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EQUATION OF MOTION OF A SCREW DISLOCATION IN A VISCO-ELASTIC CONTINUUM

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Abstract - Starting from a Lagrangean formalism for a linear visco-elastic continuum the dissipative forces are eliminated by an appropriate transformation. The equation of motion is then obtained by variation of the action integral in terms of Green's functions. The result can be expressed as a string equation with a damping term. The contribution of the shear viscosity to the damping constant is calculated on the basis of a Grüneisen constant for shear. Effective mass and line tension are also given.

I - INTRODUCTION

In several papers Brailsford /1/ has shown how the motion of dislocations is impeded by phonon and electron viscosities. He decomposed the bulk and the dislocation displacements into its Fourier components and calculated the damping as a function of the wave vector k. Bross /2/ and Stenzel /3/ on the other hand proposed to start from the equation of motion of an elastic continuum and derived an equation of motion for the screw dislocation, using Green's functions. In the following it is shown how this procedure can be implemented to include viscous effects.

II - DYNAMICS OF THE VISCO-ELASTIC CONTINUUM

Starting from the Lagrangean energy density $L = T - V$ and the dissipative function $\psi$ the equations of motion for a visco-elastic continuum read:

$$\frac{\partial}{\partial t} \frac{\partial L}{\partial \dot{\varepsilon}_{ij}} + \frac{\partial}{\partial x_j} \frac{\partial L}{\partial \varepsilon_{ij}} = \frac{\partial}{\partial x_j} \frac{\partial \psi}{\partial \ddot{\varepsilon}_{ij}}$$ (1)

Here $\varepsilon_{ij}$ and $\dot{\varepsilon}_{ij}$ denote the components of the total displacement and velocity, respectively, of a volume element at $(x; t)$, $\varepsilon_{ij}$ and $\dot{\varepsilon}_{ij}$ represent elastic and plastic deformation, respectively, and $\ddot{\varepsilon}_{ij} = \dot{\varepsilon}_{ij}$. Finally, $T = (1/2) \ddot{\varepsilon}_{ij} \ddot{\varepsilon}_{ij}$; $V = (1/2)C_{jmn} \varepsilon_{ij} \varepsilon_{mn}$; $\psi = (1/2)n_{jmn} \ddot{\varepsilon}_{js} \ddot{\varepsilon}_{mn}$, and $\rho$ denotes mass density. $C_{jmn}$ and $n_{jmn}$ are tensor of elasticity and viscosity, respectively.
In the absence of dissipation (1) can be obtained by minimizing the action integral:
\[ \delta W = \delta \int L \, \delta x \, dt = 0. \quad (2) \]

Using the dyadic Green's function \[ G_{ij}(x-x', t-t') \] Stenzel /3/ has shown how in this case an equation of motion can be derived from (1) and (2). In the present work his procedure is extended to the case of a viscous medium.

Since (2) holds for energy conserving systems only it cannot be applied directly in the case of a viscous medium. We therefore seek a transformation with respect to time such that the right-hand side of (1) vanishes. With \( t \) replaced by \( \Theta(t) \), and \( \phi(t) \equiv \Theta/dt \), (1) gives us
\[
\rho \frac{\partial^2 s_1}{\partial \Theta^2} + \rho \frac{\partial s_1}{\partial \Theta} \frac{d\phi}{dt} = \frac{\partial}{\partial x_j} \frac{\partial V}{\partial e_{1j}} = \phi \frac{\partial}{\partial \Theta} (\eta_{ljmn} \partial_j \partial_m s_n). \quad (3)
\]

Thus, if it is possible to find a transformation \( \Theta(t) \) such that the sum of the first order terms \( \partial/\partial \Theta \) vanishes, the problem will be reduced to the dissipation-free case:
\[
\rho \frac{\partial s_1}{\partial \Theta} - \phi \eta_{ljmn} \partial_j \partial_n \frac{\partial s_m}{\partial \Theta} = 0 \quad (4)
\]

Now (4) can indeed be satisfied, if \( s_1 \) is decomposed into plane waves
\[
s(x) = \Sigma s^*(k,x); \quad s^*_1 = u_1 \exp(ikx) \quad (5)
\]
and if
\[
\phi/\Theta = -\alpha \quad (6)
\]
is independent of \( t \). In the isotropic continuum with \( \eta, \zeta \) longitudinal and shear viscosity, respectively, we have for
longitudinal modes: \( \alpha = k^2(\eta + 2\zeta)/\rho \) \quad (7a)
transverse modes : \( \alpha = k^2\zeta/\rho \).

Introducing the operator
\[
\frac{\partial^2}{\partial t^2} \equiv \frac{\partial^2}{\partial \Theta^2} = \frac{\partial^2}{\partial \tau^2} + \alpha \frac{\partial}{\partial \tau} \quad (8)
\]
(1) may be broken down to
\[
\rho \frac{\partial^2 s_1}{\partial \tau^2} - \frac{\partial}{\partial x_j} \frac{\partial V}{\partial e_{1j}} = 0 \quad (9)
\]
which permits application of the minimum action principle (2).

II- SCREW DISLOCATIONS

Let the screw dislocation extend over \(-L/2 \leq x_3 \leq L/2\) and move in the \( x_1, x_3 \) plane, where it is represented by \( y(x_3, t) \) with \( |y| \ll L \). In order to avoid divergencies at \( x_1 = y \) we write its plastic distortion /3/:
\[
B_{23}(x_2,t) = -\frac{b}{2}[1+\text{erf}(\frac{x_1-y(x_3,t)}{r_0})] \delta(x_2) \quad (10)
\]
where \( r_0 \) is the width of the dislocation core and \( \delta(x_2) \) is the \( \delta \) function. Applying the Stenzel procedure to (8) under an applied external stress \( \sigma_{23}(x_3,t) \) one obtains the equation of motion for the screw dislocation:
Here, $K_0$, $K_1$, $K_{1/4}$ are modified Bessel functions of the second kind, $u$, $v$ elastic constants, and $C_T$, $C_L$ transverse and longitudinal sound velocity, respectively.

Since the screw dislocation produces only shear, $\alpha(k)$ according to (7b) is given by $K_2\zeta(k_1)/\rho$. Brailsford /1/ has given expressions for the longitudinal viscosity $\eta(k)$. He has listed four effects: scattering of phonons and electrons (high frequency modes) and thermoelastic effects due to phonons and electrons (low frequency modes). The effects considered by Brailsford are all due to density changes, they cannot be applied to screw dislocations.

Brailsford’s calculation of the phonon effects is based on a Grüneisen constant

$$\gamma = \frac{\Sigma c_V(k) \gamma(k)}{\Sigma c_V(k)}; \quad \gamma(k) = \frac{\partial \ln \omega(k)}{\partial \Delta}$$

where $c_V(k)$ is the contribution to the specific heat of the wave vector $k$, $\omega(k)$ are the eigenfrequencies, and $\Delta$ is volume dilatation. Bond changes entailing frequency changes are produced not only by volume dilatation, but also by shear. Therefore, a Grüneisen constant may be defined also for shear:

$$\gamma_s = \frac{\Sigma c_s(k) c_V(k)}{\Sigma c_s(k)} \gamma_s(k) = \frac{\partial \ln \omega(k)}{\partial \Gamma}$$

where the effective or octahedral shear

$$\Gamma = \frac{1}{3} [\varepsilon_1 - \varepsilon_2]^2 + (\varepsilon_2 - \varepsilon_3)^2 + (\varepsilon_3 - \varepsilon_1)^2]$$

has been introduced, and $\varepsilon_1$, $\varepsilon_2$, $\varepsilon_3$ are the principal strains. The phonon scattering due to screw dislocations may then be calculated in the same way as for edge dislocations /1/, yielding for the damping constant

$$B = \frac{3}{8} \frac{\pi}{2} \frac{Y_s}{B} k_T \frac{T}{C_T} + O(r_0 L)$$

Expanding to the same order of magnitude, (12) and (14) give us
\[ A = \left( \frac{\rho b^2}{4\pi} \right) \left[ \ln \left( \frac{L}{r_0} \right) + \left( C_E + 3 \ln 2 \right)/2 \right] \]
\[ C = \frac{\mu b^2}{4\pi} \left[ \frac{1 + \nu}{1 - \nu} \ln \frac{L}{r_0} + \frac{1}{2} \left( C_E + 3 \ln 2 \right) + \frac{3}{4} \right] \]

\[ C_E = 0.577... \] (Euler's constant).

IV - DISCUSSION

The present theory confirms the string equation for the motion of the dislocation. It contains a damping term which, for screw dislocations, is solely due to phonon scattering. The expressions (19), (20), are essentially identical with those given by Stenzel /3/. Also the formula for B shows only minor, though still noteworthy, differences with the corresponding term in Brailsford's theory for edge dislocations:

\[ B = \frac{9}{16} (6\pi^2)^{1/3} \left( \gamma^2 k_B T/b^2 c_L \right) \left( c_T^4/c_L^4 \right) \]

In the present treatment the factor \( c_T^4/c_L^4 \) disappears in the damping constant, while the width \( r_0 \) of the dislocation core comes in.

The appearance of \( r_0 \) in (18) causes B to diverge as \( r_0 \to 0 \) which seems plausible, as it is also found in A and C.

REFERENCES

/2/ H. Bross, phys. stat. sol. 5 (1964) 329.

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