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To cite this version:

HAL Id: jpa-00225856
https://hal.archives-ouvertes.fr/jpa-00225856
Submitted on 1 Jan 1986

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GAIN CONDITION FOR LONG-LASER-PULSE PRODUCED PLASMA

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Abstract:

The inversion condition for laser-produced plasmas is analysed theoretically for carbon fibre targets in relation to the intrinsic intensity and duration of heating laser pulses. The spatial and temporal dependence of the gain for the 3 → 2 and 4 → 3 transitions including radiation trapping effects will be discussed.

1. Introduction

The theoretical prediction of the possibility to produce amplification of XUV or soft X-ray radiation in laser-produced plasmas [1] stimulated intense efforts in laser-plasma interaction research for over the last decade. Calculations [2-4] have shown that under appropriate plasma conditions large gains and amplification can be achieved. Recently, a conclusive demonstration of such an amplification conducted at the Novette laser-target irradiation facility was reported on [5,6]. Population inversions in neonlike ions obtained in cited above calculations and experiments were explained by use of a laser-driven electron-collisional-excitation scheme. Other pump mechanisms[7] are charge-exchange [8], intermediate optical pumping [9] or the recombination scheme [10-14].

In this paper we present some theoretical results concerning the generation of population inversions in laser-produced plasmas through recombination pumping. A one-dimensional gasdynamic code with nonlinear heat conduction simulates the interaction of laser light (λ_L = 1.06 μm, I_L = 10^{11} - 10^{13} W cm^{-2}, t_L ~ 10 ns) with a cylindrical carbon plasma. The calculated plasma characteristics such as the time-evolving electron and ion temperatures or densities are further used to estimate population densities in hydrogenlike carbon ions. Analysis of the population inversions for inner-shell transitions, especially for the 3 → 2 and 4 → 3 emission processes, show that a gain optimum is reached for a relatively long time interval (~ 5 ns) after the end of the laser pulse in a defined spatial region. The results will be discussed and compared with experimental and theoretical findings for carbon plasmas in the literature [15].
2. Gas dynamics and heat flow

The plasma is assumed to consist of a charge-neutral mixture of electrons (e) and various species of carbon ions (i). Electrons will be treated as one subsystem, with internal energy $\mathcal{E}_e$ per unit mass, temperature $T_e$, pressure $p_e$ and so on. The ions form the second subsystem with a corresponding set of thermodynamic variables including specific volume $V = 1/\rho = 1/N_i m_i$ ($N_i$ - ion density, $m_i$ - averaged ion mass). Charge neutrality requires that the two subsystems share the same velocity $u$. The following set of gas-dynamic equations in lagrangian coordinates will be used to simulate laser heating of carbon plasmas with axial symmetry:

\[
\frac{\partial v}{\partial t} = \frac{\partial (ru)}{\partial m} \tag{1}
\]

\[
\frac{\partial u}{\partial t} = - r \frac{\partial P}{\partial m} \tag{2}
\]

\[
\frac{\partial r}{\partial t} = u \tag{3}
\]

\[
\frac{\partial \mathcal{E}_e}{\partial t} = -P_e \frac{\partial v}{\partial t} - \frac{\partial (rW_e)}{\partial m} - Q_{ei}(T_e - T_i) - \frac{\partial (rQ_i)}{\partial m} \tag{4}
\]

\[
\frac{\partial \mathcal{E}_i}{\partial t} = -P_i \frac{\partial v}{\partial t} + Q_{ei}(T_e - T_i) \tag{5}
\]

Besides work of the summary pressure $- \frac{\partial P}{\partial t}$ the equation of the electron internal energy contains electron heat flow

\[
W_e = - x_e \frac{\partial T_e}{\partial m} \tag{6}
\]

(with coefficient of the classical electron heat conduction $x_e = x_0 T_{e0}^2$).

and the absorbed energy of laser light per mass interval. Pressure $P$ includes gas-kinetic pressure $p = p_e + p_i = N_i (ZT_e + T_i)$, where $Z$ is the averaged charge of the ions, and an artificial viscous pressure $\Omega = \mu \frac{\partial v}{\partial m}^2$, $\mu$ is a constant. It will be assumed that both subsystems - electrons and ions - behave as perfect gases.

In our calculations only absorption by the inverse Bremsstrahlung mechanism was taken into consideration. Long intense laser pulses ($t_L \approx 5 - 500$ $\mu s$) with trapezoidal shape were supposed.

In difference to the MEDUSA code we solve the gas-dynamic equations by means of a complete implicit difference scheme on an inhomogeneous lagrangian mesh [16]. It is a complete conservative and
Fig. 1
Plasma density $N$ as function of the distance from the target axis for several times.

Fig. 2
Plasma temperature $T_e$ as function of the distance from the target axis for several times.

Fig. 3
Plasma density $N$ and temperature $T_e$ as function of the distance from target axis for different times in the interesting spatial region; laser intensity $I_L = 1.7 \cdot 10^{13} \, \text{W/cm}^2$.

Fig. 4
As in Fig. 3, laser intensity $I_L = 4.0 \cdot 10^{12} \, \text{W/cm}^2$. 
Fig. 5
Temporal behaviour of the density for different distances from the target for laser intensities $I_L = 6.7 \times 10^{13}$ W/cm$^2$ and $1.7 \times 10^{13}$ W/cm$^2$.

Fig. 6
Temporal behaviour of the temperature for different distances from the target for laser intensities $I_L = 6.7 \times 10^{13}$ W/cm$^2$ and $1.7 \times 10^{13}$ W/cm$^2$.

Fig. 7
Relative population density of the pump level, $\phi_4 = \frac{N_4}{N}$, as a function of the density $N$ and the temperature $T_e$. 
owing to the introduced artificial viscosity— a homogeneous scheme[17]. Complete conservation means that not only conservation laws for mass, impulse and full energy but also a lot of other important physical relations like equations for internal and kinetic energies or the law of volume change are preserved in the space of mesh functions.

Some typical results of the numerical calculations are presented in Figs. 1-4. In addition to spatial distributions of the ion density and electron temperature for different times the temporal behaviour of these plasma parameters for defined distances from the target axis is shown in Figs. 5-6. It can be seen that for longer laser pulses (some ns) the density distribution— as is well known— strongly decreases with increasing distance from the target. Compared to it, the temperature rises near the critical interface, where the maximum of laser energy will be deposited in the plasma, and holds approximately constant with growing distance. When the laser heating breaks off, the temperature begins to fall off in the high-density region of plasma caused by the remaining strong heat flow. Concerning the temporal behaviour of plasma properties during the free expansion regime, we can conclude from Figs. 1-4 that in regions not so far from the target axis (≤ 1 mm) the density keeps constant in time, whereas the temperature rapidly decreases.

3. Gain calculations

The gain coefficient concerning the transition \( q \rightarrow p \) in hydrogenic ions is

\[
G_{qp} = \frac{A_{qp} c^2 q (\nu - \nu_{qp})}{2 \pi \nu_{qp}^2} \left( N_q - \frac{q^2}{p^2} N_p \right),
\]

where \( A_{qp} \) is the spontaneous emission rate and \( \nu_{qp} \) the transition frequency. The factor \( g (\nu - \nu_{qp}) \) describes the normalized line shape. In the case of Gaussian line shape (Doppler broadening) this function is given by

\[
g(\nu - \nu_{qp}) = \frac{2 \sqrt{\ln 2}}{\pi \Delta \nu_{qp}} e^{-\frac{4 \ln 2 (\nu - \nu_{qp})^2}{\Delta \nu_{qp}^2}}
\]

Therefore, the gain coefficient for resonant transition becomes

\[
G_{qp} = \frac{A_{qp} c^2 2 \sqrt{\ln 2}}{8 \pi \nu_{qp}^2 \Delta \nu_{qp}^2} N_q \left( 1 - \frac{q^2}{p^2} \frac{N_p}{N_q} \right).
\]

An analytical calculation of the population densities \( N_q \) \( (q=1,2,3) \), assuming a simplified four-level system and quasistationarity gives

\[
N_1 \simeq \left\{ A_{41} + A_{42} + W_{6c}^c + (A_{43} + W_{9c}^c) \frac{A_{31} + A_{32} + W_{32}^c}{A_{31} + A_{32} + W_{32}^c + W_{3c}^c} \right\} N_0 T_1
\]
where $W_{ij}^c$ denote collisional excitation and deexcitation rates, $T_1$ is the transition time from the hydrogen - to the helium - like state.

It can be seen immediately that the gain for transitions $3 \rightarrow 2$ and $4 \rightarrow 3$ is proportional to the population density $N_4$ of the pump level and the inversion factor $(1 - \frac{N_3}{N_4})$. With rate coefficients for collisional and radiative processes given explicitly as functions of the plasma state $(N,T_e,Z)$ we obtain, for example for $N_3$:

$$N_3 = 9 \cdot 10^{-2} \frac{1 + 1.3 \cdot 10^{-12} \frac{N}{T_e} Z^{-5}}{1 + 2.48 \cdot 10^{-14} \frac{N}{T_e} Z^{-5} (1 + 8.43 e^{-0.856 Z^2/T_e})} N_4. \quad (13)$$

Then, for the interesting gain factors we can write

$$G_{32} = 2.05 \cdot 10^{-14} Z^{-3/2} \frac{N}{T_e} \frac{1 + 1.3 \cdot 10^{-12} \frac{N}{T_e} Z^{-5}}{1 + 2.48 \cdot 10^{-14} \frac{N}{T_e} Z^{-5} (1 + 8.43 e^{-0.856 Z^2/T_e})} (1 - 1.48 \cdot 10^{-14} \frac{N}{T_e} Z^{-5} - \beta_A) f_4 \quad (14)$$

for the transition $3 \rightarrow 2$ and

$$G_{43} = 1.37 \cdot 10^{-12} Z^{-3/2} \frac{N}{T_e} (1 - 0.16) \frac{1 + 1.3 \cdot 10^{-12} \frac{N}{T_e} Z^{-5}}{1 + 2.48 \cdot 10^{-14} \frac{N}{T_e} Z^{-5} (1 + 8.43 e^{-0.856 Z^2/T_e})} \frac{f_1}{\beta A} \quad (15)$$

for the transition $4 \rightarrow 3$.

The term

$$\beta A = (1 + 4.34 \cdot 10^{-14} \frac{N}{T_e} Z^{-5}) \cdot 2 \cdot 10^{-14} \frac{N}{T_e} \frac{1}{Z} f_1 L \quad (16)$$

describes the effect of the $L_4$ -absorption ($L$ - absorption length, $f_1 = N_1/N$ -relative population density of the ground level).
Temporal behaviour of the gain coefficient for the transition $3\rightarrow 2$ at different distances from the target. Laser intensity $I_L = 1.7 \times 10^{13}$ W/cm$^2$

As in Fig. 8, the laser intensity is $4.0 \times 10^{12}$ W/cm$^2$

Temporal behaviour of the gain for the $4\rightarrow 3$ transition at different distances from the target. Laser intensity $I_L = 1.7 \times 10^{13}$ W/cm$^2$

As in Fig. 10, laser intensity is $4.0 \times 10^{12}$ W/cm$^2$
The relative population density of the pump level in our simplified model

\[
\rho_4 = \frac{N_4}{N}
\]  

(17)

was calculated numerically solving the full system of rate equations for population densities in the quasistationary approach. Resulting functional dependence on density and temperature for \( \rho_4 \) is shown in Fig. 7.

If we know the values for density and temperature in dependence on the space coordinate and time from gas-dynamic calculations, we can determine \( \rho_4 \) by use of Fig. 7 and finally estimate gain coefficients with formulas (14) and (15). Some results are presented in Figs. 8-11. 

4. Discussion

Corresponding to equations (14) and (15) the gain is proportional as the most important dependence - to the term \( \frac{N}{Te} \) and the relative population of the pump level, \( \rho_4 \), which is also a function of \( N \) and \( T_e \). Therefore, high values of \( \frac{N}{Te} \), \( \rho_4 \) and the inversion factor \( \rho \) (\( \geq 0.2 \)) are necessary for an optimal gain.

As we can see from Fig. 7, the relative population density of the pump level \( \rho_4 \) increases - as expected - with increasing ion density \( N \) and decreasing temperature \( T_e \). Considering the inversion limit, there exist a maximum value \( \rho_4(\text{max}) = 2 \times 10^{-3} \) for the \( 3 \rightarrow 2 \) and \( \rho_4(\text{max}) = 1.2 \times 10^{-4} \) for the \( 4 \rightarrow 3 \) transitions.

Reabsorption of the resonance line \( L_{\lambda} \) lowers the inversion limit of the \( 3 \rightarrow 2 \) transition, and therefore, value \( \rho_4(\text{max}) \) will be decreased too.

The gain curves in Figs. 8-11 can be understood in connection with spatial and temporal variations of density and temperature in the plasma heated by long laser pulses. So we can see that there exist a spatial region from approximately 300 to 3000 \( \mu \text{m} \) measured from the axis of the cylindrical target, in which the inversion condition is optimally performed. This optimal situation occurs after the laser pulses (\( \geq 10 \) ns) and will exist for a relatively long time (5 to 10 ns duration).

3 \( \rightarrow \) 2 transition

The spatial region with high gain is small (200 to 300 \( \mu \text{m} \)) for the transition \( 3 \rightarrow 2 \). This is caused by the fact that inversion could be reached at sufficiently low temperatures, which will be obtained after the laser pulse in a small spatial region not so far from the target. The maximum gain values for the \( 3 \rightarrow 2 \) transition are 5 cm\(^{-1}\). However, take into account the influence of the \( L_{\lambda} \) -absorption, this value is valid only for \( \rho_1 \approx 10^{-5} \) and this means that for a population of the ground state of \( \rho_1 = 0.01 \) the thickness of the spatial region, in which a maximum gain may be detected, is only 10 \( \mu \text{m} \). For a thickness of 100 \( \mu \text{m} \) we get a gain of 1 cm\(^{-1}\) for a time interval of some ns. However, because \( \rho_1 \) increases in time (\( \rho_1 \geq 0.1 \)), the effect of \( L_{\lambda} \) -absorption on the \( 3 \rightarrow 2 \) transition is reduced for shorter pulses.
To summarize the theoretical results concerning the X-ray gain in cylindrical carbon plasmas for the $3 \rightarrow 2$ transition using relatively long laser pulses ($t_L \sim 10$ ns), we expect an optimal gain of $< 5$ cm$^{-1}$ in a small spatial region ($< 100 \mu$m) at distances from the target of 500 to 1000 $\mu$m. Maximum gain will be obtained after the end of the laser pulse. It could be detected during a time interval of approximately 5 ns.

$4 \rightarrow 3$ transition

The inversion condition $N = N_{\text{inv}}(T_e)$ shows a nonlinear dependence on the temperature for $T_e \geq 10$ eV. Therefore, higher temperatures seem to be favoured to give an optimal gain. Moreover, for long laser pulses the temperature is relatively high and nearly constant over a large spatial range far from the target. So, a high gain for the $4 \rightarrow 3$ transition with values of $\approx 0.05$ cm$^{-1}$ ($C_{\text{max}} = 0.08$ cm$^{-1}$) will be reached for a wide spatial region (1000 to 3000 $\mu$m) in a time interval of about 10 ns beginning at the end of the laser pulse. The influence of the laser intensity (10$^{12}$ to 10$^{13}$ W/cm$^2$) is small. It gives rise to only slightly different spatial and temporal behaviour.

We conclude that for the $4 \rightarrow 3$ transition longer laser pulses can generate a gain in a wide spatial range for long times. However, the maximum gain value reached is small ($\approx 0.08$ cm$^{-1}$).

To test our theoretical model we compared the results with some experimental data [15]. For $N = 5 \cdot 10^{16}$ cm$^{-3}$, $T_e = 9$ eV and $N = 10^{17}$ cm$^{-3}$, $T_e = 14$ eV, the experimental gain factors are 0.01 and 0.02 cm$^{-1}$ respectively. Corresponding theoretical values 0.018 and 0.025 cm$^{-1}$ calculated with help of Eq. 15 are in good agreement with the experimental findings. For the population density of the pump level $N_4$ a value of $N_4(\text{exp.}) = 1.2 \cdot 10^{12}$ cm$^{-3}$ was measured. Theoretically we get from Fig. 7 $N_4(\text{theor.}) = 1.6 \cdot 10^{12}$ cm$^{-3}$.

In conclusion we note that analogous investigations for Al ($Z = 13$) and higher $Z$ values are in preparation.

References