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THE BORDONI RELAXATION - THREE VALLEYS MODEL

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Résumé - Une méthode analytique du calcul du frottement intérieur dû à la
formation des doubles décrochements est développée à partir de la théorie
cinétique dans le cas de trois vallées de potentiel. Le modèle permet de
décrire correctement les résultats expérimentaux, à condition que la
dislocation ne franchisse pas de trop nombreuses vallées de potentiel.

Abstract - The analytical description of internal friction related to the
double-Kink relaxation is obtained applying the kinetic theory in the case
of three potential valleys. This model can describe experimental curves in
the case in which dislocations do not pass by many potential valleys.

I - INTRODUCTION

The aim of the paper is to obtain an analytical description of the relaxation
related to a thermally activated process of double-kink formation on dislocations
(Bordoni relaxation). The starting point of our analysis is based on results
obtained from application of the kinetic theory.

The model of two potential valleys (i=2) is oversimplified and does not give
complete description of the experimental data /1/. Therefore we decided to elaborate
a three potential valleys model which enables to obtain an analytical expression for
internal friction Q−1 for a case in which the applied stress σ is much smaller than
internal stresses Σ_i.

II - DESCRIPTION OF THE THREE POTENTIAL VALLEYS MODEL

In the case of three potential valleys the system of the kinetic theory equations
may be written as:

\[ \begin{align*}
\dot{n}_0 &= -n_0 k'_{01} + n_1 k'_{10} \\
\dot{n}_1 &= n_0 k'_{01} - n_1 (k'_{10} + k'_{12}) + n_2 k'_{21} \\
\dot{n}_2 &= n_1 k'_{12} - n_2 k'_{21}
\end{align*} \]  

(1)

where \( n_i \) is the occupation probability of a i-th potential valley by the
dislocation.

The jump frequencies \( k'_{i,j} \) from i-th to j-th valleys are given by:

\[ \begin{align*}
k'_{i,i-1} &= k_0 \exp \left[ -\frac{(E_a(i) + V_a(i)\sigma)}{kT} \right] \\
k'_{i,i+1} &= k_0 \exp \left[ -(E_c - V_c \sigma)/kT \right] \\
k_0 &= \frac{\nu_D}{100}
\end{align*} \]  

(2)

in which \( \nu_D \) is the Debye frequency and \( \sigma \) the applied stress (\( \sigma \ll \Sigma_i \)), \( E_a, V_a \) are
energy and volume for double-kink formation respectively and \( E_a, V_a \) the double-kink
annihilation energy and volume.
Solution of the equations system (1) may be written as:

\[ n_i = \frac{\theta_1 \gamma_1}{\alpha_i} \exp \left(-\frac{t}{\theta_1}\right) \alpha_i^i, \quad i,1 = 0,1,2 \]  

(3)

where:

\[ \alpha = 1, \quad \alpha = \frac{k't_{01} \theta_1 - 1}{k't_{10} \theta_1} \]

The relaxations time \( \theta_0^{\text{eq}} \) corresponds to the equilibrium state \( \left(n_i=0\right) \) for \( \sigma \neq 0 \) in which \( n_i = n_i^e \). The other two relaxation times \( \theta_1 \) and \( \theta_2 \) are equal:

\[ \theta_1 = 2/(A + \sqrt{A^2 - 4B}), \quad \theta_2 = 2/(A - \sqrt{A^2 - 4B}) \]  

(4)

where:

\[ A = k't_{01} + k't_{10} + k't_{12} + k't_{21}, \quad B = k't_{01}k't_{12} + k't_{21}k't_{10} + k't_{01}k't_{21} \]

Three constants \( \gamma_i \) are determined from the initial condition

\[ n_0(0) = n_0^e, \quad n_1(0) = n_1^e, \quad n_0(\infty) + n_1(\infty) + n_2(\infty) = 1 \]

where \( n_i^e \) are the equilibrium \( n_i \) values for \( \sigma = 0 \).

Having the solution of the equations system (1) we can write an expression for anelastic deformation \( \epsilon_{an} \) related to the double-kink formation.

\[ \epsilon_{an}(t) = N \frac{\theta_1 \gamma_1}{\alpha_i} (V_C + V_d(t))(n_1(t) - n_1^e) \]  

(5)

where \( N \) is the number of dislocations in a unit volume. After insertion of (3) in (5) we obtain:

\[ \epsilon_{an}(t) = N(V_C + V_d(1))\left[\gamma_1 (1 - \gamma_1 \alpha_1^1) \left(1 - e^{-t/\theta_1}\right) + \gamma_2 (1 - \gamma_2 \alpha_2^2) \left(1 - e^{-t/\theta_2}\right)\right] \]  

(6)

where \( \gamma_1 = \frac{V_C + V_d(2)}{V_C + V_d(1)} \)

Thus we obtained two relaxation processes related to \( \theta_1 \) and \( \theta_2 \).

Using a linear approximation for coefficient \( k'_i \),i, expressions for efficiencies \( \Delta_1 \) and \( \Delta_2 \) of the two relaxation processes related to two relaxation times \( \theta_1 \) and \( \theta_2 \) may be obtained:

\[ \Delta_i = N \frac{\theta_0 \gamma_1 \left(V_C + V_d(1)\right)}{(\theta_2 - \theta_1)^2} \left[k_0 \theta_2 - (n_1^e + n_1^c + n_1^x V_1) \right]_i \left[\frac{k_0 \theta_2 - 1}{k_2 \theta_2 - 1} n_i^e \right] \]  

(7)

\[ \Delta_2 = N \frac{\theta_0 \gamma_1 \left(V_C + V_d(1)\right)}{(\theta_2 - \theta_1)^2} \left[(n_1^e + n_1^c + n_1^x V_1) - k_0 \theta_2 \right]_i \left[\frac{k_0 \theta_2 - 1}{k_2 \theta_2 - 1} n_i^e \right] \]

Internal friction \( Q^{-1} \) may be then calculated from the formula:

\[ Q^{-1} = \frac{\omega \theta_1}{1 + \omega^2 \theta_1^2} + \frac{\omega \theta_2}{1 + \omega^2 \theta_2^2} \]  

(8)

where \( \omega \) - the angular frequency.
III - RESULTS AND DISCUSSION

Four I.F. curves calculated from the formula (8) are presented in Fig. 1 (IF, temperature $T'$ and dislocation strength $L'$ are normalized). We can note that in cases 1 and 4 only two valleys play a significant role and the probability of occupation of the third one "0" or "2" can be neglected. For cases 2 and 3, all three valleys play relatively significant role and total curve is composed of two processes. Fig. 2 presents the variations of three normalized parameters characterizing the IF peak - its height $Q^{-1}_{MN}$, maximum temperature $T'M$ and its broadening factor $\alpha$ as a function of $L'$. The suitable energetic diagrams are presented schematically.

It can be seen that if a Paré condition is fulfilled the broadening factor $\alpha \approx 1$; the peak temperature $T'M$ decreases when $C$ increases and is constant when $L'$ increases. For a given loop length $Q^{-1}_{MN}$ decreases with $\sigma_1$ which is connected with the reduction of the area swept out by the dislocations as the number of double kinks increases /1/. For constant $\sigma_1$, $Q^{-1}_{MN}$ increases as a function of $L'$ according, in the first approximation, to relation $Q^{-1}_{MN} \sim (V_C + V_a(1))^2$.

The minimum of $Q^{-1}_{MN}$ represents a deviation from the Paré condition. At this point $\alpha$ assumes its maximum value and $T'M$ is a function of $L'$ and $\sigma_1$. The evolution of those parameters is related to the variation of energetic diagrams, so it is connected with a distribution of relaxation times. From Fig. 2 it can be seen that results obtained using formula (8) are consistent with results obtained in paper /2/ for $\sigma_1$ and $L$ relatively small. So, calculations of IF curves from the formula (8) show that:

1. The intensity of the total $Q^{-1}/T'$ curve passes through a maximum for the fulfilled Paré condition ($E_C=E_a(i)$).
2. If the first Paré condition ($E_C=E_a(1)$) is fulfilled then the total curve will be controlled by the longer relaxation time, i.e. $\theta_2$ ($\Delta_2 >> \Delta_1$).
3. If the second Paré condition ($E_C=E_a(2)$) is fulfilled then the total curve will be controlled by the shorter relaxation time i.e. $\theta_1$ ($\Delta_1 >> \Delta_2$).
4. If $E_a(2) < E_C < E_a(1)$ the total curve consists of a composition of two processes and in some particular case two separate maxima can occur.

It seems that formula (8), after integration with a distribution of internal stresses and/or length of dislocation segments may be used to describe experimental curves determined for relatively large deformations - $\sigma_1$ then relatively large, $L$ small and dislocations forming double-kink do not pass through many potential barriers. In this case using a fitting method of theoretical model parameters we should obtain some information about parameters of loop lengths and internal stresses distributions.

REFERENCES

Fig. 1. - IF curves calculated using formula (8) for a double-kink energy $2W_k=0.1\text{eV}$, $F=100\ \text{Hz}$ and $L: L'=5$, $C=g_1g_2=0.1$; $2: L'=5.7$, $C=0.1$; $3: L'=5$, $C=0.118$; $4: L'=5$, $C=0.132$. The suitable energetic diagrams are presented schematically.

Fig. 2. - Variation of $Q^{-1}$, $T_\text{MN}$, $u$ with $L'$ ($C=0.1$, $2W_k=0.1\text{eV}$, $F=100\ \text{Hz}$).