c, i.e. the density falls below $p^0$ throughout. One recognizes density depressions (bubbles) in the interior and some areas of higher density at this time initiating the disassembly of the system. At a later stage these density islands are capable to recover back to saturation density sucking in the matter of their local environment. It turns out that all islands which have at least a mass of two nucleons remain stable and therefore form ultimately stable soliton-like fragments. Those wave components which do not stabilize but rather disperse with a density dropping below our cluster identification cut-off are registered as evaporated nucleons. A major part of them leaves the interaction zone at very early times. Altogether, however, the whole performance cannot be regarded as a surface evaporation process. It is a boiling dynamics, with bubble formation in the interior while first fragments fly away from the surface.

4. Mass Spectra and Multiplicity Distributions

Let us now come to the results of a representative ensemble of events characterized by a given excitation energy, fig.2. All events have a central density of about 1.5 times $p^0$. One realizes that the shapes of the resulting mass spectra change quite significantly in the displayed range of excitation energies. While at $E^*/A = 10$MeV always only a few light particles (below a mass of 5) are emitted leaving a heavy residue close to the total mass of 40, the mass spectrum becomes very broad at $E^*/A = 12$MeV. There, all masses appear with similar probabilities and the final configurations consist of about 3 fragments with masses larger than 2 units. With even higher excitation energies the mass spectrum becomes increasingly steeper, the multiplicity of fragments grows.

It is interesting to remark that a similar sequence of mass spectra is obtained if one looks into the distribution of cluster sizes as a function of time, fig. 3. At early times there is always a big connected cluster. Then light particles (essentially with a mass below 2), namely the fastest ones are evaporated. Later one comes into the stage of multifragment-
tation where about three fragments coexist. If the excitation energy is still sufficient these ones will further fragments enriching the content of lighter fragments. Depending on the total excitation energy this performance proceeds on a different time scale and ceases at a correspondingly evolved situation. The whole picture seems to resemble a kind of Markovian decay chain where excited prefragments evaporate light particles and multi-fragment into new prefragments of less excitation energy until their excitation energy has dropped so much as to stop the process. This is a picture studied phenomenologically in ref. [11].

Relative to the earlier calculations without anti-symmetrization [6] one obtains the onset of multifragmentation at lower excitation energies. The reason is that for the same force a system of Fermions is less bound than the corresponding Boltzmann system due to the additional Pauli pressure. The inclusion of the Coulomb repulsion will even further lower the corresponding excitation energy.

\[ \frac{E}{A} = 15.171 \text{ MeV} \]
\[ \frac{P}{A} = 11.13 \text{ MeV} \]
\[ t = 96 \text{ fm/c} \]
\[ t = 144 \text{ fm/c} \]

Fig. 3. Distribution of cluster sizes as a function of time \( t \) for two excitation energies.

5. The Role of Initial Fluctuations

The picture employed is such that given the initial configuration the final outcome of the fragmentation dynamics is determined deterministically by solving the equations of motion. To this extent the description preserves the entropy, certainly a limitation of the model as it omits possible collision terms. In turn, however, given this model, one can ask the question which pieces of information contained in the initial state actually determine the fragmentation. In our case the initial state is entirely given by the initial positions and momenta (centroids of the corresponding Wigner distribution) of all the single-particle wave functions at \( t=0 \). For this purpose we neglected anti-symmetrization and performed the following analysis: during the evolution each wave function fractions and goes with a certain probability into any of the fragments or into the back ground which does not belong to any of the fragments. However, it turns out that more than 50% of its probability goes into only one fragment or into the back ground density. We then looked into the
distribution of all classical initial positions of those wave functions which go essentially into one fragment, and likewise we did for the initial momenta. As a result of this analysis we stated a strong correlation among the distribution of the initial momenta, fig. 4, and the final fragmentation, while such a correlation could not be established for the initial positions. Evidently a coalescence or percolation picture employed in momentum space may be a much better phenomenological approach to describe the dynamics than employed in coordinate space or actually the product space of both. Yet, a definite answer cannot yet be given as we may not have performed the analysis at a time which one likes to call the moment of freeze-out. This still has to be established. In a way it is a work similar to that of analysing an experiment, and a lot of experience is still needed to come behind the secrets of multifragmentation.

![The Correlation Between Momenta and Fragmentation](image)

**Fig. 4.** Distribution of the initial momenta of the single particle wave functions of one event and their assignment to the different final fragments. The + denote those wave functions which essentially go into the background, i.e. into no stable fragment. Else the initial momenta of wave functions going into the same fragment are plotted by the same symbol.

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### 6. Conclusions

Within a time-dependent field theoretical model we have shown that a series of numerical experiments with random initial conditions corresponding to hot nuclear systems leads to final stable fragments in a quite natural way. The presented study accounted correctly for the Fermionic nature of the nucleons. As a main conclusion we realized that the transition from an evaporation picture, over binary fragmentation to multifragmentation occurs in a very limited range of excitation energies; in our two-dimensional study around 12 MeV/nucl. We expect that the so far not considered Coulomb interaction will further reduce this energy.
Certainly there are a lot of open questions relative to the presented approach. One concerns the neglect of the residual interactions in our approach. There are physical arguments in favour that they may not be that important once one considers a statistical ensemble, which exploits all the possible fluctuations the system can take. Second concerns the definition of the initial state. Here a coupling to another dynamical theory for the initial entropy generating phase is certainly desirable. The third concerns the generalisation to a genuine 3+1 dimensional model.

In conclusion, we presented a full scale micro-dynamical model which opens a challenging perspective of studying many of the pending questions on micro-dynamical instabilities. Furthermore we expect some insight from the comparison of this model study with quasi-static, i.e. freeze-out consideration to the same dynamical system. The correlation analysis presented here is one step in this direction. A detailed account of this work will be given elsewhere [10].

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