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STABILIZING EFFECT OF EMITTED CHARGES ON THE CONE-LIKE SHAPES OF ELECTRIFIED MENISCI

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Abstract: When a meniscus is formed at the free end of a capillary tube and is subjected to the effect of a sufficiently intense electric field, the meniscus may display the shape of a cone. Depending upon the experiment conditions, a field effect charge emission, the production of droplets or corona discharge may be observed at the apex of the cone. The cone shape is essential to generate the first two phenomena. In this study, the profiles of electrified menisci were calculated taking account of the regulating effect that charge emission may exert on the electric field at the apex of the cone. The calculations particularly show that conical meniscus equilibrium is possible over a whole range of electric potential and hydrostatic pressure values.

1. INTRODUCTION

In 1914, Zeleny (1) studied corona discharges produced from liquid menisci, using an assembly in which a capillary tube (fig.1) was fed with liquid under a variable pressure, close to zero. A voltage of a few thousand volts was then applied between the meniscus formed at the free end of the capillary tube and a plate. Zeleny observed that droplets could be emitted under the action of the electrical field. Among the operating modes which he described (2), the most interesting is certainly that in which the meniscus takes the shape of a cone, with its apex extended by a very fine filament, the breakage of which results in charged microdroplets.

It would seem that these phenomena were not the subject of any further study until they were rediscovered (3) in the early fifties. However, they have since been the subject of many works (4). In particular, several authors (5,6) have attempted to use this process for electrical propulsion in space. Thus, in 1961, as part of a study concerning the production of charged droplets in a vacuum, using a liquid metal, Krohn (7) observed that metal ions were also emitted by the cone-shaped meniscus. Various authors have attempted to obtain this phenomenon without there being simultaneous emission of droplets. The ion sources thus created were studied for various applications, such as ionic propulsion, ion implantation, ionic microscopy, microlithography, etc... (8,9,10,11).
In the case of both ion emission and that of a fine spray, the cone-like shape of the meniscus is essential as, in the former case, it allows ions to be extracted by field effect and, in the second case, it allows a liquid filament to be created, the diameter of which is much smaller than that of the capillary tube.

Further, in all theoretical studies concerning calculation of the shape of cone-like menisci, no author has considered specific phenomena occurring at the apex of the cone and, notably, the presence of the space charge (12).

The first work carried out to theoretically justify the existence of cone-like menisci was that of Taylor (13). This author considered a specific geometrical configuration formed by a strictly conical meniscus (i.e., with straight generating lines), supported by a semi-infinite cone trunk. He demonstrated that, at zero hydrostatic pressure, capillary and electrostatic pressures can be balanced at all points, for a critical value of electrical potential, providing the half-angle of the cone apex be 49.3 degrees. This model corresponds to a very particular case and is still the subject of some discussion (14,15). However, more sophisticated studies have since been carried out. This is notably the case of the works of Basaran and Scriven (16) who, in 1981, obtained transient forms of cone-shaped menisci using a finite element method.

However, the above works do not cover certain experiments facts concerning stable cone-shaped menisci:

- firstly, these cone-shaped menisci can exist through a whole area of electrical potential and hydrostatic pressure values,
- secondly, they have never been observed without a simultaneous charge emission due, depending upon circumstances, to a corona discharge, the production of charged droplets or ion emission. In all these cases, a space charge is created which exerts a regulating effect on the electrical field at the apex of the cone and which seems indispensable for equilibrium to be maintained.

Therefore, in this work, this effect has been modelled and used in a method (17), which allows electrified menisci shapes to be calculated with no simplification assumptions. For reasons discussed later in the text, this modelling has been applied to the case in which a corona discharge occurs at the apex of the cone. In fact, the calculations performed actually result in the observed cone-like shapes.
2. THEORY

2.1 Equations

The problem treated is that of a meniscus formed at the end of a vertical capillary tube (fig.1) set to electrical potential $U_0$ in relation to a semi-infinite horizontal plate, located at a distance $D$ from the exit section of the capillary tube, of radius $R$. The shape of the meniscus is axisymmetrical and is represented by the function $Z(R)$, where $R$ is the radial coordinate normalised in relation to $R$. The meniscus is formed by a liquid of density $\rho_L$ and with surface tension $\sigma$.

The balance equation for the surface, considered as conducting, is written in a non-dimensional form (18,19):

$$B(K_H + \varepsilon Z) + \left(\frac{1}{R_1} + \frac{1}{R_2}\right) + \frac{E_S^2}{2K_0} \cdot \frac{\varepsilon_0 U_0^2}{AD} = 0 \quad (1)$$

In the first term, which represents hydrostatic pressure, $B$ is equal to $\rho g R^2/\sigma$, where $g$ is the acceleration due to gravity. $K_H$ is equal to $H/R$, where $H$ is the height (always counted positively upwards from the capillary tube exit section) of a column of the same liquid as that forming the meniscus. The constant $\varepsilon$ is equal to 1 for a pendent meniscus and -1 for a sessile meniscus.

In the second term, which represents capillary pressure, $R_1$ and $R_2$ are the main radii of curvature normalised in relation to $R$. In cylindrical symmetry, they are calculated as a function of the first derivative $Z$ and second derivative $Z''$ of $Z$, in relation to $R$.

In the third term, which represents electrostatic pressure, $E_S$ is the field at the surface normalised in relation to $U_0/D$. $K_0$ is equal to $D/R$, $\varepsilon_0$ is the dielectric constant of the surrounding medium.

To solve equation (1), $E_S$ must be known at each point on the surface, which means, in principle, that the distributions of field $E$ and charges $\rho$ must be calculated for all the space between the surface of the liquid and the plate.

For this, Poisson's equation and the current conservation equation must be simultaneous solved. Considering the electrical mobility of the ions to be a constant $K$, this is written as follows:

$$\nabla \cdot E = -\rho \phi \quad (2)$$

$$\nabla \times E = \nabla \times (\frac{1}{\rho}) \quad (3)$$

where $\phi = \frac{\rho e_0 D^2}{\varepsilon_0 U_0^2}, \rho e_0$ being a characteristic value of the density of the charges emitted at the meniscus surface.

The equation system (1) (2) and (3), in which the unknowns are $Z$, $E$ and $\rho$, must therefore be solved.

2.2 Boundary conditions

The boundary conditions used for the meniscus profile are:

$Z(1) = 0$ and $Z(R)(0) = 0$

The conditions adopted for potential $U$ are represented in figure 2.
Finally, as demonstrated by Felici (20), a condition fixing the value of emitted charges density $\rho_e$, must be imposed on the meniscus surface. This density will be expressed using an equation modelling charge emission.

### 2.3 Charge emission law

In the case of ion emission, it is very difficult to define a suitable charge emission equation. In addition, extremely fine meshing would be required to represent the tip of the cone apex, which is extremely pointed.

In the case of spraying, complex hydrodynamic phenomena should also be considered.

This is the reason for which it was preferred to model the regulating effect of charges in the case of corona effect. In order to cover certain characteristic properties of the corona discharge, the following assumptions were made:

- charges are only emitted locally for a field at the surface, $E_s$, above a certain critical value, $E_c$, which is an increasing function of the surface curvature $\xi$,
- the density of the emitted charges, $\rho_e$, increases with the difference between the surface field, $E_s$, and the critical field, $E_c$.

The equation adopted is written as follows:

$$\rho_e = \phi \frac{e_d}{D} [E_s - E_c(\xi)]$$  \hspace{1cm} (4)
where \[ E_C(\xi) = 31 + GV \xi \text{ kV/cm} \] (5)

In this expression, the constant \( G \) will be between 9 and 14 (21,22).

3. RESULTS

Equations (1), (2) and (3) are transformed into discrete format using finite difference formulae in variable pitch meshing (fig. 2).

Equation (2) has been handled using a point by point over-relaxation method and equation (3) using the characteristics method (19). Equation (1) was solved using Newton's method.

The calculations give the distribution of potentials and charges in space, transferred current, \( I \), and distribution of surface charges on the meniscus.

This method gives results basically similar to experiment observations concerning both meniscus cone-like shapes and the values of the potential applied and the current for which these menisci are obtained.

As an example, figure 3 shows the profile calculated for one of the capillary geometries adopted. The radius of curvature at the apex of the cone is, in this case, limited by the pitch of the meshing used. Except around the apex, increased meshing density has virtually no effect on the shape of the cone, the generating line of which is practically straight, in this particular case.

Such cone-shape menisci are obtained over a significant range of hydrostatic pressure and electrical potential values.

Concerning the potential, it should be pointed out that the extent of this range varies considerably with the shape of the capillary tube used.

Generally, theoretical results demonstrate the main types of phenomena observed in experiments (18).

4. CONCLUSION

The results obtained using the method developed to determine the shape of electrified menisci emitting charges justify the existence of stable cone-shape menisci observed during experiments.

The emission equation used models the case of a corona discharge. The theory gives a correct qualitative description of the main phenomena observed, notably, cone-shape menisci may be obtained over a significant range of potential and hydrostatic pressure values.

Whichever process is used for charge emission, the cone-shape menisci obtained are of similar form. A meniscus shape calculation more suitable for the ion source case could be achieved, using a charge emission equation corresponding more accurately to the field emission phenomenon, in the method described above, and by considering the fact that ion speed is then a function of local electrical potential.
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