HIGH FREQUENCY AMPLIFICATION BY LASER WIGGLER BEAT WAVES IN A PLASMA

J. Bobin

To cite this version:
J. Bobin. HIGH FREQUENCY AMPLIFICATION BY LASER WIGGLER BEAT WAVES IN A PLASMA. Journal de Physique Colloques, 1986, 47 (C6), pp.C6-159-C6-164. <10.1051/jphyscol:1986622>. <jpa-00225865>

HAL Id: jpa-00225865
https://hal.archives-ouvertes.fr/jpa-00225865
Submitted on 1 Jan 1986

HAL is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers.

L’archive ouverte pluridisciplinaire HAL, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d’enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.
HIGH FREQUENCY AMPLIFICATION BY LASER WIGGLER BEAT WAVES IN A PLASMA

J.L. BOBIN

L.P.O.C. Université Pierre et Marie Curie, T-12, E-5, 4, Place Jussieu, F-75252 Paris Cedex 05, France

Résumé - Un rayonnement haute fréquence peut être amplifié par combinaison d'une onde laser et d'un faisceau d'électrons relativistes, se propageant collinairement à travers un plasma, à l'intérieur d'un onduleur.

Abstract - By combining a laser wave and a relativistic electron beam propagating colinearly through a plasma inside a wiggler, high frequency radiation can be amplified.

1- Introduction

Beating between a laser beam and a wiggler in presence of a relativistic electron beam is a well-known process which has been successfully used in the Free Electron Laser (F.E.L.)[1] and is considered as an acceleration mechanism in the Inverse Free Electron Laser [2]. The microscopic phenomenon used in such devices is Stimulated Compton Scattering:

\[
\begin{align*}
\text{incoming photon (laser)} & \rightarrow \text{stimulated scattering} \\
& \leftarrow \text{pseudo photon (wiggler)}
\end{align*}
\]

Whenever the particle density in the beam is large enough, electrons interact collectively with the field by Stimulated Raman Scattering [3]. In principle the amplified wavelength can be adjusted to any value by varying the electron beam energy. However, it turns out that Free Electron Lasers operate in the infrared with the noticeable exception of the Orsay Storage Ring experiment in which visible and near U.V light were amplified [4].
When the interaction takes place within a plasma inside the wiggler, specific properties appear [5] which are to be investigated in the following sections.

2- Wavematching in a Beam Plasma system

The laser wave and the wiggler are resonantly coupled to an electron plasma wave if and only if an electron beam propagates in the same direction as the light driving beam (Fig 1). There exist two branches in the longitudinal wave dispersion relation

\[ \mathcal{E}(\omega, k) = 1 - \left( \frac{\omega_p^2}{\omega^2} \right) - \left( \frac{\omega_p^2}{\gamma_b^2} \right) (k\nu_b - \omega)^2 = 0 \]

where \( \nu_b \) is the beam velocity ( \( \sim c \) ) and \( \omega_p, \omega_{pb} \) are the relativistically invariant plasma frequencies of the motionless plasma and the beam respectively. Two couplings may occur between the laser \( (\omega_1, k_1) \), the wiggler \( (0, k_2 << k_1) \) and the plasma wave \( (\omega_1, k_0 + k_1 + k_2) \).

The phase velocity of the longitudinal beat is

\[ \nu_R = \frac{\omega_1}{k_1 + k_2}. \]

Thus, either the phase velocity is larger than the velocity of the beam

\[ \nu_b = \nu_R - \left( \frac{\omega_p^2}{\gamma_b^2} \right) k_0 \left( 1 - \frac{\omega_{p}^2}{\omega_1^2} \right)^{1/2} \]

and energy can flow from the field to the electrons (inverse Cerenkov regime), or the beam velocity is larger.

Fig. 1. Matching of E.M. and plasma waves. a) Inverse Cerenkov regime; b) Cerenkov regime.
\[ y_b = \frac{1}{\omega_b / Y_b \lambda_0 (1 - \omega_p^2 / \omega_1^2)^{1/2}} \]

and energy flows from the electrons to the field (Cerenkov regime).

In both cases \( Y_R \) is given by

\[ Y_R^2 = \frac{k_0^2}{(k_0^2 - \omega_1^2/c^2)} = \frac{k_0^2}{(2k_1 + k_2 - \omega_p^2/c^2)} \]

which since \( \omega_1^2 = \omega_p^2 + k_1^2c^2 \), implies a divergence for

\[ \omega_p^2 = 2c \omega_1 k_2 - c^2 k_2^2 \]

i.e., for the electron density in the plasma

\[ n_0 = \frac{\varepsilon_0 m_0 \omega_p^2}{e^2}. \]

Given \( \omega_1 \) and \( \lambda_2 (= 2\pi/k_2) \), there is an upper boundary for the plasma density inside the wiggler. Conversely, given \( \lambda_2 \) and the plasma density, i.e., \( \omega_p \), there is a lower boundary for the frequency \( \omega_1 \) of the laser light. In general, given \( Y_R \) and \( k_2 \), \( \omega_p^2 \) is a function of \( \omega_1 \), viz

\[ \omega_p^2 = \frac{1}{(Y_R^2 - 1) + 2ck_2 \omega_1 Y_R/(Y_R^2 - 1)^{1/2} - c^2 k_2^2} \]

and the minimum value of \( \omega_1 \) for which both \( Y_R \) and \( Y_b \) tend to infinity, is

\[ \omega_{\text{min}} = \frac{(\omega_p^2 + c^2 k_2^2)}{2ck_2}. \]

The way \( Y_b \) depends upon the laser frequency is displayed on fig.2.

**Fig. 2.** Plot of \( \lambda_b \) versus \( \omega_1/\omega_p \) in the Cerenkov regime: \( n_0 = 2 \times 10^{15} \text{ cm}^{-3} \) (a & b), \( 2 \times 10^{17} \text{ cm}^{-3} \) (c & d); \( \lambda_2 = 3 \text{ cm} \) (a & c). 10 cm (b & d).
For large $\omega_1$, it varies as the square root of $\omega_1$. Some representative values are listed in the following table ( $n_{eb}$ is the electron density of the beam in the moving frame):

<table>
<thead>
<tr>
<th>$n_{eb}$ (cm$^{-3}$)</th>
<th>$\lambda_2$ (cm)</th>
<th>$\nu_R$</th>
<th>$\omega_1$ (s$^{-1}$)</th>
<th>$\hbar \omega_1$ (e.V.)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$10^{13}$</td>
<td>10</td>
<td>$10^3$</td>
<td>$4 \times 10^{16}$</td>
<td>20</td>
</tr>
<tr>
<td>$10^{14}$</td>
<td>1</td>
<td>$10^4$</td>
<td>$4 \times 10^{19}$</td>
<td>$2 \times 10^4$</td>
</tr>
</tbody>
</table>

3. Negative energy waves

The curves on fig. 2 have an interesting property. They correspond to negative energy longitudinal waves as expected since the coupling takes place on the middle branch of the dispersion relation (fig. 1b). This can be shown in a more precise way. Let $\omega_p$ and $c$ be the unit frequency and the unit velocity respectively. Then, setting

$$\Omega = \omega / \omega_p \quad \Omega_{pb} = \omega_{pb} / \omega_p \quad U_b = \nu_b / c \quad K = ck / \omega_p$$

the plasma dielectric constant is rewritten

$$\varepsilon(\Omega, K) = 1 - 1 / \Omega^2 - \Omega_b^2 (1 - U_b^2) / (\Omega - KU_b)^2$$

The energy of the mode has the sign of $\partial \varepsilon / \partial \Omega$ (see e.g. Landau & Lifshitz *Electrodynamics of continuous media*). Now,

$$\partial \varepsilon / \partial \Omega = (2 / \Omega^3) (\Omega^3 - KU_b) / (\Omega - KU_b).$$

For the lower branches of the dispersion relation, one has $\omega / k < \nu_b$ i.e. $\Omega < KU_b$. Accordingly the derivative $\partial \varepsilon / \partial \Omega$ is negative when $\Omega^3 > KU_b$, that is also $\omega > \omega_p 2/3(KU_b)^{1/3}$.

which indeed corresponds to the middle branch on fig. 1b.
In the laser wiggler beating inside a plasma, radiation amplification may result from Raman scattering as in an ordinary F.E.L. But, in the laboratory reference frame the laser wave and the longitudinal wave have the same frequency. Furthermore, the latter has a negative energy. These are the condition for the onset of another mechanism which is known among plasma physicists as an explosive instability (fig. 3)[6]. Three waves are coupled in such a way that the long time behaviour of the amplitudes \( a_1, a_2, a_0 \), obeys the equations (in complex notation)

\[
\begin{align*}
\frac{\partial a_0}{\partial t} &= -g_0 a_1 a_2^* \frac{\partial \mathcal{E}_0}{\partial \Omega} \\
\frac{\partial a_1}{\partial t} &= g_1 a_0 a_2^* \frac{\partial \mathcal{E}_1}{\partial \Omega} \\
\frac{\partial a_2}{\partial t} &= g_2 a_0 a_1^* \frac{\partial \mathcal{E}_2}{\partial \Omega}
\end{align*}
\]

in which the \( g_0 \) are coupling constants. All three right hand sides are positive, allowing the simultaneous growth of the three amplitudes. Such a system of equations can be solved analytically or on a computer for given initial conditions, e.g. a large \( a_2(0) \), a small \( a_1(0) \).

---

**Fig. 3** In the Raman back scattering, a pseudo photon (wiggler) combines with a quantum of longitudinal oscillation (plasmon) in order to create a photon (laser); in the explosive instability, a plasmon decays into a photon and a pseudo photon.
and $a_0(0)=0$. A typical result is shown on fig. 4. The divergence occurs for a time proportional to the reciprocal of the initial amplitude $a_1(0)$. Saturation may be due to finite length effects which act as a damping [7] or to a relativistic correction (cubic) subsequent to the growth of the plasma wave amplitude.

The explosive instability is similar to the behaviour of recently found solutions of the F.E.L. dynamical equations [8],[9], in which the laser light is amplified while electron bunching increases. A random noise can start the process (known as Self Amplification of Spontaneous Emission: S.A.S.E.).

4- Conclusion

It has been shown in this presentation that passing the electron beam through a plasma pushes the working conditions of the F.E.L. towards higher frequencies. In such a scheme light amplification is associated with an explosive instability and is expected to occur in bursts. Operation in the X-ray range of frequencies requires high electron densities in the plasma and in the relativistic beam as well. Further detailed studies are needed in order to investigate the dynamics and check the compatibility with the technology of the near future.

3- V. Granstein et al., Appl. Phys. Lett. 30 (1977) 384
5- J.L. Bobin, Optics Comm. 55 (1985) 413
6- see e.g. J. Weiland, H. Wilhelmsson, "Coherent Nonlinear Interaction of Waves in Plasmas" (1977) Pergamon
7- D. Pesme et al., Phys. Rev. Lett. 31 (1975) 203
8- R. Bonifacio, F. Casagrande, Optics Comm. 50 (1984) 251