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MULTIFRAGMENTATION OF HOT NUCLEI

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Abstract - Highly excited nuclei may disintegrate into many fragments of different sizes. A statistical liquid drop theory for the multifragmentation of finite nuclei is developed. A crack temperature is defined for the threshold of the multifragmentation process. This temperature is much lower than the critical temperature for the liquid-gas phase transition.

1. INTRODUCTION

When a piece of matter suddenly breaks into many smaller parts the process is called multifragmentation. This process is well known from everyday experience with condensed matter such as solids or liquids but the phenomenon has not been studied much in detail so far. The reason is that the complexity of the process is overwhelming for a macroscopic body. In fig. 1 we show the projection of the break up of a piece of crystalline solid after external mechanical strain of varying strength. It is characteristic that as the external strain increases the number of fragments grows (and the bits become smaller). Although this is a trivial fact for us a detailed description of the dynamics of the formation of all these bits is clearly practically impossible. We therefore have to start any description of a fragmentation with a much lower level of ambition.

Fig. 1 - Multifragmentation of a piece of sugar after three different degrees of external pressure of increasing strength.

Fig. 2 - Calculated central density as function of time in a cascade model for the collision between two heavy nuclei at intermediate energy.
Also atomic nuclei can fragmentate. Here the problem is more practicable because of the limited number of nucleons in the nucleus, but also in this case one cannot yet make very detailed models of multifragmentation processes. We shall in this lecture discuss the multifragmentation of nuclei within a statistical model which was recently developed. The multifragmentation of nuclei at excitation energies above the nuclear binding came into focus a few years ago when accelerators for nuclear collisions at intermediate energy became available.

The statistical thermodynamical theory for breakup of nuclei was introduced by Randrup and Koonin /1/ and later together with Fai /2/-/3/. The crucial role of the long range Coulomb force for the fragmentation was emphasized by Gross et al. /4/. In this lecture we shall in particular discuss the model of Bondorf et al. /6/-/8/ the basic physical principles of which were introduced in /5/. This model puts emphasis on realistic breakup volumes and the finite size of the fragmenting nuclei as well as a liquid drop formulation of the energies. Other statistical model descriptions have been made by Mekijian /9/, who treats mainly fragmentation at higher energies. Aichelin and Hüfner /10/ treat the fragmentation in a statistical way with no thermodynamic requirements. The treatment of fragmentation in a dynamic, more or less collective description has been done by several authors, with Hartree-Fock by Knoll et al. /11/, Bonche et al. /12/, progressing breakup by Biró et al. /13/, cascade by Cugnon, Boal et al. /14/, Vlasov equation by Remaud et al. /15/, Uehling-Uhlingbeck (Boltzmann) model by Bertsch et al. /16/.

The fragmentation relates to speculations on the stability of the nuclear liquid and the liquid gas phase transition for the matter, Mosel et al. /17/, Lamb et al. /18/. Fragmentation of ordinary molecular condensed matter, has been studied in particular within the so called percolation theory which was taken up by Campi and Desbois /19/, by Bauer et al. /20/ and by Biró et al. /21/ for nuclear problems. The connection between the percolation and the other fragmentation models were so far not well understood but some start to attack this problem was recently suggested /22/.

Experimental studies are still scarce. This is due to the very expensive and complicated equipment which is needed and which calls for a 4m setup with the possibility to detect a large number of fragments simultaneously. Therefore the most comprehensive studies were so far made by means of emulsions, Jakobsson et al. /23/, or else in more indirect ways through semi-exclusive experiments with correlations between a few detected fragments (see experimental results /26/ mainly from BEVALAC, CERN-SC, MSU, SARA and GANIL). Also studies of non nucleonic emission from the fragmenting systems have been made theoretically and experimentally, starting with the work of Bertsch /24/, and experiments /23/ on pion emission. Such particle studies give an independent account of properties of the hot matter, and they play an increasing role.

For a more comprehensive outline of recent theoretical and experimental developments I refer to various conferences in the last couple of years, f. ex. the Visby meeting /26/. For a review see Csernai et al. /27/.

In the following we shall go through some of the most important features of the statistical multifragmentation theory, and discuss advantages and limitations. After that we show some results, and we discuss how the theory can be combined with dynamical theories for the evolution of the heavy ion collision before fragmentation. We also discuss characteristic experimental signatures.
2. PARTITIONS

The most basic concept of multifragmentation is the partition /6/ For a body of \( A_0 \) constituent identical particles we define a vector \( N(A) \) which fulfills the condition

\[
A_0 = \sum_{A=1}^{N(A)} A
\]

Thus \( N(A) \) is the number of particles of mass \( A \). Another important quantity related to the partition vector is the multiplicity

\[
M = \sum_{A=1}^{N(A)} 1
\]

In table 1 we show for \( A_0 = 4 \) the 5 possible partition vectors together with their multiplicity.

<table>
<thead>
<tr>
<th>partition vector</th>
<th>multiplicity</th>
</tr>
</thead>
<tbody>
<tr>
<td>((4,0,0,0))</td>
<td>4</td>
</tr>
<tr>
<td>((2,1,0,0))</td>
<td>3</td>
</tr>
<tr>
<td>((1,0,1,0))</td>
<td>2</td>
</tr>
<tr>
<td>((0,2,0,0))</td>
<td>2</td>
</tr>
<tr>
<td>((0,0,0,1))</td>
<td>1</td>
</tr>
</tbody>
</table>

The number of different partition vectors increases steeply with \( A_0 \). There are algorithms by which this number can easily be calculated, and in table 2 we give the number of partitions associated with various values of \( A_0 \).

<table>
<thead>
<tr>
<th>( A_0 )</th>
<th>number of different partitions</th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td>5</td>
</tr>
<tr>
<td>10</td>
<td>42</td>
</tr>
<tr>
<td>20</td>
<td>627</td>
</tr>
<tr>
<td>50</td>
<td>204 226</td>
</tr>
<tr>
<td>100</td>
<td>190 562 992</td>
</tr>
</tbody>
</table>

Since nuclei has both protons and neutrons a realistic partition vector has two indeces, \( Z \) and \( A \), and besides the baryon conservation (1) the partition vector also fulfills the charge conservation,

\[
Z_0 = \sum_{Z=1}^{N(A,Z)} Z
\]

In the following treatments we will simplify the problem to only using (1) and then assume an analytic connection \( Z=Z(A) \). As it is already implied in fig. 1 the total multiplicity \( M \) has an important physical meaning. It is therefore relevant to investigate how the partitions are distributed with respect to \( M \). In fig. 2 the full drawn curve shows the distribution of \( M \)-values for \( A_0 = 100 \). It is seen that the distribution is broad and peaked at some value between \( M = 0 \) and \( M = A_0/2 \).

From a physical point of view the partition vector, as defined in this section, is not at all sufficient for defining the complete
physical configuration of a fragmented system. This requires a complete specification of all the quantum numbers of the fragments, i. e. energies, parities, momenta and angular momenta. However, the most basic and visible concept for the fragmentation is the partition. In the following theory we shall treat only the partitioning in the most elaborate way and apply approximate methods when dealing with the other quantities of the fragmentation.

3. STATISTICAL MODEL

We want to describe fragmentation of nuclei. As the total nuclear binding energy is of the order of 7-8 MeV per nucleon it is clear that at an excitation energy \( e^* \) per nucleon of this order of magnitude the system must disintegrate. It is also evident that if the fragments are not just nucleons but nuclei, then not all of the binding energy is used for the fragmentation. This means that the fragmentation can happen at a somewhat lower excitation energy. Off hand it is not obvious that multifragmentation is at all a preferred decay mode. Successive or multiple fast evaporation could also carry excess energy away so that a multifragmentation mechanism would be surpassed. However the experimental evidence (see sect. 4) shows that in heavy ion collisions at intermediate energy, events with a number of not too small fragments are frequent. This gives a strong motivation for a multifragmentation description.

Let us consider a near central heavy ion collision in which an intermediate system has been formed with a high excitation energy. We think of an excitation energy from a few until of the order 50 MeV per nucleon so that the intrinsic nucleonic degrees of freedom are still only weakly excited. This corresponds to bombarding energy below 0.5 GeV per nucleon on a resting target.

Such a highly excited nucleus will be compressed at the first collision stages. A calculation of the central density as function of time during a collision event (as f. ex. estimated from a nuclear multi-cascade calculations) is shown in fig.2, (ref. /28/). From the figure it is seen that first the central density increases to a maximum. Then the composite compressed nucleus expands, in the beginning as one coherent body, but around some later time the density becomes so low, that the matter becomes unstable, low density zones may develop, the matter in between will condense, and the fragments are formed. This process is very chaotic, and fluctuations and complicated interaction patterns make a detailed description rather impossible. Instead we have to turn to statistical methods. This is our starting point of the statistical theory.

We simplify the composite system to a spherical nucleus of mass and charge \( A_0, Z_0 \) with an excitation energy \( e^* \) pr. nucleon like in a compound nucleus theory. The concepts of partition apply to this composite nucleus. We assume that the energy is thoroughly spread over the degrees of freedom of the system and continues to be so during the expansion. The main idea is now to assume that the system at the density of breakup consists of a mixture of nuclear droplets of varying size. We assume that this mixture is close to thermal and chemical equilibrium as permitted by the constraints on energy and mass for the finite system. Just for this density we shall assume that the droplets don’t interact any longer. Thus the expansion flow energy is neglected. Each possible partition \( (i) \) of the matter gives rise to one droplet composition the thermodynamical probability of which can be calculated as

\[
W(i) = \exp(S(i))
\]
where $S(i)$ is the entropy of partition $(i)$. We have to determine this entropy, and besides take into account the energy conservation which we write as

$$E_{\text{tot}} = \left(\frac{3}{5}\right) \cdot (Z\cdot\epsilon)^2/R_b + \Sigma N(A,Z) \cdot E(A,Z)$$

$$= E_{\text{ground}} + E_{\text{star}} = E_{\text{ground}} + \epsilon_{\text{star}} \cdot A$$ \hspace{1cm} (5)

where $R_b$ is the radius of the spatial breakup volume $V_b$. The energy $E_{\text{star}}$ is the excitation energy of the nucleus. The entropy can be expressed as a sum

$$S(i) = \Sigma N(A,Z) \cdot S(A,Z,T,V_b)$$ \hspace{1cm} (6)

It is worth noticing that in (5) and (6) both the energy and the entropy are simple sums of contributions of each fragment $A,Z$ (when the total energy is reduced by the bulk Coulomb energy). This means that (4) factorizes nicely. The energy for each fragment is:

$$E(A,Z) = E_{\text{transl}} + E_{\text{bulk}} + E_{\text{surf}} + E_{\text{sym}} + E_{\text{coul}}$$ \hspace{1cm} (7)

with

$$E_{\text{transl}} = \frac{3}{2} \cdot T \cdot \epsilon_0 \approx 16 \text{MeV}$$
$$E_{\text{bulk}} = (-B(A,Z) + T^2/\epsilon_0) \cdot A \quad \text{for} \quad A>4 \quad ; \quad B(A,Z)\approx 16 \text{MeV}$$
$$E_{\text{surf}} = (\delta - T \cdot \epsilon_0/dT) \cdot \beta \approx (0.12) \cdot \left(\frac{To^2+T^2}{To^2+T^2}\right)^{5/4}$$
$$\beta(0) \approx 18 \text{MeV} \quad ; \quad T_0 \approx 15-20 \text{ MeV}$$
$$E_{\text{sym}} = 9 - (A - 2.2)^2/2 \quad ; \quad g \approx 25 \text{ MeV}$$
$$E_{\text{coul}} = (3/5) \cdot (Z\cdot\epsilon)^2/R_0 \cdot [1 - R_0/R_{\text{cell}}]$$
$$R_0 = r_0 \cdot A^{(1/3)} \quad ; \quad r_0 \approx 1.2 \text{ fm}$$

The terms in (7) are just generalized Weissssacker mass energies. The surface energy has been given a temperature dependence (see /18/ and /6/) so that it vanishes at the critical temperature $T_0$ like in a van der Waals gas. The bulk intrinsic excitation energy of the fragments is contained in the term $T^2/\epsilon_0$ and in the temperature dependent part of the surface energy. A very important property of the model is that we determine the free volume in which the fragments move just at breakup. This is done by demanding that the average distance $2d$ between the fragments at this stage is of the order of the maximum range of the nuclear force between fragments. This gives the breakup volume $V_b$:

$$V_b = (1 + K) \cdot V_0 = V_0 + V_f$$
$$K = \left[1 + (d\epsilon_0/dT) \cdot (M^{(1/3)} - 1) \right]^{3} - 1$$
$$V_0 = 4/3 \pi R_0^3 \quad ; \quad 2d \approx 2.8 \text{ fm}$$

The free volume for the translational motion of the fragments at the breakup situation is $V_f = K \cdot V_b$ and the "cell radius" $R_{\text{cell}} = R_0 \cdot (V_b/V_0)^{1/3}$.

The entropy is calculated from the energies (7) through the free energy. The final expression for the thermodynamical probability (4) is now /6/:

$$W(i) = \prod g(A,Z) \cdot V_f \cdot A^{(3/2)} \cdot \exp \left(3/2 + S_i(A,Z) / L(T)^3 \right) ^{N(A,Z) / N(A,Z)}$$ \hspace{1cm} (9)

The intrinsic entropy of the fragments is $S_i(A,Z) = (2 \cdot T/\epsilon_0) \cdot A - (d\beta/dT) \cdot A^{(2/3)}$ for $A>4$. The statistical factor $g(A,Z)$ is specified individually for the fragments with $A>4$ and otherwise put equal to 1.
The quantity $L(T)$ in (9) is the thermal wave length for a nucleon with temperature $T$. In order to calculate the entropy $S(i)$ to be used in (4) we now in principle run through all the possible partitions. For each partition $(i)$ with given $(N(A,Z))$ a temperature $T(i)$ is determined from (5) and (7) and with that $W(i)$ is determined from (9). In practice it is very difficult to run through all the partitions for bigger nuclei because of their large number. Therefore one chooses to use a Monte Carlo method in many cases. We show some characteristic calculated results in figures 3. to 6. The mass number is $A_0 = 100$ and the charge $Z_0 = 50$.

One sees from fig. 3 how narrow the thermodynamic multiplicity distribution is compared to the unbiased one. This means, that for a Monte Carlo simulation one can very quickly learn the relevant $M$-range which governs the fragmentation, and then bias the sampling accordingly. Our investigations show that this method leads to a very good Monte Carlo simulation economy. In fig. 4 we show how the average multiplicity $M$ varies with excitation energy. At the onset of fragmentation, $M$ rises smoothly from 1. In fig. 5 we show how the temperature varies with excitation energy. The onset of fragmentation happens at a characteristic temperature, the crack temperature, which is of the order of 5-6 MeV. This is also known from other investigations, see f.ex. /12/. It is fairly constant with mass $A_0$, with a tendency to rising towards the small masses $A_0$. The figure shows how the temperature remains almost constant over a large range of excitation energy. Fig. 6 shows a few calculated mass distributions for different excitation energies. One sees that for the energy just above the crack point the mass distribution has a modified U-shape.
Fig 5. Calculated average temperature as a function of the excitation energy per nucleon.

Fig 6. Calculated average mass distributions for various excitation energies.

Some limitations.

One can make several objections to the present model. We shall now mention a few.

In the model we take spherical fragments. This is not very realistic, especially for few body breakup. In this case one might choose realistic dynamically relevant breakup shapes like in binary fission. But as soon as $A \geq 3$ the choice of shapes and relative positions of fragments becomes very difficult.

We assume thermal and chemical equilibrium. This could be unrealistic because the reaction time is so fast that all partition possibilities will not be probed during a collision. It is worth noticing that in the model there is a possibility to simulate non equilibrium in at least two ways. The first one is to increase the parameter $E_0$ so that the intrinsic fragment heat capacity is hindered. Another way is to constrain the partition space in some physically relevant way.

Another objection to the model is the neglect of the interaction between the droplets at the time of breakup. The use of an energy dependence of the surface energy corresponding to properties of drops surrounded by gas is also questionable because fragments are largely facing empty space.

For these and other reasons which will be discussed further below one should at present only consider the model as a minimum bias model.
4. REACTION DYNAMICS AND RELATION TO EXPERIMENT

The statistical multifragmentation theory is designed for the interpretation of experiments in which nuclear aggregates are excited above or of the order of the total binding energy. Such reactions can be of various kinds.

Local excitations.

The excitation of nuclei by high energy protons has been known for a long time. In such reactions the injection of energy into the nuclear volume will always be rather local. Even in the case that a nucleus is struck centrally by a relativistic proton, some of the energy is presumably first deposited along a cylinder or cone. From there it spreads to the rest of the nucleus. The uniform conditions as used in the multifragmentation theory are not easy to obtain in such a reaction. Nevertheless one can try to see if the observed inclusive mass distributions can be fitted with the theory. One can fit the observed distributions rather well in the multifragmentation theory with a temperature just above the crack temperature (ref. /22/). This, however, does not mean that the true explanation of the breakup mechanism has been found. There could be other, and very different mechanisms, leading to the same inclusive mass distributions. A breakup of the nuclei by cold multifragmentation is one possibility. Another possibility is a mechanism with even less equilibrium where some of the bigger fragments come from the large cold spectator parts in non-central collisions.

Antiproton-annihilation on a nucleus is another way to excite the nucleus locally. Dependent on the incident momentum of the antiproton, an energy of the order 2GeV will be deposited more or less in the nuclear surface. The typical decay from a slow antiproton interaction leads to emission of some pions (maybe 5-6) followed by the deexitation of the residual nucleus. Evaporation and fission are typical decay modes (Nifenecker et al. /29/), but also multifragmentation is a possibility.

Another way is the injection of one delta-resonance after a charge exchange reaction. This gives an excitation energy of the target nucleus in the range 250-350 MeV. In this case there is a considerable chance that fast nucleons dominate the deexitation in a non-equilibrium reaction, but a reaction with a more uniform spread of the energy over the nuclear volume is also possible.

The energy is well defined in the locally injected energy reactions. This means that the target mass dependence of fragmentation should be strong because of the importance of the quantity e* in the fragmentation process.

Heavy ion excitation.

In a heavy ion reaction one can excite an extended nuclear volume in a short time. This makes heavy ion collisions especially well suited for nuclear multifragmentation. Some characteristic possible reaction mechanisms for heavy ion collisions as functions of energy and impact parameter are shown in fig. 7.
Fig. 7 Schematic reaction "phase diagram" for the collision between Ar and Ag (A1=40, A2=107). The shaded zone separates the low and the intermediate energy regions. The separation curves between PS (participant-spectator) and TE (total explosion) are calculated by combining the model of /34/ with the theory of multifragmentation.

The map in fig. 7 is still highly theoretical, and many details should be filled in. The zone TE marks the "total explosion" for more central collisions where multifragmentation can take place. For small impact parameters the two nuclei can be thoroughly mixed with each other. When the energy is high enough they disassemble. We have calculated the multiplicities by combining the interacting PS model of /34/ with the statistical theory of multifragmentation and shown the onset of fragmentation by the dotted curves in the figure. At the present stage of the theory one should consider the positions of the calculated curves as qualitative, giving only the trends.

We discussed the timing of a heavy ion collision process in the beginning of this paper (fig. 2). Let us continue the discussion here. When the nuclear density takes its maximum value, the matter is compressed and hot. This means that it will start to expand but also that the fast particles near the surface may escape before real multifragmentation takes place /30/. This cools the system, and therefore the fragmentation may not be as dominant as predicted in the more simple approach leading to fig. 7. The fast cooling will shift the total explosion zone to the right in the figure. The fast cooling plays a special role when pion emission is accompanied by fragmentation /31/. In this case the pion escapes first followed by a later fragmentation.

A special effect of the early compression in the heavy ion collision is that outwards collective non-thermal flow of the expanding nucleus will occur. This "blast effect", Siemens and Rasmussen /32/, leads to fragmentation with lower temperature than in the present version of the statistical model, and it boosts up fragmentation at the expense of evaporation. The problem needs further investigation.
Confrontation between theory and experiment.

Experiments on heavy ion fragmentation are being made at several laboratories. In particular the laboratories mentioned in the introduction are active in this field. But also at accelerators which are able to excite nuclei to an energy just above the crack point, such studies are now in progress. It is not the purpose of this paper to make a comprehensive survey of available experiments. The most illustrative ones are still those made in emulsion studies (Jakobsson et al. /23/). These studies seem to show directly that the multifragmentation process exists. The dramatic change in the formation of coherent pieces of reaction products when the bombarding energy rises from 27 to 44 MeV per nucleon in some heavy ion collisions made here at GANIL /33/ could be a signature of the onset of fragmentation. Multiparticle-detectors with a close to 4π-geometry are necessary in this connection.

The multiparticle nature of the reaction events is a strong challenge to both theoreticians and experimentalists in heavy ion physics. In many ways the problems are similar to or even more complicated than the problems in high energy physics. Also there many-particle events are dominant. We have to get used to more sophisticated signals than the more or less inclusive data which are now in use.

In this respect an event simulator like the one by Fai and Randrup /3/ is an important tool. By this method one can bias, on the event level, the possible outcomes of individual collisions and in this way simulate experiments. The confrontation between theory and experiment now consists in comparison between averages over events with the following coordinate types:

- **theory** \((x_1, x_2, x_3, \ldots, A, B, C, \ldots, \alpha, \beta, \ldots)\)
- **experiment** \((x_1, x_2, x_3, \ldots, A, B, C, \ldots)\)

Theoretical parameters:
- **variables**: experimental
- **of reaction**: input variables
- **products**: (projectile, target, (mass, charge, energy, \ldots))
- **state, momentum**: \(\ldots\)

One has to integrate both the experimental counts and the theoretical probabilities or generated events over so many coordinate intervals that a comparison can be made in a constrained variable-space which is common for theory and experiment. Some variable-averaging of this kind is almost always done a priori for technical reasons, and sometimes in a way which makes it difficult to find a common variable-space. The theoretical parameters can be determined by variation in the fitting procedures.

These remarks are in a way trivial and should always be followed by physicists. The enormous complexity of the heavy ion collision, however, forces us to to be more disciplined on this point. Also the present paper does not quite live up to these ideals.

5. CONCLUSIONS

The statistical model of multifragmentation is one of several theories which can be applied to understand the "total explosion" of a highly excited nucleus.
The model is a phase space approach which uses the assumption that the reaction time is so long that there is established an approximate thermal equilibrium in the expanding nuclear matter just before the freeze out of the fragments. The theory enables a prediction of the crack temperature around 5–6 MeV, above which the nuclear matter breaks up into several pieces. This temperature is lower than the liquid gas phase transition temperature by a factor of the order 2–3. Through the possible choice of the intrinsic heat capacity of the nuclear fragments the model can simulate both hot and some kind of cold fragmentation. The model takes full account of the fluctuations in the partition space, and is in this way similar to the nuclear percolation theory.

In order to compare the theory with experiments it is necessary to combine it with realistic dynamic models of the collision process. This includes decay by evaporation of the primary fragments.

ACKNOWLEDGEMENTS


REFERENCES

see also
see also
GANIL preprints 85.12 and 86.01.
/30/ Nifenecker, H. and Polikanov, S., private communication 1986.