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CLASSICAL ANALOGUES OF THE RPA OPERATORS AND THE VLASOV EQUATION

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Abstract: The linearized Vlasov equation for finite nuclei is discussed.

The nuclear Vlasov equation describes the time evolution of the classical distribution function \( f(x,p,t) \) for a system of fermions interacting through short range forces. The generator \( h(x,p,t) \) of the time displacement is itself a functional of the distribution function. Assuming two-body and three-body forces, we have

\[
\begin{align*}
\frac{\partial f}{\partial t} + \frac{\partial f}{\partial x} \cdot \frac{\partial h}{\partial p} - \frac{\partial f}{\partial p} \cdot \frac{\partial h}{\partial x} &= 0, \\
\end{align*}
\]

where

\[
\begin{align*}
h(x,p,t) &= h[f] - \frac{P^2}{2m} + \frac{1}{2} \int v(x,x') f(x',p',t) \, dx' \\
&\quad + \frac{1}{2} \int w(x,x',x'') f(x',p',t) f(x'',p'',t) \, dx'' \, dx' \\
&\quad + \frac{1}{2} \int v(x,x') f(x',p',t) f(x'',p'',t) \, dx'' \, dx'.
\end{align*}
\]

The energy of the system is

\[
E[f] = \int \frac{P^2}{2m} f(x,p,t) \, dx + \frac{1}{2} \int v(x,x') f(x',p',t) f(x'',p'',t) \, dx'' \, dx' \\
+ \frac{1}{2} \int w(x,x',x'') f(x',p',t) f(x'',p'',t) \, dx'' \, dx'.
\]

We observe that the saturation properties of nuclear matter require that \( v(x,x') \) is attractive and \( w(x,x',x'') \) is repulsive. Canonical transformations play an important role in Vlasov dynamics. The evolution in time is given by a canonical transformation. A state of equilibrium \( f_0(x,p) \) possesses an important property. The inequality

\[
E[f_0] < E[f]
\]

holds for all distribution functions \( f(x,p) \) which are related to \( f_0(x,p) \) by a canonical transformation. It follows that

\[
\begin{align*}
\frac{\partial f_0}{\partial x} \cdot \frac{\partial h_0}{\partial p} - \frac{\partial f_0}{\partial p} \cdot \frac{\partial h_0}{\partial x} &= 0, \\
\end{align*}
\]

where

\[
h_0(x,p) = h[f_0].
\]

By \( \{A,B\} \) we denote the Poisson bracket of \( A \) and \( B \).

The time evolution of systems close to equilibrium is described by the linearized Vlasov equation, which reads

\[
\begin{align*}
\frac{\partial f}{\partial t} + \{\delta f, h_0\} + \{f_0, \delta h\} &= 0.
\end{align*}
\]
Here, \( \delta f \) and \( \delta h \) are, respectively, the fluctuating parts of \( f \) and \( h \). It is clear that

\[
\delta h(x,p,t) = \int d\Gamma' v_{\text{eff}}(x,x') \delta f(x',p',t),
\]

(8)

where

\[
v_{\text{eff}}(x,x') = v(x,x') + \int d\Gamma'' w(x,x',x'') f_0(x'',p'').
\]

(9)

Brink et al. /1/ have studied eq. (7) by an elegant Fourier transform technique similar to the Laplace transform method considered by Landau /2/ for plasma oscillations. The solution obtained by these authors involves a typical integration over a singularity of the form usually associated with dissipative time-dependent processes. This is the source of the attenuation of small disturbances known as Landau damping.

An alternative solution of the plasma oscillations problem as a normal mode expansion has been obtained by Van Kampen /3/. Analogously, a complete set of normal mode generators (classical images of RPA operators), suitable to express the general solution of the nuclear Vlasov equation for infinite systems has recently been obtained by Piza et al. /4/.

The equivalence of the Landau and Van Kampen treatments of the initial value problem for plasma oscillations has been demonstrated by K. M. Case /5/. Since the equivalence of both treatments remains valid in the nuclear case (D. M. Brick and J. da Providência, unpublished), it follows that Landau damping is just an interference effect between normal mode amplitudes and is, therefore, strongly dependent on the initial conditions.

It is convenient to introduce the generator \( S \) of the distribution function fluctuations by

\[
\delta f = \{ S, f_0 \}.
\]

(10)

An equation for \( S \) is easily obtained. By the Jacobi identity and eq. (5) it follows that \( \{ \{ S, f_0 \}, h_0 \} = \{ \{ S, h_0 \}, f_0 \} \). The linearized Vlasov equation may therefore be written

\[
\{ (\partial_t S + \{ S, h_0 \}) - \delta h \}, f_0 \} = 0.
\]

(11)

Thus, in order to solve eq. (7), it is enough to require

\[
\partial_t S + \{ S, h_0 \} - \delta h = 0.
\]

(12)

Following ref. /4/ we look for the stationary solutions of eq. (12):

\[
i \omega_n S + \{ S_n, f_0 \} - \delta h_n = 0.
\]

(13)

Here, \( \delta h_n \) is obtained from \( \delta h \) by writing \( \delta f_n = \{ S_n, f_0 \} \) for \( \delta f \) in eq. (8).

The eigenvalues \( \omega_n \) in eq. (13) are real. To see this, multiply by \( f_0 \) the Poisson bracket of eq. (13) and \( S_0^* \) and integrate over the whole phase space. We find

\[
-i \omega_n \int d\Gamma \{ S_0^*, S_n \} f_0(x,p) = \int d\Gamma \{ S_0^*, \{ S_n, h_0 \} \} f_0(x,p)
- \int d\Gamma \{ S_0^*, \delta h_n \} f_0(x,p).
\]

(14)

It follows from (4) that the change in energy

\[
\delta E[S] = \frac{1}{2} \int d\Gamma \{ S, \{ S, h_0 \} \} f_0(x,p)
- \frac{1}{2} \int d\Gamma d\Gamma' v_{\text{eff}}(x,x') \{ S, f_0^* \}(x,p) \{ S, f_0 \}(x',p')
\]

(15)
is positive (more precisely, non negative) for any real function $S(x,p)$. Taking, in sequence, $S = S_n + S^*_n$ and $S = i(S_n - S^*_n)$, we prove that the right hand side of eq. (14) is positive (non negative). Since the normalization integral

$$i \int d\Gamma \{S_n, S^*_n\} f_o(x,p)$$

is real, it follows that $\omega_n$ cannot be complex. It may be seen that if $S_n$ and $\omega_n$ are some eigensolution and the corresponding eigenvalue, also $S^*_n$ and $-\omega_n$ are an eigensolution and the corresponding eigenvalue.

Finally, it may be seen that "distinct" eigensolutions are orthogonal and may be normalized:

$$i \int d\Gamma \{S_n, S^*_n\} f_o(x,p) = \frac{\omega_n}{|\omega_n|} \delta_{nm}.$$ (16)

Although it would be more appropriate to label the eigenmodes by continuous parameters, we find it more suggestive to use discrete indices as labels. However, the reader may, if he wishes, understand the Kronecker $\delta_{nm}$ as a Dirac $\delta$-function $\delta(n-m)$ and summations $\sum_n$ as integrations $\int d\omega$.

The eigensolutions are complete. Any generator $D(x,p)$ (restricted, at zero temperature, to the Fermi surface $h_0(x,p) = \frac{5}{4}$) may be expressed as a linear combination of eigensolutions /4/

$$D(x,p) = \sum_{n, \omega_n > 0} (d_n S_n(x,p) + d^*_n S^*_n(x,p)),$$ (17)

where, if $\omega_n > 0$,

$$d_n = i \int d\Gamma \{D, S^*_n\} f_o(x,p).$$ (18)

We are now in a position to determine the generator $D(x,p,t)$ which at time $t=0$ satisfies $D(x,p,0) = D(x,p)$, that is, to solve the Vlasov initial value problem. We have

$$D(x,p,t) = \sum_{n, \omega_n > 0} (d_n S_n(x,p) e^{i\omega_n t} + d^*_n S^*_n(x,p) e^{-i\omega_n t}).$$ (19)

Physically, the coefficients $d_n$ in this expansion represent transition amplitudes from the quantal ground state $|0\rangle$ to the excited state $|n\rangle$, induced by the transition operator $D$,

$$d_n = <n|D|0>.$$ (20)

The square of these amplitudes satisfy the energy weighted sum rule

$$\sum_{n, \omega_n > 0} \omega_n d^*_n d_n = \frac{1}{2} \int d\Gamma \{D, \dot{D}\} f_o(x,p) = \delta E[D].$$ (21)

Classically, $d^*_n d_n$ measures the energy fraction located at the $n$th eigenmode.

The exact solution of the Vlasov equation for finite nuclei appears to be a difficult problem. In principle, the generator of the distribution function fluctuations may be expressed as a linear combination of infinitely many degenerate solutions appropriate to the corresponding (translationally invariant) extended system. However, the weight function is determined by an awkward boundary condition at the nuclear surface (D. M. Brink, private communication). It seems, therefore, convenient to resort to a suitable approximate scheme. Variational methods appear to be particularly well adapted to this purpose. We will briefly discuss several types of trial generators which have been considered by various authors. It is convenient to split the generator $S$ into the time-even and the time-odd parts,

$$S(x,p,t) = Q(x,p,t) + F(x,p,t)$$ (22)
where
\[
Q(x,p,t) = \frac{1}{2} (S(x,p,t) + S(x,-p,t)) \tag{23}
\]
\[
P(x,p,t) = \frac{1}{2} (S(x,p,t) - S(x,-p,t)) \tag{24}
\]

Variational choice I (see ref. /6/):
\[
Q = \phi(x,t) + \frac{1}{2} \sum_{k\ell} p_k p_\ell \phi_{k\ell}(x,t) \tag{25}
\]
\[
P = \sum_k p_k s_k(x,t) \tag{26}
\]

Variational choice II (see refs. /7,8/):
\[
Q = \phi(x,t) + \frac{1}{2} \sum_{k\ell} p_k p_\ell \phi_{k\ell}(x,t) \tag{27}
\]
\[
P \text{ such that}
\[
f_U(x,p,t) = f_0 + \{P, f_0\} + \frac{1}{2} \{P, \{P, f_0\}\} + \ldots
\]
\[
= \theta (\lambda - h_0(x,p) - W(x,t) - \frac{1}{2} \sum_{k\ell} p_k p_\ell \chi_{k\ell}(x,t)). \tag{28}
\]

Variational choice III (see ref. /9/):
\[
Q \text{ such that}
\[
f_U(x,p,t) = f_0 + \{Q, f_0\} + \frac{1}{2} \{Q, \{Q, f_0\}\} + \ldots
\]
\[
= \theta (\lambda - h_0(x,p) - m u(x,t)), \tag{29}
\]
\[
P = \sum_k p_k s_k(x,t). \tag{30}
\]

In table I we present the values of the energies and the percentages of the \(m_1\) sum for electric eigenmodes of a nucleus with \(A=208\) as predicted by variational calculations based on the trial generators listed.

The variational choice III leads to the elegant fluid dynamical scheme of Holzwarth and Eckart /10/. This is the simplest choice which yields realistic results for the giant resonances. The fields \(s\) and \(u\) are canonically conjugate pairs.

Although the variational choice I provides apparently a richer parametrization of the distribution function, it does not seem to constitute an improvement over choice III. The reason for this may be related to the fact that the fields \(\phi, \phi_{k\ell}\) and \(s_k\) cannot be grouped into canonically conjugate pairs. This implies that the information contained in \(\phi_{k\ell}\) is to some extent redundant, as far as it does not affect much the dynamics.

The variational choice II gives a good description not only of the giant resonances, but also of the low lying modes. This feature is connected with a proper treatment of the surface dynamics. Classically, contact forces are unable to produce surface tension, which may therefore be regarded a quantal effect. Accordingly, surface modes come out at zero energy. If a reasonable value of the surface tension is added by hand to the model /11/, satisfying values for the energies of the surface modes are obtained (see last column of table I).

The fields \(\phi\) and \(\phi_{k\ell}\) are canonically conjugate, respectively, to the fields \(W\) and \(\chi_{k\ell}\).

Variational solutions of the linearized Vlasov equation for hot nuclei have also been obtained by J. P. da Providência /12/.
Table I

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References